Procedure for Hypothesis Testing

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Quantitative Techniques and Research Methodology

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Lecture-2

Concept:

According to Karl Pearson, a scientific study has the following elements;

1. Careful Observation – unbiased, reliable and valid

2. Classification of Facts

3. Relationships and Sequences

4. Discovery of Scientific Law

5. Self Criticism – reliability and validity

6. Universal Acceptance

To test a hypothesis means to certify whether or not the hypothesis seems to be valid. In hypothesis testing the main question is: whether to accept or reject the null hypothesis? Procedure for hypothesis testing , therefore refers to all those steps that we undertake for making a choice between the rejection and acceptance of a null hypothesis. The various steps involved in hypothesis testing include;

1. Formulation of Hypothesis: *The first step consists of making a formal statement of the null hypothesis (H0) and the alternative hypothesis (H1 or Ha).The hypothesis should be clearly stated considering the nature of the research problem. A null hypothesis represents the hypothesis we are trying to reject and alternative hypothesis represents all other possibilities.*

Suppose: To test whether all the students got 60 levels of merits, the hypothesis may be;

H0: mean (meu) $==$ 60 Null hypothesis of no difference

H1: mean (meu) $=/=$ 60 other possibility

It is an important step; hence it must be accomplished with care in accordance with the object and nature of the problem under consideration.

2. Determining the Level of Significance: *The hypotheses are tested on a pre-determined level of significance. Hence, after the formulation of hypothesis one has to determine the level of significance at which the hypothesis would be tested. It means we have to determine the level of confidence at certain degree of freedom with which a particular hypothesis is accepted or rejected (Tabulated value).It is always in some percentage (usually 5%) which should be chosen with great care, thought and reason. If we take the significance level at 5 per cent then it implies that H0 will be rejected when the sampling result (observed value) has less than 0.05 per cent probability of occurring (H0 is true).It means a researcher is willing to take as much as 5 per cent risk of rejecting the null hypothesis when it (H0) happens to be true.*

Thus the significance level is the maximum value of probability of rejecting H0 when it is true and is usually determined in advance before testing the hypothesis. Generally 5% or 1% levels of confidence are adopted for this purpose- meaning thereby our decision to accept or reject a hypothesis correct to 95 or 99 times out of 100 and chances of going wrong only 5 or 1 per cent respectively. The factors which affect the level of significance are – the magnitude of difference between sample means, size of the samples, the variability of measurements within the samples and whether the hypothesis is directional or non-directional.

Decision Rule - *Rule to accept or reject the hypothesis.*

It one or two items out of 10 are defective we have frame rule whether we are going to accept the hypothesis or vice-versa.

Type -1 and Type -11 Errors – *In testing the hypothesis there are two types of errors we can make. We may reject H0 when H0 is true and we may accept H0 when in fact H0 is not true. The former is known as Type-1 error and the later is Type-11 error. In other words, Type-1 error means rejection of hypothesis which should have accepted and Type-11 error means accepting the hypothesis which should have been rejected.*

Type-1 error is denoted by alpha – alpha error and Type-11 error is denoted by beta – beta error.

Decision

	Accept H0	Reject H0
H0 (True)	Correct decision	Type- 1 error
H0 (False)	Type-11 error	Correct decision

The probability of Type – 1 error is usually determined in advance and is understood as level of significance of testing the hypothesis. If Type – 1 error is fixed at 5 per cent it means that there are about 5 chances in 100 that we will reject H0 when H0 is true. We may control Type- 1 error just by fixing it at lower level of 1 per cent where the maximum probability of committing Type-1 error will be only 0.01.

3. Test Statistics: *A test statistics is a statistics computed from a sample (Calculated value). After the level of significance, the next step in hypothesis testing is to determine the appropriate sampling distribution. Test statistics selected on the basis of nature of study undertaken. Statistical methods are used for any testing hypothesis on the basis of variables and suitability.*

When statistics are calculated under the assumption that the data follow some common distribution used in hypothesis testing (normal distribution) we call them parametric statistics as   - t- test, x2-test, ANOVA, linear regression etc. This step includes selecting a random sample and computing an appropriate value from the sample data concerning the statistics from the relevant distribution. Various sampling methods are used to draw samples in this process.

4. Comparison of the Statistic with Probability: *In this step the calculated probability is compared with the specified value of alpha (Type-1 error), the significance level. If the calculated probability is equal to or smaller than the alpha value in case of one- tailed test (alpha / 2 in case of two –tailed test), then the null hypothesis is rejected i.e. alternative*

hypothesis is accepted but if the calculated probability is greater, then the null hypothesis is accepted.

Steps involved in Hypothesis Testing: 1. Formulation of Hypothesis – Null hypothesis and Alternative hypothesis 2. Selecting Significance Level 3. Sampling Method 4. Data Collection – Survey or Field Work – Tabulation and Classification 5. Test Statistic – Probability Distribution 6. Probability Test and Comparison (Calculated Probability with Expected Probability) and 7. Findings

Example-1: One Sample Hypothesis Testing

Hypothesis Testing Examples: #1

A principal at a certain school claims that the students in his school are above average intelligence. A random sample of thirty students IQ scores have a mean score of 112. Is there sufficient evidence to support the principal's claim? The mean population IQ is 100 with a standard deviation of 15. IQ scores are normally distributed.

$$H_0 : u = 100 \quad \alpha = 0.05$$
$$H_1 : u > 100$$

A principal at a certain school claims that the students in his school are above average intelligence. A random sample of thirty students IQ scores have a mean score of 112. Is there sufficient evidence to support the principal's claim? The mean population IQ is 100 with a standard deviation of 15.
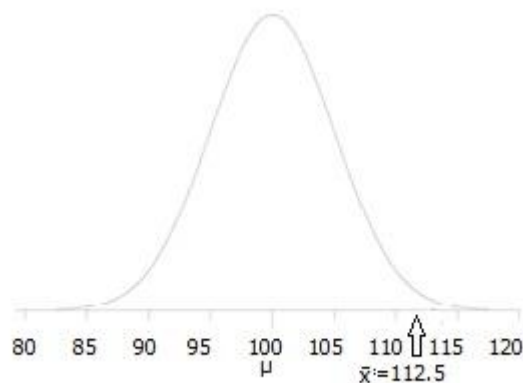
Step 1: State the Null hypothesis. The accepted fact is that the population mean is 100, so: H_0: $\mu=100$.
Step 2: State the Alternate Hypothesis. The claim is that the students have above average IQ scores, so:
H_1: $\mu > 100$.
The fact that we are looking for scores "greater than" a certain point means that this is a one-tailed test.
Step 3: Draw a picture to help you visualize the problem.

Step 4: State the alpha level. If you aren't given an alpha level, use 5% (0.05).
Step 5: Find the rejection region area (given by your alpha level above) from the z-table. An area of .05 is equal to a z-score of 1.645.

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Step 6: Find the test statistic using this formula:
For this set of data: z= (112.5 – 100) / (15/√30) = 4.56.
Step 7: If Step 6 is greater than Step 5, reject the null hypothesis. If it's less than Step 5, you cannot reject the null hypothesis. In this case, it is greater (4.56 > 1.645), so you can reject the null.

Example – 2

Blood glucose levels for obese patients have a mean of 100 with a standard deviation of 15. A researcher thinks that a diet high in raw cornstarch will have a positive or negative effect on blood glucose levels. A sample of 30 patients who have tried the raw cornstarch diet

have a mean glucose level of 140. Test the hypothesis that the raw cornstarch had an effect.
Step 1: State the null hypothesis: $H_0: \mu = 100$
Step 2: State the alternate hypothesis: $H_1: \neq 100$
Step 3: State your alpha level. We'll use 0.05 for this example. As this is a two-tailed test, split the alpha into two.
0.05/2=0.025
Step 4: Find the z-score associated with your alpha level. You're looking for the area in one tail only. A z-score for 0.75(1-0.025=0.975) is 1.96. As this is a two-tailed test, you would also be considering the left tail (z = 1.96)

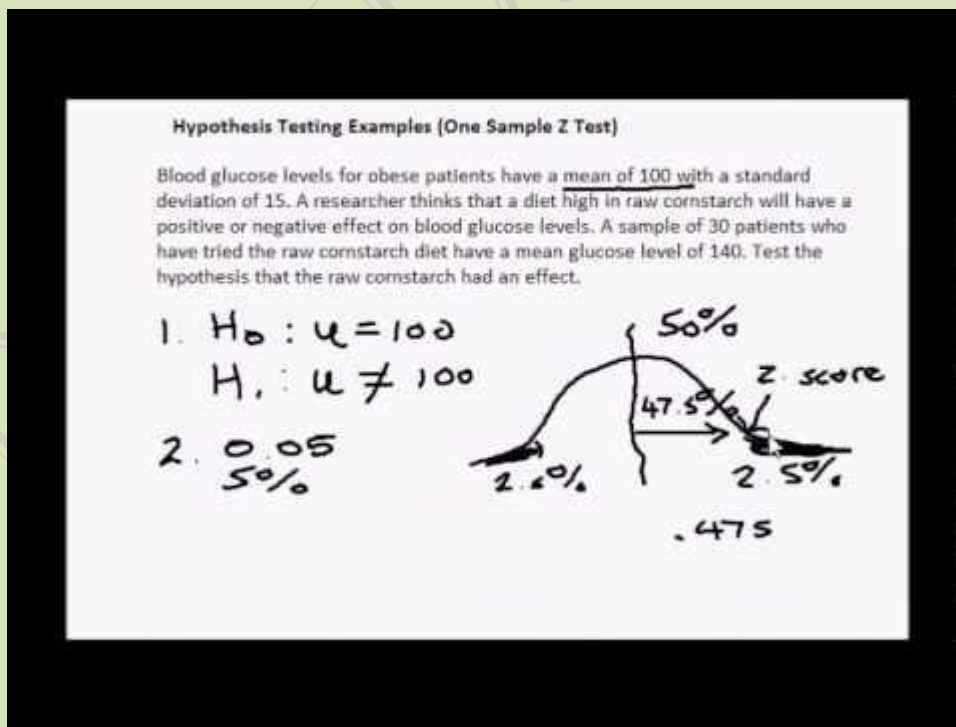$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Step 5: Find the test statistic using this formula:
z = (140 – 100) / (15/√30) = 14.60.
Step 6: If Step 5 is less than -1.96 or greater than 1.96 (Step 3), reject the null hypothesis. In this case, it is greater, so you can reject the null.
*This process is made much easier if you use a TI-83 or Excel to calculate the z-score (the "critical value").
See:

Hypothesis Testing Examples (One Sample Z Test)

Blood glucose levels for obese patients have a mean of 100 with a standard deviation of 15. A researcher thinks that a diet high in raw cornstarch will have a positive or negative effect on blood glucose levels. A sample of 30 patients who have tried the raw cornstarch diet have a mean glucose level of 140. Test the hypothesis that the raw cornstarch had an effect.

1. $H_0 : \mu = 100$
   $H_1 : \mu \neq 100$

2. 0.05
   5%

50%

47.5%

z score

2.6%

2.5%

.475

FLOW DIAGRAM FOR HYPOTHESIS TESTING

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FLOW DIAGRAM FOR HYPOTHESIS TESTING

FORMULATION OF HYPOTHESIS
Statement of H_0 and H_1

↓

Level of Significance
(α value)

↓

Decision of Correct Sampling Dist^n
Test Statistics

↓

Random Samples

↓

Comparision of Test Statistics (Probability)
from Expectations

↓

Is the Observed / Calculated value equal to or Smaller
than the expected / tabulated value ?

↓   ↓

yes   No

Reject H_0 / Accept H_1 Accept H_0 / Reject H_1
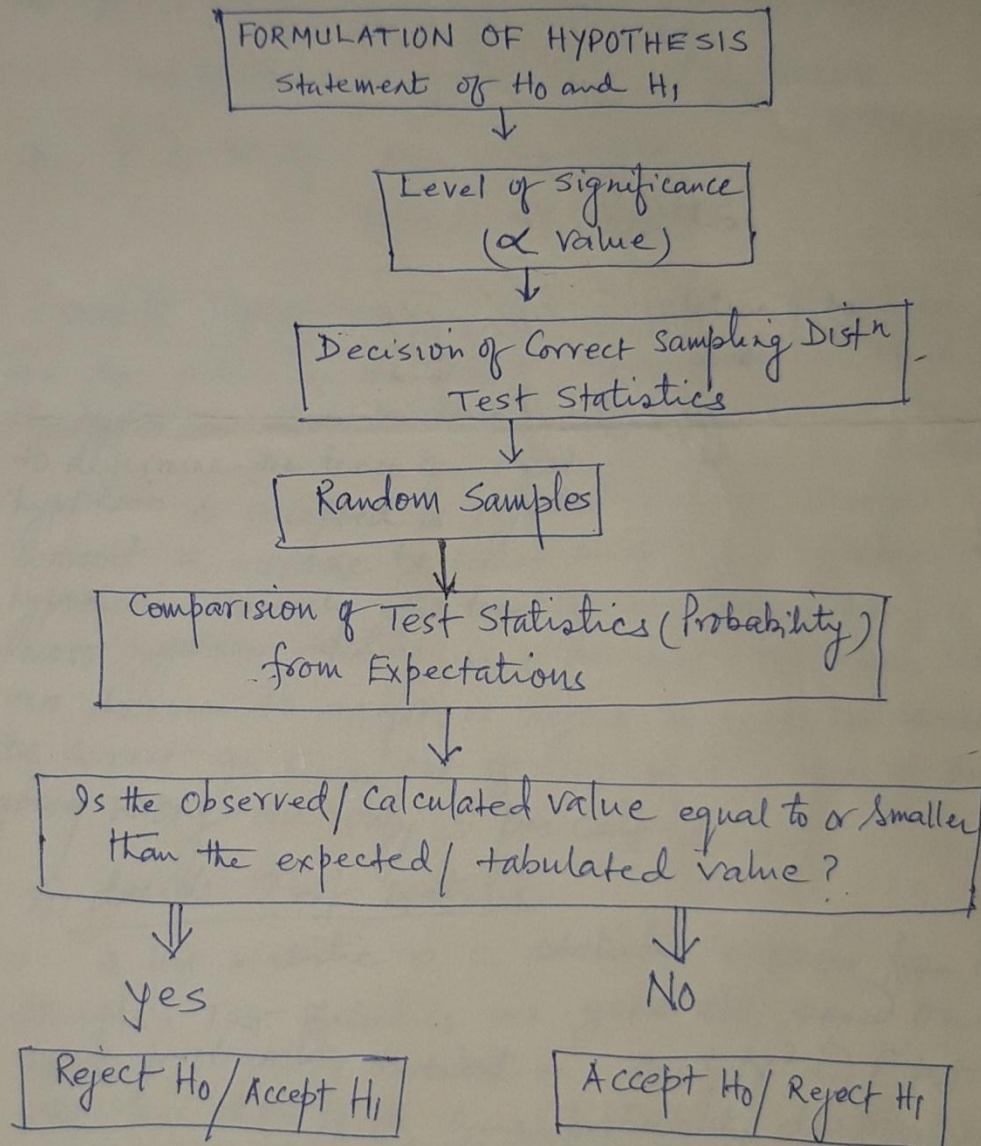
Five Steps in Hypothesis Testing:

1. *Specify the Null Hypothesis*

2. *Specify the Alternative Hypothesis*

3. *Set the Significance Level (a)*

4. *Calculate the Test Statistic and Corresponding P-Value*

5. *Drawing a Conclusion*

Step 1: Specify the Null Hypothesis
The null hypothesis (H_0) is a statement of no effect, relationship, or difference between two or more groups or factors. In research studies, a researcher is usually interested in disproving the null hypothesis.

Examples:

- *There is **no difference** in intubation rates across ages 0 to 5 years.*

- *The intervention and control groups have **the same** survival rate (or, the intervention **does not improve** survival rate).*

- *There is **no association** between injury type and whether or not the patient received an IV in the prehospital setting.*

Step 2: Specify the Alternative Hypothesis
The alternative hypothesis (H_1) is **the statement that there is an effect or difference.** This is usually the hypothesis the researcher is interested in proving. The alternative hypothesis can be **one-sided** (only provides one direction, e.g., lower) **or two-sided.** We often use two-sided tests even when our true hypothesis is one-sided because it requires more evidence against the null hypothesis to accept the alternative hypothesis.

Examples:

- *The intubation success rate **differs** with the age of the patient being treated **(two-sided)**.*

- *The time to resuscitation from cardiac arrest **is lower** for the intervention group than for the control **(one-sided)**.*

- *There is **an association** between injury type and whether or not the patient received an IV in the prehospital setting **(two sided)**.*

Step 3: Set the Significance Level (a)

The significance level (denoted by the Greek letter alpha— a) is generally set at 0.05. **This means that there is a 5% chance that you will accept your alternative hypothesis when your null hypothesis is actually true.** The smaller the significance level, the greater the burden of proof needed to reject the null hypothesis, or in other words, to support the alternative hypothesis.

Step 4: Calculate the Test Statistic and Corresponding P-Value

In another _section_ we present some basic test statistics to evaluate a hypothesis. Hypothesis testing generally uses a test statistic that **compares groups or examines associations between variables.** When describing a single sample without establishing relationships between variables, a _confidence interval_ is commonly used.

The p-value **describes the probability of obtaining a sample statistic** as or more extreme by chance alone if your null hypothesis is true. This p-value is determined based on the result of your test statistic. Your conclusions about the hypothesis are based on your p-value and your significance level.

Example:

- P-value = 0.01 This will happen 1 in 100 times by pure chance if your null hypothesis is true. Not likely to happen strictly by chance.

Example:

- P-value = 0.75 This will happen 75 in 100 times by pure chance if your null hypothesis is true. Very likely to occur strictly by chance.

Cautions About P-Values

your **sample size directly impacts your p-value.** Large sample sizes produce small p-values even when differences between groups are not meaningful. **You should always verify the practical relevance of your results.** On the other hand, a sample size that is too small can result in a failure to identify a difference when one truly exists.

Plan your sample size ahead of time so that you have enough information from your sample to show a meaningful relationship or difference if one exists. See _calculating a sample size_ for more information.

Example:

- Average ages were significantly different between the two groups (16.2 years vs. 16.7 years; $p = 0.01$; n=1,000). Is this an important difference? Probably not, but the large sample size has resulted in a small p-value.

Example:

- *Average ages were not significantly different between the two groups (10.4 years vs. 16.7 years; p = 0.40, n=10). Is this an important difference? It could be, but because the sample size is small, we can't determine for sure if this is a true difference or just happened due to the natural variability in age within these two groups.*

If you do a large number of tests to evaluate a hypothesis (called multiple testing), then you need to control for this in your designation of the significance level or calculation of the p-value. For example, if three outcomes measure the effectiveness of a drug or other intervention, you will have to adjust for these three analyses.

Step 5: Drawing a Conclusion

1. *P-value <= significance level (a) => Reject your null hypothesis in favor of your alternative hypothesis. Your result is statistically significant.*

2. *P-value > significance level (a) => Fail to reject your null hypothesis. Your result is not statistically significant.*

*Hypothesis testing is not set up so that you can absolutely prove a null hypothesis. Therefore, when you do not find evidence against the null hypothesis, you fail to reject the null hypothesis. When you do find strong enough evidence against the null hypothesis, you reject the null hypothesis. **Your conclusions also translate into a statement about your alternative hypothesis.** When presenting the results of a hypothesis test, include the descriptive statistics in your conclusions as well. Report exact p-values rather than a certain range. For example, "The intubation rate differed significantly by patient age with younger patients have a lower rate of successful intubation (p=0.02)." Here are two more examples with the conclusion stated in several different ways.*

Example:

- *H_0: There is no difference in survival between the intervention and control group.*

- *H_1: There is a difference in survival between the intervention and control group.*

- *a = 0.05; 20% increase in survival for the intervention group; p-value = 0.002*

Conclusion:

- *Reject the null hypothesis in favor of the alternative hypothesis.*

- *The difference in survival between the intervention and control group was statistically significant.*

- *There was a 20% increase in survival for the intervention group compared to control (p=0.001).*

Example:

- *H_0: There is no difference in survival between the intervention and control group.*

- *H_1: There is a difference in survival between the intervention and control group.*

- *a = 0.05; 5% increase in survival between the intervention and control group; p-value = 0.20*

Conclusion:

- *Fail to reject the null hypothesis.*

- *The difference in survival between the intervention and control group was not statistically significant.*

- *There was no significant increase in survival for the intervention group compared to control (p=0.20).*