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Topic: Exponential population growth

CC V- Environmental Science

POPULATION GROWTH

- Population growth refers to the changes in the size of the population i.e. how the number of individuals in a population increases or decreases with time.

- The density of a population in a given habitat during a given period fluctuates due to change in four basic processes.

  - **Natality**
  - **Mortality**
  - **Immigration**
  - **Emigration**

- **Natality-** It refers to the birth of individual in a population. Natality rate(or birth rate) is the number of individuals produced or born in a given period per unit time.

- **Mortality-** It refers to the death of individuals in a population. Mortality rate(or death rate) is the number of individuals died during a given period.

- **Immigration-** Number of individuals of the same species that have come into the habitat from elsewhere during a time period under consideration.

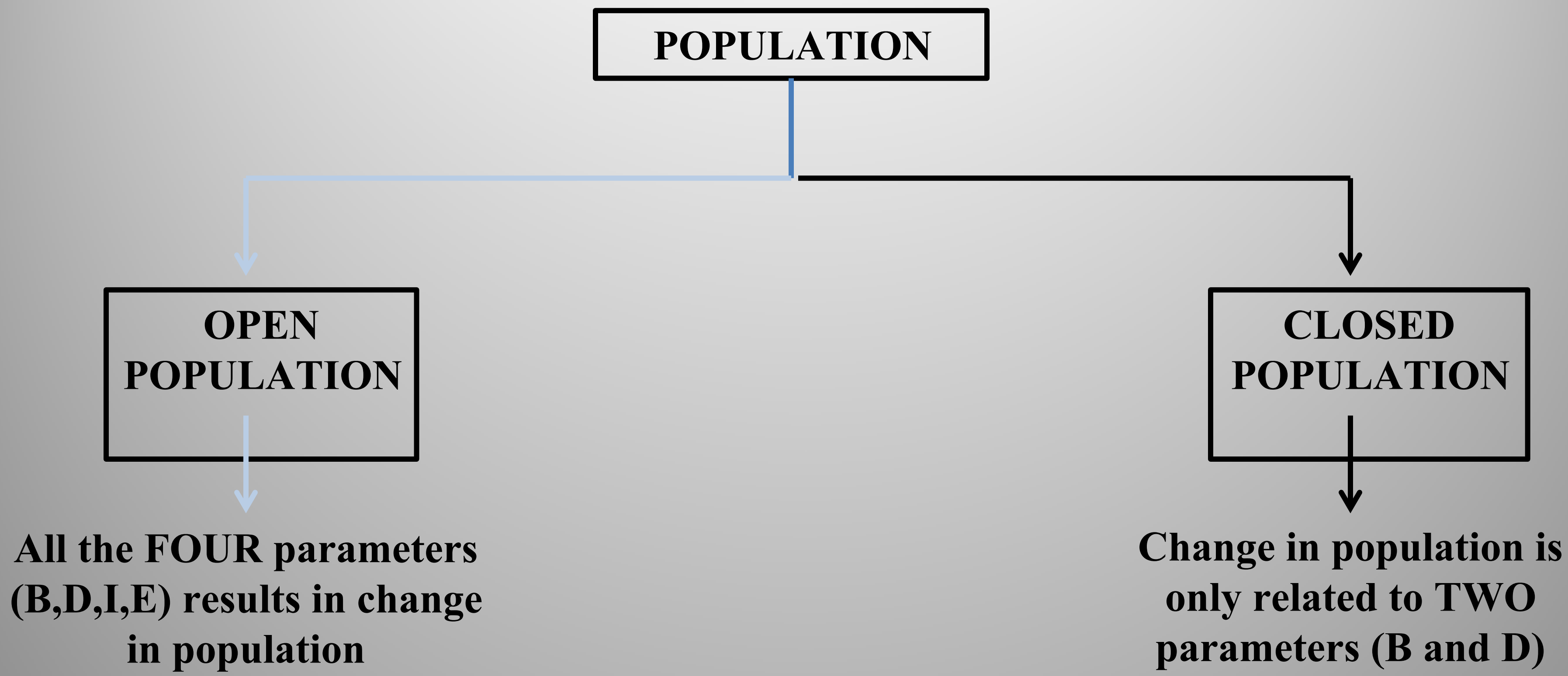
- **Emigration-** Number of individuals who left the habitat and gone elsewhere during a time period under consideration.

So, Population density at time ‘t’ -

$$N_t = N_0 + (B + I) - (D + E)$$

Population Gain (B + I) **Population Loss** (D + E)

Based on the above four parameters , population can be divided into two parts -



POPULATION GROWTH MODELS

- A population model is a type of mathematical model that is applied to the study of population dynamics.

- Population growth models.

  - ***Exponential Growth Model***

  - Logistic Growth Model

Exponential Growth

- Thomas Malthus (1798) wrote 'An Essay On The Principle Of Population' in which he wrote population growth occurs exponentially. During exponential growth the number increases in the geometric progression $2^0, 2^1 \ldots 2^n$.

  He examined the relationship between population growth and resources.

Conditions for Exponential Growth

- **Resources should be unlimited**
- **No immigration and emigration**
- **No mortality**
- **Then birth rate alone will account for the change in population number. Under this condition population growth will stimulate compound interest, a continued increase for exponential growth.**

Mathematical Expression

- In an unlimited closed population the exponential form may be represented by -

$$\Rightarrow \frac{dN}{dt} = (b-d)N \quad Or \quad \frac{dN}{dt} = rN \text{ ———— } (i)$$

$$\left(\begin{array}{c} Where\ N = Population\ size/density \\ b = per\ capita\ birth\ rate(no.of\ births\ per\ individul\ per\ unit\ of\ time) \\ d = per\ capita\ death\ rate(no.of\ deaths\ per\ individul\ per\ unit\ of\ time) \\ \frac{dN}{dt} = rate\ of\ change\ of\ population\ size \\ r = intrinsic\ rate\ of\ natural\ increase\ (r = b - d) \end{array} \right)$$

- The equation shows that the rate of increase or decrease of the population is directly proportional to the population size and the growth rate.

- The most useful equation for calculating the exponential growth is the integrated form and the curve is ‘J-Shaped curve’ and depends upon the value of ‘r’.

$$\Rightarrow \frac{dN}{dt} = rN \quad \text{———— (i)}$$

$$\Rightarrow \frac{dN}{N} = rdt \quad \text{———— (ii)}$$

Integrating on both sides of equation (ii)

$$\Rightarrow \int \frac{dN}{N} = \int rdt$$

$$\Rightarrow c_1 + lnN = rt + c_2 \qquad Since \begin{pmatrix} \int \frac{dx}{x} = lnx + c \qquad [Integration\ Formula] \\ where\ c_1\ and\ c_2\ are\ constants\ for\ integration \end{pmatrix}$$

$$\Rightarrow lnN = rt + (c_2 - c_1)$$

$$\Rightarrow lnN = rt + c \qquad \begin{pmatrix} Combining\ the\ constants\ of\ integration\ c_1\ and\ c_2 \\ into\ a\ single\ constant\ 'c' \end{pmatrix}$$

Now, each side of the equation is raised to the power of ‘e’ to get rid of the logarithm.

$$\Rightarrow e^{\ln N} = e^{rt+c} \quad \left(\begin{array}{c} e = natural\ logarithm\ base. \\ Also\ known\ as\ 'Euler's\ Number' \end{array}\right)$$

$$\Rightarrow N = e^{c} \times e^{rt} \text{ ——— (iii)}$$

Now at t=0 (the initial time we observe the population)

$$\Rightarrow N_0 = e^{c} \times e^{r \times 0}$$

$$N_0 = e^{c} \times 1$$

$$N_0 = e^{c} \text{ ——— (iv)} \quad (e^{c}\ is\ the\ initial\ population\ size)$$

Substituting the value of e^c in the equation (iii)

$At\ \ t = 0,\ N = N_0$ $\qquad$ $At\ \ t = t,\ N = N_t$

$$\Rightarrow N_t = e^{c} \times e^{rt}$$

$$\Rightarrow N_t = N_0 \times e^{rt} \quad \left(\begin{array}{c} Integral\ Equation\ for\ the\ exponential\ growth \\ of\ the\ population \end{array}\right)$$

Equations for Exponential Growth

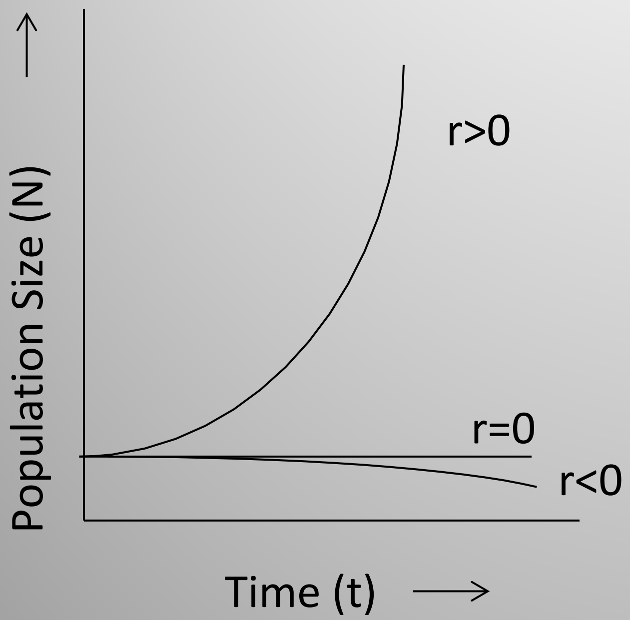
$$\Rightarrow \frac{dN}{dt} = rN \qquad (Differential\ Equation)$$

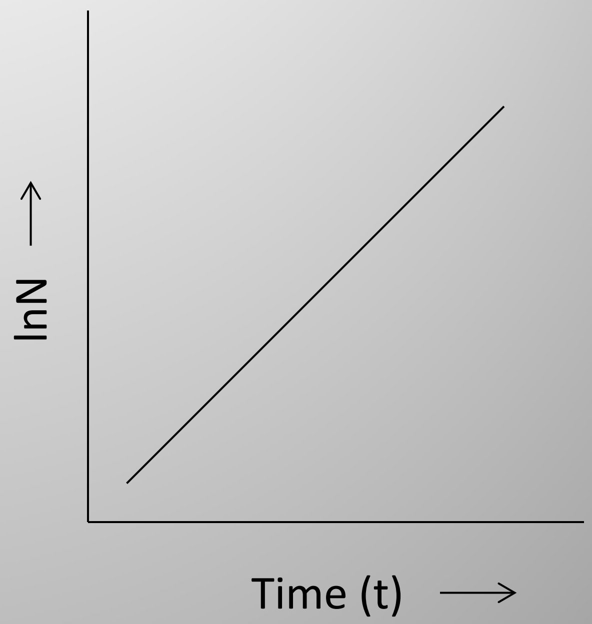
$$\Rightarrow N_t = N_0\, e^{rt} \qquad (Integral\ Equation)$$

- When resources (food and space) in a habitat are unlimited, all the members of a species have the ability to grow exponentially.

- The population size that increases exponentially at a constant rate, results in a J-shaped growth curve when population size (N) is plotted over time(t).

Exponential Growth Curve





- The growth curve is J-shaped and depends upon the value of ‘r’
  - If r>0 (means population increases exponentially)
  - r<0 (exponential decline in the population)
  - r=0 (No change in the population size)

Example of exponential growth

- There really are no example of positive exponential growth as such in the physical universe. Things grows exponentially for a while but they always hit some limit.

- One such example of exponential growth is observed in bacteria. It takes bacteria roughly an hour to reproduce through prokaryotic fission.

- A population cannot grow exponentially forever. Over time it will exceed its carrying capacity.

THANKS