Department of Zoology
Patna University, Patna

Dr. Anupma Kumari
Associate Professor

M.Sc. Semester II
Core Course (CC-5): Environmental Science

Topic: Population Growth

(Exponential growth, Verhulst-Pearl logistic growth model)

LOGISTIC GROWTH

- No population continues to grow indefinitely. In particular, populations that exhibit exponential growth eventually confront the limits of the environment.

- As a population’s density changes, interactions mediated by the environment occur among members of the populations and tend to regulate the population’s size. These interactions include a wide variety of mechanisms relating to physiological, morphological and behavioral adaptations.

Assumptions of Logistic growth pattern

1) Abiotic factors (density dependent factors) do not affect birth and death rate.

2) Carrying capacity is constant for a system.

3) Population growth is not affected by age distribution.

4) Interaction between population and K of the environment is instantaneous.

5) Birth and death rate change linearly with population size.

- The concept of decline of the exponential growth rate as the size of the population decreases is described by the Mathematician -

  PIERRE FRANCOSIS VERHULST and RAYMOND PEARL

- He called the growth logistic because of its 'logarithmic exponential form' that can be described as-

$$\Rightarrow \frac{dN}{dt} = rN$$ (by adding a variable to describe the effect of density)

$$\Rightarrow \frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

$$\Rightarrow \frac{dN}{dt} = rN(1 - \frac{N}{K})$$

Where
$$\begin{pmatrix} \frac{dN}{dt} = Spontaneous\ rate\ of\ change\ of\ population \\ \frac{K-N}{K} = Unutilized\ resource\ for\ population\ growth \end{pmatrix}$$

This equation says that the rate of increase of population over a unit of time is equal to the potential increase of the population times the unutilized portion of the resources.

$$\frac{dN}{dt} = rN \left| \frac{K-N}{K} \right.$$

Reduces population growth

CONDITION 1

When the value of 'N' (population density) is low i.e. when N<<K

Then $$\frac{K-N}{K} = 1$$

Equation would be

$$\frac{dN}{dT} = rN$$

Population will grow *exponentially* i.e. most of the resources are unutilized.

CONDITION 2

When the value of 'N' (population density) is equal to 'K' N=K

Then $$\frac{K-N}{K} = 0$$

Equation would be

$$\frac{dN}{dT} = 0$$

Population growth is slow i.e. all the resources are Utilized.

CONDITION 3

When the value of 'N' (population density) is greater than 'K' i.e. N>>K

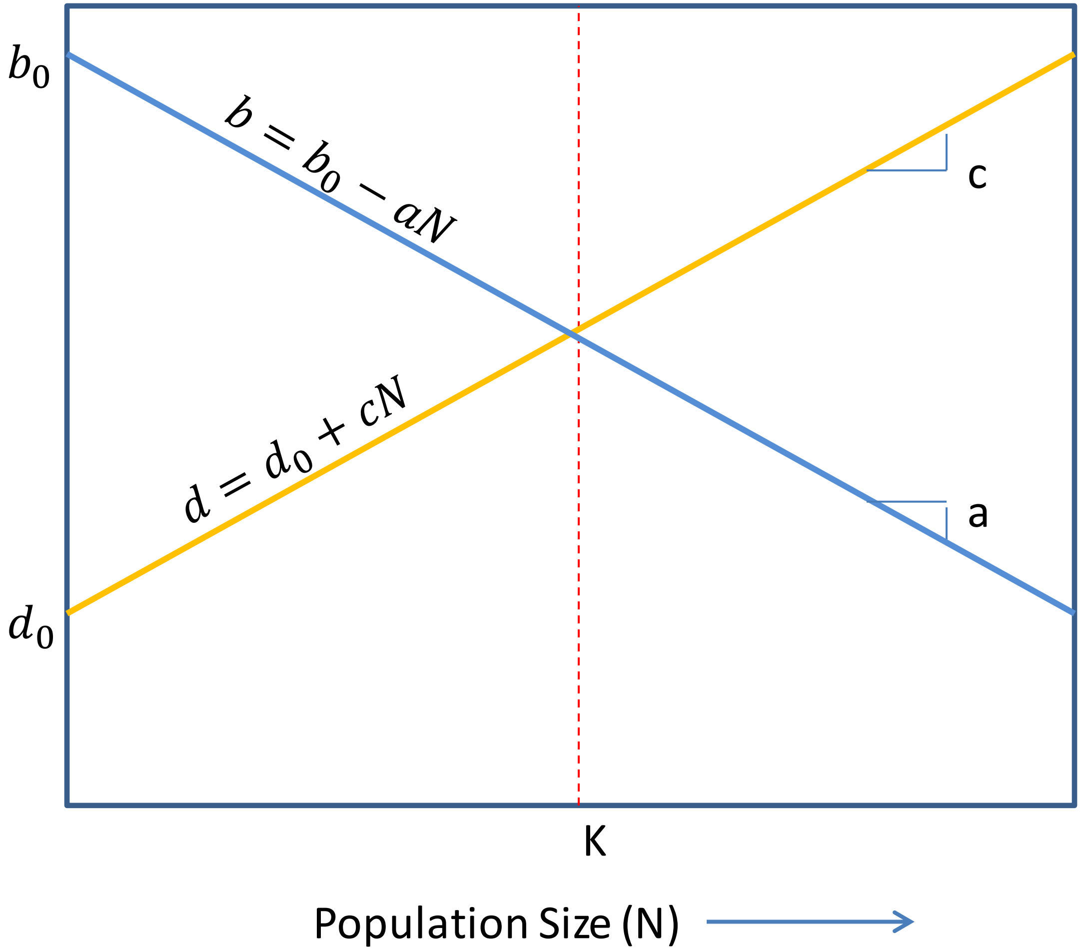
Then equation would be

$$\frac{dN}{dT} = Negative$$

The Environment Functions to Limit Population Growth

- Exponential model of population growth based on several assumptions:
- 1. Essential resources (space, food, etc.) are unlimited.
- 2. Environment is constant.
- In real situation, environment is not constant and resources are limited. As the density of a population increases, demand for resources increases. If the rate of consumption exceeds the rate at which resources are replenished, then the resource base will shrink. Shrinking resources and the potential for an unequal distribution of those resources results in increase mortality or decreased fecundity or both.
- The simplest form of representing changes in birth rates and death rates with increasing population is a straight line (linear function).
- The graph in Figure (1) presents an example in which the per capita birth rate (b) decreases with increasing population size and per capita death rate (d) increases with population size.

Population Regulation —— (Intra specific Competition)



Graph (1): <u>Birth rate and Death rate are represented as a linear function of population size</u>

- This Graph (1) shows that the rate of birth decreases with increasing population size and death rate increases with population size.

- Then, we can represent the change in birth rate as a function of population size -

$\Rightarrow$ $b = b_0 - aN$

$$\left(\begin{array}{c} b_0 \; is \; when \; N_0 = 0 \\ a = slope \; of \; the \; line \; represented \; by \; \frac{\Delta b}{\Delta N} = a \end{array} \right)$$

In this equation $'b_0' is \; the \; intercept$ and $'a'$ is the slope of the line

The intercept $'b_0'$ represents birth rate achieved under ideal conditions whereas 'b' is the actual birth rate which is reduced as the population size increases.

- Similarly we can represent change in death rate as a function of population size.

$\Rightarrow$ $d = d_0 + cN$

$$\left(\begin{array}{c} d_0 \; is \; when \; N_0 = 0 \\ c = slope = \frac{\Delta d}{\Delta N} = c \end{array} \right)$$

Mathematical Derivation

- The logistic equation can be derived from exponential equation considering the condition that resources are limited.
- Then we can represent change in birth rate as a function of population size.

$$\Rightarrow b = b_0 - aN$$

$$\Rightarrow d = d_0 + cN$$

$$\Rightarrow \frac{dN}{dt} = rN$$

$$\Rightarrow \frac{dN}{dT} = (b - d)N$$

$$\Rightarrow \frac{dN}{dT} = [(b_0 - aN) - (d_o + cN)]N \quad \text{——— (i)}$$

The pattern of population growth now defers form the original 'exponential model' as N increases, the rate of birth declines and the rate of death increases. Finally it result in slowing of the rate of the population growth

- When the death rate exceeds the value of the birth rate, population growth is negative and the population size declines.
- When the rate of birth equals to the rate of death, the rate of population change is zero.

The value of population size where the rate of birth is equal to the rate of death represents the maximum sustainable population size under the prevailing environmental conditions.

$$\Rightarrow \frac{dN}{dT} = [(b_0 - aN) - (d_o + cN)]N \qquad \text{(i)}$$

Equating the equation (i) with zero

$$\Rightarrow [(b_0 - aN) - (d_o + cN)]N = 0$$

$$\Rightarrow b_0 - d_0 = aN + cN$$

$$\Rightarrow b_0 - d_0 = (a + c)N$$

$$\Rightarrow N = \frac{b_0 - d_0}{a + c}$$

$$\Rightarrow N = K$$

(Where N=Population Size

K= Carrying Capacity)

Carrying Capacity

- **Carrying capacity** can be defined as-

  - maximum sustainable population size under prevailing environmental condition or

  - the no. of individuals of a particular species that a particular environment can support indefinitely.

- Carrying capacity is not fixed but varies over time and space with the abundance of limiting resources.

- It is determined by various factors including- predation, competition and climatic conditions.

- So, growth of the population eventually slows as the population reaches the carrying capacity .

From *Equation (i)*

$$\Rightarrow \frac{dN}{dT} = [(b_0 - aN) - (d_o + cN)]N$$

$$\Rightarrow \frac{dN}{dT} = [(b_0 - d_0) - (aN + cN)]N$$

$$\Rightarrow \frac{dN}{dT} = [(b_0 - d_0) - (a + c)N]N$$

Multiplying and dividing by $(b_0 - d_0)$

$$\Rightarrow \frac{dN}{dT} = \frac{b_0 - d_0}{b_0 - d_0}[(b_0 - d_0) - (a + c)N]N$$

$$\Rightarrow \frac{dN}{dt} = b_0 - d_0\left[\frac{b_0 - d_0}{b_0 - d_0} - \frac{(a + c)N}{b_0 - d_0}\right]N$$

$$\Rightarrow \frac{dN}{dt} = b_0 - d_0\left[1 - \frac{N}{K}\right]N$$

Since $\left(K = \frac{b_0 - d_0}{a + c}\right)$

$$\Rightarrow \frac{dN}{dt} = rN\left[1 - \frac{N}{K}\right]$$

[*Where* $r = b_0 - d_0$]

Equation For Logistic Growth

The logistic equation may be written in several ways -

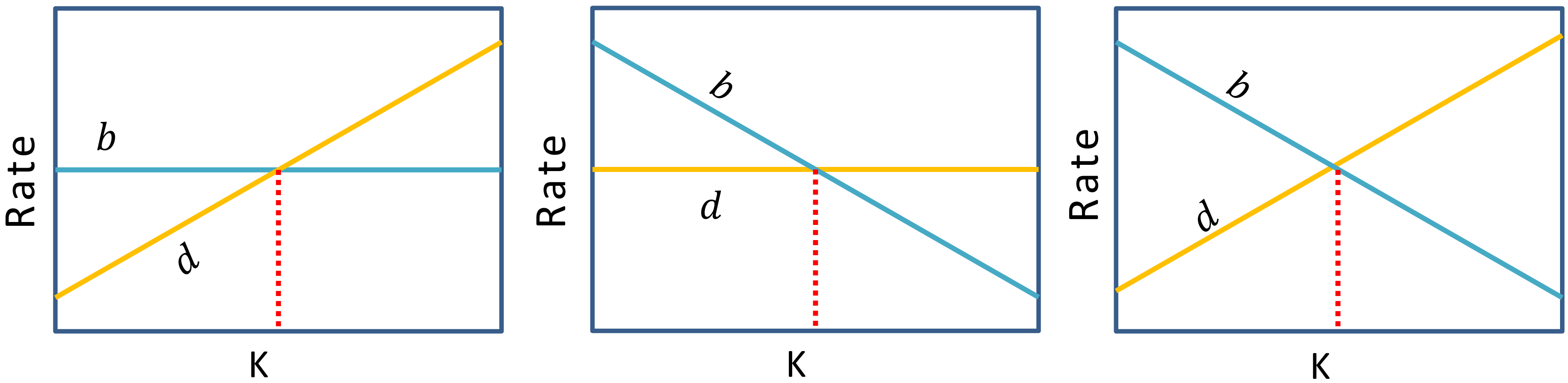
$$\Rightarrow \frac{dN}{dt} = rN\left[\frac{K - N}{K}\right]$$

$$\Rightarrow \frac{dN}{dt} = \frac{rNK}{K} - \frac{rN^2}{K}$$

$$\Rightarrow \frac{dN}{dt} = rN\left[\frac{K}{K} - \frac{N}{K}\right]$$

$$\Rightarrow \frac{dN}{dt} = rN\left[1 - \frac{N}{K}\right]$$

Three Different Scenario(s) →



A

- Birth rate is constant

- Birth rate is independent of population size.

B

- Death rate is constant. Only birth takes place.

C

- Birth rate decreases, death rate increases as the population size increases.

- Population regulation involves density dependent. The concept of 'Carrying Capacity' suggests a negative feedback between population increase and resources available in the environment.

- As population density increases, the per capita availability of resource declines. The decline in per capita resources eventually reaches at some crucial level at which it functions to regulate population growth.

- This model of population regulation is density dependent.

- Density dependent affects influence a population in proportion to its size. It functions by slowing the rate of increase.

- So, regulation in population size occur in three different situations-

Fig (A): the birth rate is independent of population density, as indicated by the horizontal line.

Only the death rate increases with population size. At K, equilibrium is maintained by increasing mortality.

Fig (B) : The situation is reversed, mortality is independent but birth declines with population size. At K, the decreasing birth rate maintains equilibrium.

Fig (C) : represents full density dependent regulation. Both birth rate and mortality are density dependent.

- The logistic model of population growth produces a sigmoid (S-Shaped) growth curve when population size is plotted over time.

- In **SIGMOID CURVE**-

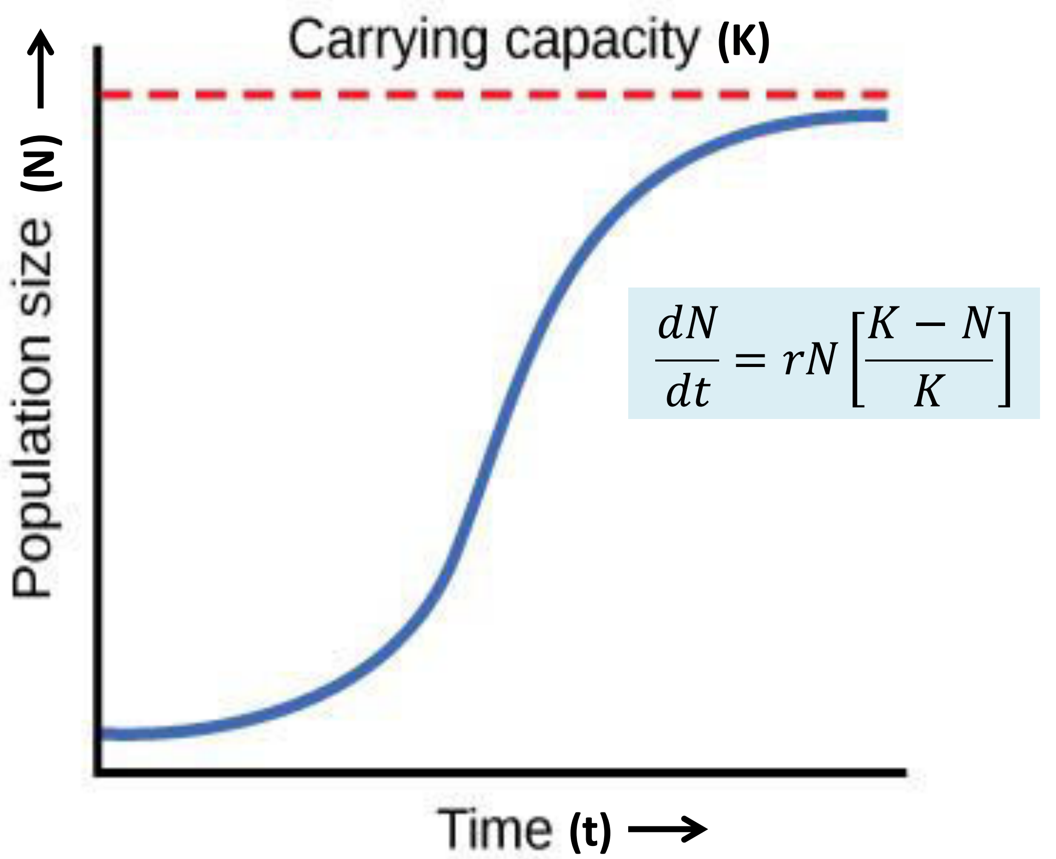
  - the population increases slowly at first(acceleration) then more rapidly (perhaps approaching a logarithmic phase) but then slows down gradually as the environment resistance increases , until equilibrium is reached and maintained.

- Implicit in the concept of ‘K’ is competition among individuals for essential resources. Competition occurs when individuals use a common resource i.e. in short supply relative to the number seeking it.

- Competition among the individuals of the same species is referred to as intraspecific competition.

- The intensity of intraspecific competition is usually density dependent it increases gradually, at first affecting growth and development. Later it affects individual survival and reproduction.

LOGISTIC GROWTH CURVE



<u>Graph (2) – Sigmoid (S-Shaped) Logistic Growth Curve</u>

THANKS