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PROBABILITY AND DISTRIBUTION

Unit-5

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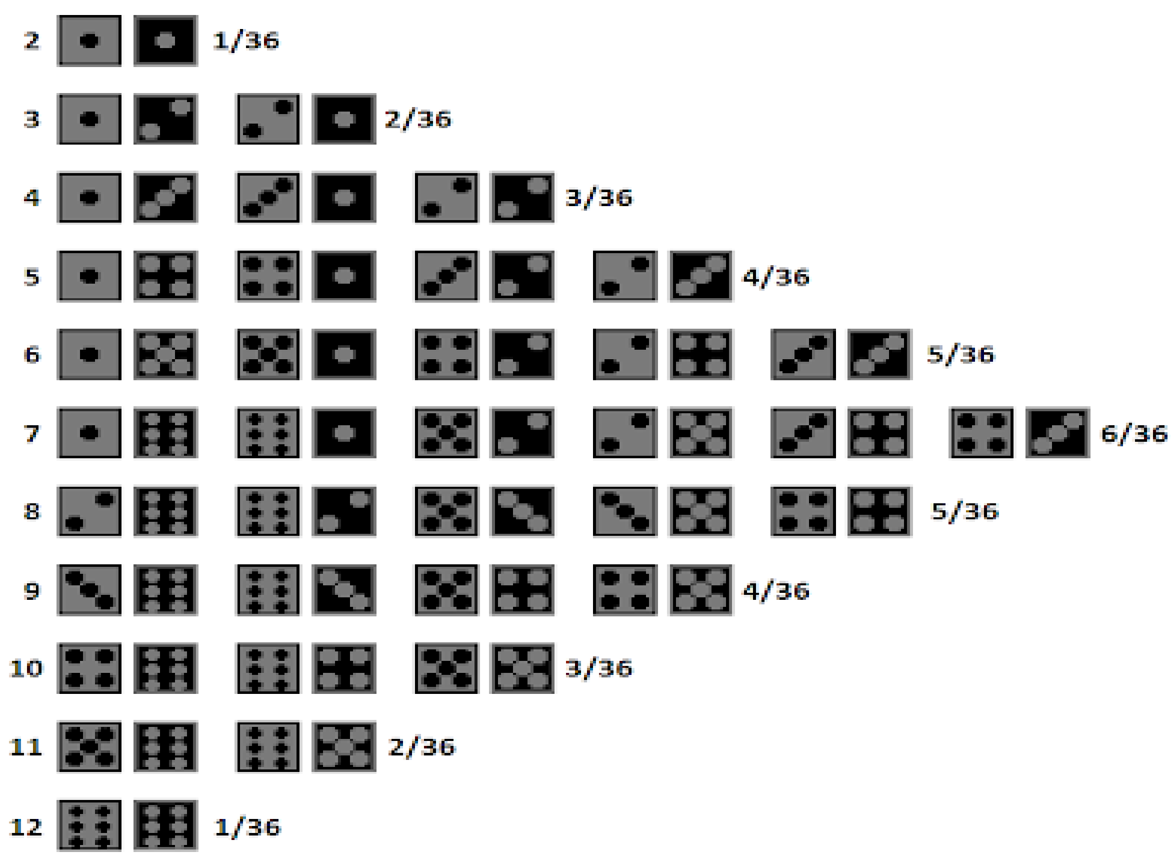
PROBABILITY

Probability is the branch of mathematics concerning numerical descriptions of how likely an event is to occur or how likely it is that a proposition is true. Probability is a number between 0 and 1, where, roughly speaking, 0 indicates impossibility and 1 indicates certainty.[note 1][1][2] The higher the probability of an event, the more likely it is that the event will occur. A simple example is the tossing of a fair (unbiased) coin. Since the coin is fair, the two outcomes ("heads" and "tails") are both equally probable; the probability of "heads" equals the probability of "tails"; and since no other outcomes are possible, the probability of either "heads" or "tails" is 1/2 (which could also be written as 0.5 or 50%).



The probabilities of rolling several numbers using two dice.

Types of probability

There are three major types of probabilities:

Theoretical Probability

Experimental Probability

Axiomatic Probability

- Theoretical Probability

It is based on the possible chances of something to happen. The theoretical probability is mainly based on the reasoning behind probability. For example, if a coin is tossed, the theoretical probability of getting head will be ½.

- Experimental Probability

It is based on the basis of the observations of an experiment. The experimental probability can be calculated based on the number of possible outcomes by the total number of trials. For example, if a coin is tossed 10 times and heads is recorded 6 times then, the experimental probability for heads is 6/10 or, 3/5.

- Axiomatic Probability

In axiomatic probability, a set of rules or axioms are set which applies to all types. These axioms are set by Kolmogorov and are known as Kolmogorov's three axioms. With the axiomatic approach to probability, the chances of occurrence or non-occurrence of the events can be quantified. The axiomatic probability lesson covers this concept in detail with Kolmogorov's three rules (axioms) along with various

Types of Events

The toss of a coin, throw of a dice and lottery draws are all examples of random events.

Events

When we say "Event" we mean one (or more) outcomes.

Example Events:

Getting a Tail when tossing a coin is an event

Rolling a "5" is an event.

An event can include several outcomes:

Choosing a "King" from a deck of cards (any of the 4 Kings) is also an event

Rolling an "even number" (2, 4 or 6) is an event

Events can be:-

Independent (each event is not affected by other events),

Dependent (also called "Conditional", where an event is affected by other events)

Mutually Exclusive (events can't happen at the same time)

Let's look at each of those types.

Independent Events

Events can be "Independent", meaning each event is not affected by any other events.

This is an important idea! A coin does not "know" that it came up heads before ... each toss of a coin is a perfect isolated thing.

Example: You toss a coin three times and it comes up "Heads" each time ... what is the chance that the next toss will also be a "Head"?

The chance is simply 1/2, or 50%, just like ANY OTHER toss of the coin.

What it did in the past will not affect the current toss!

Some people think "it is overdue for a Tail", but really truly the next toss of the coin is totally independent of any previous tosses

<u>Dependent Events</u>

But some events can be "dependent" ... which means they can be affected by previous events.

Example: Drawing 2 Cards from a Deck

After taking one card from the deck there are less cards available, so the probabilities change!

Let's look at the chances of getting a King.

For the 1st card the chance of drawing a King is 4 out of 52

But for the 2nd card:

If the 1st card was a King, then the 2nd card is less likely to be a

King, as only 3 of the 51 cards left are Kings.

If the 1st card was not a King, then the 2nd card is slightly more likely to be a King, as 4 of the 51 cards left are King.

Mutually Exclusive

Mutually Exclusive means we can't get both events at the same time.

It is either one or the other, but not both

Examples:

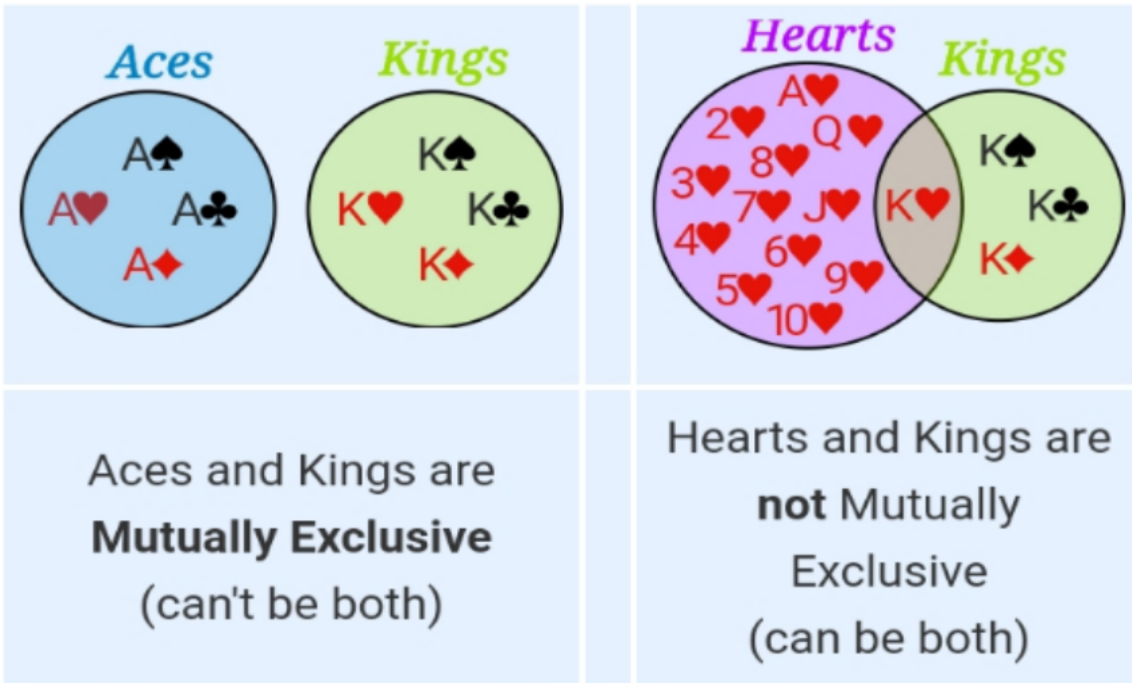
Turning left or right are Mutually Exclusive (you can't do both at the same time)

Heads and Tails are Mutually Exclusive

Kings and Aces are Mutually Exclusive

What isn't Mutually Exclusive

Kings and Hearts are not Mutually Exclusive, because we can have a King of Hearts!



Basic rules of probability

If A and B are two events, then;

$P (A \cup B) = P (A) + P (B) - P (A \cap B)$

$P (A \cap B) = P (B) \cdot P (A | B)$

Question : - Find the probability of rolling a ‘3 with a die.’

Solution:

Sample Space = {1, 2, 3, 4, 5, 6}

Number of favourable event = 1

Total number of outcomes = 6

Thus, Probability, P = 1/6

Question : - Draw a random card from a pack of cards. What is the probability that the card drawn is a face card?

Solution:

A standard deck has 52 cards.

Total number of outcomes = 52

Number of favourable events = 4 x 3 = 12 (considered Jack, Queen and King only)

Probability, P = Number of Favourable Outcome/Total Number of Outcomes = 12/52= 3/13.

Question : - A vessel contains 4 blue balls, 5 red balls and 11 white balls. If three balls are drawn from the vessel at random, what is the probability that the first ball is red, the second ball is blue, and the third ball is white?

Solution: The probability to get first ball is red or the first event is 5/20.

Now, since we have drawn a ball for the first event to occur, then the number of possibilities left for the second event to occur is 20 – 1 = 19.

Hence, the probability of getting second ball as blue or the second event is 4/19.

Again with the first and second event occurred, the number of possibilities left for the third event to occur is 19 – 1 = 18.

And the probability of the third ball is white or third event is 11/18.

Therefore, the probability is 5/20 x 4/19 x 11/18 = 44/1368 = 0.032.

Or we can express it as P = 3.2%.

Question : - Two dice are rolled, find the probability that the sum is:

equal to 1

equal to 4

less than 13

Solution:

1) To find the probability that the sum is equal to 1 we have to first determine the sample space S of two dice as shown below.

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }

1) Let E be the event "sum equal to 1". Since, there are no outcomes which where a sum is equal to 1, hence,

P(E) = n(E) / n(S) = 0 / 36 = 0

2) Three possible outcomes give a sum equal to 4 such as;

E = {(1,3),(2,2),(3,1)}

Hence, P(E) = n(E) / n(S) = 3 / 36 = 1 / 12

3) From the sample space, we can see all possible outcomes, E = S, give a sum less than 13. Like:

(1,1) or (1,6) or (2,6) or (6,6). So you can see the limit of an event to occur is when both dies have number 6, i.e. (6,6). Hence,

P(E) = n(E) / n(S) = 36 / 36 = 1

The Addition Rule

The addition rule states the probability of two events is the sum of the probability that either will happen minus the probability that both will happen

Addition Law

A or B will occur is the sum of the probabilities that A will happen and that B will happen, minus the probability that both A and B will happen. The addition rule is summarized by the.
formula: $P(A \cup B)=P(A)+P(B)-P(A \cap B)$

Proof. Let n be the total number of equally likely cases and let m_1 be favourable to the event A and m_2 be favourable to the event B. Then the number of cases favourable to A or B is $m_1 + m_2$. Hence the probability of A or B happening as a result of the trial

$$= \frac{m_1 + m_2}{n} = \frac{m_1}{n} + \frac{m_2}{n} = P(A) + P(B).$$

(2) *If A, B, are any two events* **(not mutually exclusive),** *then*

$$\mathbf{P(A + B) = P(A) + P(B) - P(AB)}$$

or

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If the events A and B are any two events then, there are some outcomes which favour both A and B. If m_3 be their number, then these are included in both m_1 and m_2. Hence the total number of outcomes favouring either A or B or both is

$$m_1 + m_2 - m_3.$$

Thus the probability of occurrence of A or B or both

$$= \frac{m_1 + m_2 - m_3}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m_3}{n}$$

Hence $$P(A + B) = P(A) + P(B) - P(AB)$$

or $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Consider the following example. When drawing one card out of a deck of 52 playing cards, what is the probability of getting heart or a face card (king, queen, or jack)? Let H denote drawing a heart and F denote drawing a face card. Since there are 13 hearts and a total of 12 face cards (3 of each suit: spades, hearts, diamonds and clubs), but only 3 face cards of hearts, we obtain:

$P(H)=13/52$

$P(F)=12/52$

$P(F∩H)=3/52$

Using the addition rule, we get:

$P(H∪F)=P(H)+P(F)−P(H∩F)=13/52+12/52−3/52$

The reason for subtracting the last term is that otherwise we would be counting the middle section twice (since H and F overlap).

Addition Rule for Disjoint Events

Suppose A and B are disjoint, their intersection is empty. Then the probability of their intersection is zero. In symbols:

$P(A∩B)=0$

. The addition law then simplifies to:

$P(A∪B)=P(A)+P(B)$

when $A∩B=∅$

The symbol ∅ represents the empty set, which indicates that in this case A and B

do not have any elements in common (they do not overlap).

Key Points

The addition rule is:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

The last term has been accounted for twice, once in

P(A) and once in P(B), so it must be subtracted once so that it is not double-counted.If A and B are disjoint,

then $P(A \cap B) = 0$

, so the formula becomes

$P(A \cup B) = P(A) + P(B)$.

Key Terms

probability: The relative likelihood of an event happening.

EXAMPLE:

Suppose a card is drawn from a deck of 52 playing cards: what is the probability of getting a king or a queen? Let A represent the event that a king is drawn and B represent the event that a queen is drawn. These two events are disjoint, since there are no kings that are also queens. Thus:

$P(A \cup B) = P(A) + P(B) = 4/52 + 4/52 = 8/52 = 2/13$

The Multiplication Rule

The multiplication rule states that the probability that A and B both occur is equal to the probability that B occurs times the conditional probability that A occurs given that B occurs

The multiplication rule can be written as:

$P(A \cap B) = P(B) \cdot P(A|B)$.

We obtain the general multiplication rule by multiplying both sides of the definition of conditional probability by the denominator.

sample space: The set of all possible outcomes of a game, experiment or other situation.

The Multiplication Rule

(2) Multiplication law of probability *or* **Theorem of compound probability.** *If the probability of an event A happening as a result of trial is P(A) and after A has happened the probability of an event B happening as a result of another trial (i.e.,* **conditional probability of B given A**) *is P(B / A), then the probability of* **both** *the events A and B happening as a result of two trials is* **P(AB) or P(A ∩ B) = P(A) . P(B/A).**

Proof. Let n be the total number of outcomes in the first trial and m be favourable to the event A so that $P(A) = m/n$.

Let n_1 be the total number of outcomes in the second trial of which m_1 are favourable to the event B so that $P(B/A) = m_1/n_1$.

Now each of the n outcomes can be associated with each of the n_1 outcomes. So the total number of outcomes in the combined trial is nn_1. Of these mm_1 are favourable to both the events A and B. Hence

$$P(AB) \text{ or } P(A \cap B) = \frac{mm_1}{nn_1} = P(A) \, . \, P(B/A).$$

Similarly, the *conditional probability of A given B is P(A/B).*

$$\therefore \quad P(AB) \text{ or } P(A \cap B) = P(B) \, . \, P(A/B)$$

Thus $$P(A \cap B) = P(A) \, . \, P(B/A) = P(B) \, . \, P(A/B).$$

Switching the role of A and B, we can also write the rule as:

$P(A \cap B) = P(A) \cdot P(B|A)$

We obtain the general multiplication rule by multiplying both sides of the definition of conditional probability by the denominator. That is, in the equation

$P(A|B) = P(A \cap B) P(B)$

, if we multiply both sides by

$P(B)$, we obtain the Multiplication Rule.

The rule is useful when we know both

$P(B)$ and $P(A|B)$, or both $P(A)$ and $P(B|A)$.

EXAMPLE:-

Suppose that we draw two cards out of a deck of cards and let A be the event the the first card is an ace, and B be the event that the second card is an ace, then:

$P(A) = 4/52$

And:

$P(B|A) = 3/51$

The denominator in the second equation is 51 since we know a card has already been drawn. Therefore, there are 51 left in total. We also know the first card was an ace, therefore:

$P(A \cap B) = P(A) \cdot P(B|A) = 4/52 \ . \ 3/51 = 0.0045$

Independent Event

Note that when A and B are independent, we have that

$P(B|A)=P(B)$, so the formula becomes

$P(A \cap B)=P(A)P(B)$

, which we encountered in a previous section. As an example, consider the experiment of rolling a die and flipping a coin. The probability that we get a 2 on the die and a tails on the coin is

$1/6 \cdot 1/2 = 1/12$

, since the two events are independent

Independence

To say that two events are independent means that the occurrence of one does not affect the probability of the other.

Independent Events

In probability theory, to say that two events are independent means that the occurrence of one does not affect the probability that the other will occur. In other words, if events A and B are independent, then the chance of A occurring does not affect the chance of B occurring and vice versa. The concept of independence extends to dealing with collections of more than two events.Two events are independent if any of the following are true:

$P(A|B)=P(A)$

$P(B|A)=P(B)$

$P(A \text{ and } B)=P(A) \cdot P(B)$

To show that two events are independent, you must show only one of the conditions listed above. If any one of these conditions is true, then all of them are true.

Translating the symbols into words, the first two mathematical statements listed above say that the probability for the event with the condition is the same as the probability for the event without the condition. For independent events, the condition does not change the probability for the event. The third statement says that the probability of both independent events A and B occurring is the same as the probability of A occurring, multiplied by the probability of B occurring.

As an example, imagine you select two cards consecutively from a complete deck of playing cards. The two selections are not independent. The result of the first selection changes the remaining deck and affects the probabilities for the second selection. This is referred to as selecting “without replacement” because the first card has not been replaced into the deck before the second card is selected.

However, suppose you were to select two cards “with replacement” by returning your first card to the deck and shuffling the deck before selecting the second card. Because the deck of cards is complete for both selections, the first selection does not affect the probability of the second selection. When selecting cards with replacement, the selections are independent.

EXAMPLE:-

When flipping a coin, what is the probability of getting tails 5 times in a row?

Recall that each coin flip is independent, and the probability of getting tails is 1/2 for any flip. Also recall that the following statement holds true for any two independent events A and B:

$P(A \text{ and } B)=P(A)\cdot P(B)$

Finally, the concept of independence extends to collections of more than 2 events.Therefore, the probability of getting tails 4 times in a row is:

$1/2 \cdot 1/2 \cdot 1/2 \cdot 1/2 = 1/16$