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Topic: Lotka-Volterra model

M.Sc. IV Semester , Elective Paper-IC

Lotka-Volterra model

Lotka-Volterra model is an interspecific competition theory, named after the two mathematician, the American Alfred Lotka and the Italian Vittora Volterra.

In 1920's they independently derived the mathematical expression to describe the relationship between two different species using the same resource, by modifying the logistic equation with addition of the term "competition coefficient" to take into account of competition effect of one species on the population growth of other species.

For two species (species 1 and species 2) with population N_1 and N_2, the competition coefficient is expressed as αN_2 and βN_1 respectively.

Hence, Lotka-Volterra model is the extension of the sigmoidal logistic growth model.

Derivation of Equation

The logistic equation for population growth of single species coexist Independently is

$$\frac{dN}{dt} = \gamma N \left(\frac{K - N}{K} \right)$$

Now, modifying the logistic equation with addition of competition Coefficient αN_2 and βN_1, the change in the population size for two Competing species can be represented as

$$\frac{dN_1}{dt} = \gamma_1 N_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1} \right) \quad \text{— (I)}$$

$$\frac{dN_2}{dt} = \gamma_2 N_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2} \right) \quad \text{— (II)}$$

Where,

α = competition coefficient as effect of species 2 on species 1

β = competition coefficient as effect of species 1 on species 2

r1 & r2= intrinsic growth rate of species 1 and species 2

K_1 & K_2= carrying capacity of species 1 and species 2

ASSUMPTIONS

Some major outcomes that lie behind the Lotka-Volterra model are

1. The environment is homogeneous and stable, without any fluctuations.
2. Migration is unimportant
3. The effect of competition is instantaneous
4. Coexistence require a stable equilibrium
5. Competition is a only important biological interaction.

Joint Dynamics

Based on the assumptions, the modified logistic equation (I& II) can be used to describe the joint dynamics as the ‘ combined population effect’.

In absence of any interspecific competition either α or N_2=0 in equation (1) and β or N_1=0 in equation (II), population of each species grows logistically in equlibrium at ‘K’ (carrying capacity) and population growth (dN_1/dt or dN_2/dt) approaches zero.

i.e. $\mathbf{N_1 = K_1}$ **&** $\mathbf{dN_1/dt = 0}$

$\mathbf{N_2 = K_2}$ **&** $\mathbf{dN_2/dt = 0}$

Inherent in the logistic equation is the inhibitory effect of each individual on its own species’ population growth. This effect is represented by $1/K_1$ for species 1 and $1/K_2$ for species 2.

In the presence of competition, the picture changes.

In competing populations, the inhibitory effect of each N_2 individual on N_1 is α / K_1. Similarly, the inhibitory effect of each N_1 individual on the population growth of species 2 is β / K_2.

For example, the carrying capacity for species 1 is K_1 and as N_1 approaches K_1 , the population growth (dN_1 / dt) approaches zero. However, species 2 is also vying for the limited resource that determines K_{1}, so we must consider the impact of species 2. Because if α is the per capita effect of species 2 on species 1, the total effect of species 2 on species 1 is αN_2. so we must consider the effects of both species in calculating population growth. As the combined population effect ($N_1 + \alpha N_2$) approaches K_1, the growth rate of species 1 will approach zero. The greater the density of the competing speciec(N_2), the greater the reduction in the growth rate of species 1(dN_1 / dt).

The outcome of competition depends upon the relative values of K_1, K_2, α and β.

So, considering the effect of both species the combined population effect for species 1 is represented as

$$\mathbf{N_1 + \alpha N_2 = K_1 \ \& \ dN_1/dt = 0}$$

Similarly, the combined population effect for species 2 is represented as

$$\mathbf{N_2 + \beta N_1 = K_2 \ \& \ dN_2/dt = 0}$$

With the help of graphs, we can better understand the equations.

In each case the ordinate will represent the population size of species 1 and the abscissa the population size of species 2.

Two lines are plotted on each graph: one representing each of the two species.

The diagonal line for species 1 represents the combined population densities of species 1 and 2 that equal K_1 and therefore $dN_1/dt=0$.

The diagonal line for species 2 represents the combined population densities of species 1 and 2 that equal K_2 and therefore $dN_2/dt=0$.

For any point on the species 1 line, $N_1 + \alpha N_2 = K_1$. When $N_{1=} K_1$, then N_2 must be zero. Because α is the per capita effect of species 2 on species 1, the population density of species 2 that is equivalent to the carrying capacity of species 1 ($\alpha N_2 = K_1$) will be $N_2 = K_1/\alpha$. Therefore, when $N_2 = K_1/\alpha$ then N_1 must be equal to zero. . If $N_2 = K_1 / \alpha$ then N_1 can never increase.

For any point on the species 2 line, $N_2 + \beta N_1 = K_2$. When $N_{2=} K_2$, then N_1 must be zero. Because β is the per capita effect of species 1 on species 2, the population density of species 1 that is equivalent to the carrying capacity of species ($\beta N_2 = K_2$) will be $N_1 = K_2/ \beta$. Therefore, when $N_1 = K_2/ \beta$ then N_2 must be equal to zero. If $N_1 = K_2 / \beta$ thenN_2 can never increase.

JOINT DYNAMICS IN TERMS OF ZERO ISOCLINES & PARAMETERS

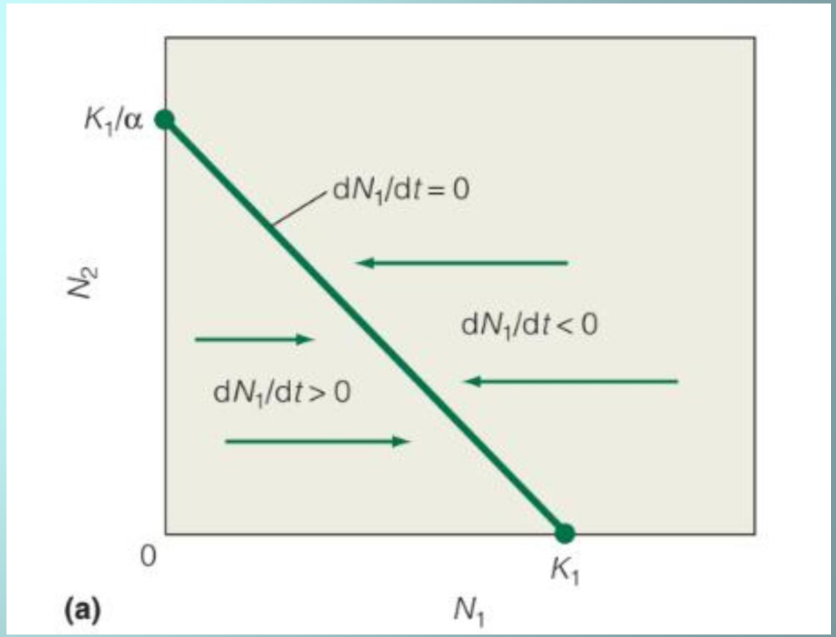
"Zero-isoclines" are the points on a diagonal line showing zero Population growth ($dN/dt=0$) when a graph of N_2 in y axis against N_1 in x axis is plotted

Species- 1

1. $dN_1/dt=0$
 when $N_1=K_1$ (if $N_2=0$)
 $N_2=K_1/\alpha$ (if $N_1=0$

ii. $dN_1/dt >0$
When $N_1+ \alpha N_2 < K_1$
[Below zero-isocline represented by
 in increasing value of N_1 ➡

ii. $dN_1/dt <0$
When $N_1+ \alpha N_2 >K_1$
[Above zero-isocline represented by
 in decreasing value of N_1 ⬅

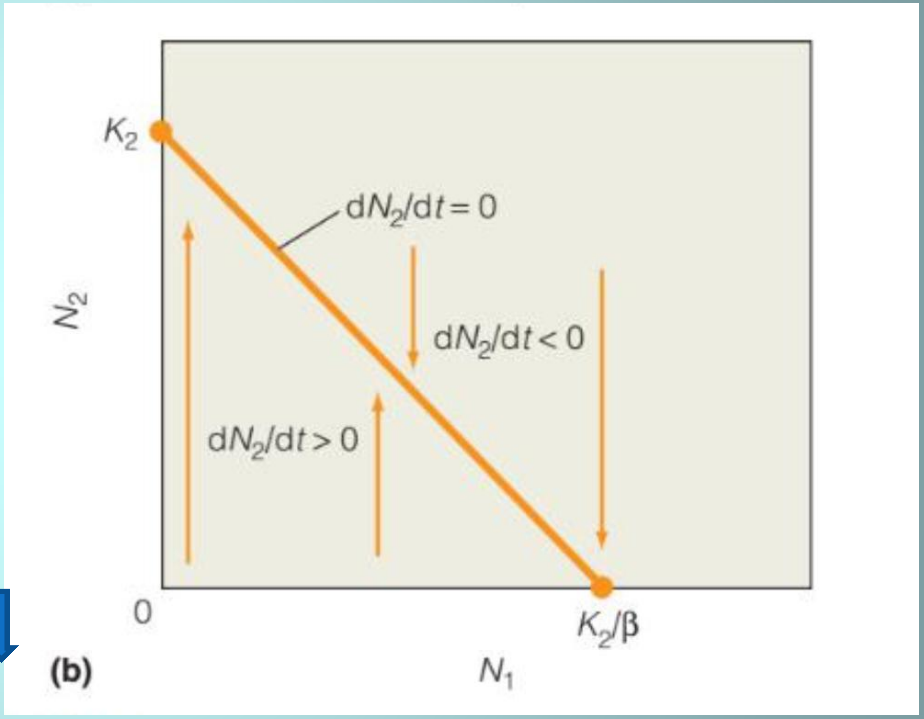


Species -2

1.$dN_2/dt=0$
when $N_2=K_2$ (if $N_1=0$)
$N_1=K_2/\beta$ (if $N_2=0$)

ii. $dN_2/dt > 0$
When $N_2+ \beta N_1 < K_2$
[Below zero-isocline represented by ↑
in increasing value of N_2

ii. $dN_2/dt < 0$
When $N_2+ \beta N_2 > K_2$
[Above zero-isocline represented by ↓
in decreasing value of N2



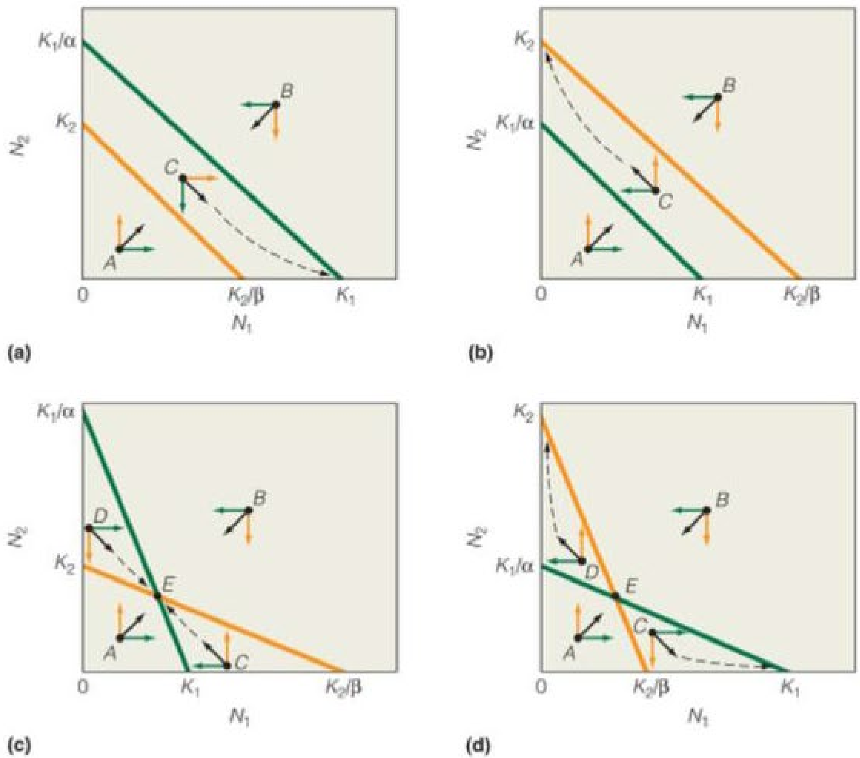


Figure 13.2 Four possible arrangements of the zero-growth isoclines for *species 1* and *species 2* using the Lotka-Volterra model of competition. In case (a), the isocline of *species 1* falls outside the isocline of *species 2*. *Species 1* always wins, leading to the extinction of *species 2*. In case (b), the situation is the reverse of (a). (c) Each species inhibits the growth of its own population more than that of the other by intraspecific competition. The species coexist. (d) The isoclines cross. Each species inhibits the growth of the other more than its own growth. The more abundant species often wins.

In figure c, the isocline of species 1 is parallel to, and lies inside, the Isocline of species 2. In this case, even when the population of species 1is at its carrying capacity (K_1), its density cannot stop the population of Species 2 from increasing ($K_1 < K_2 /\beta$). As species 2 continues to increase, Species 1 eventually extinct.

Figure c. The isocline of species 1 fall inside the isocline of species 2. Species 2 always wins, leading to the extinction of species 1

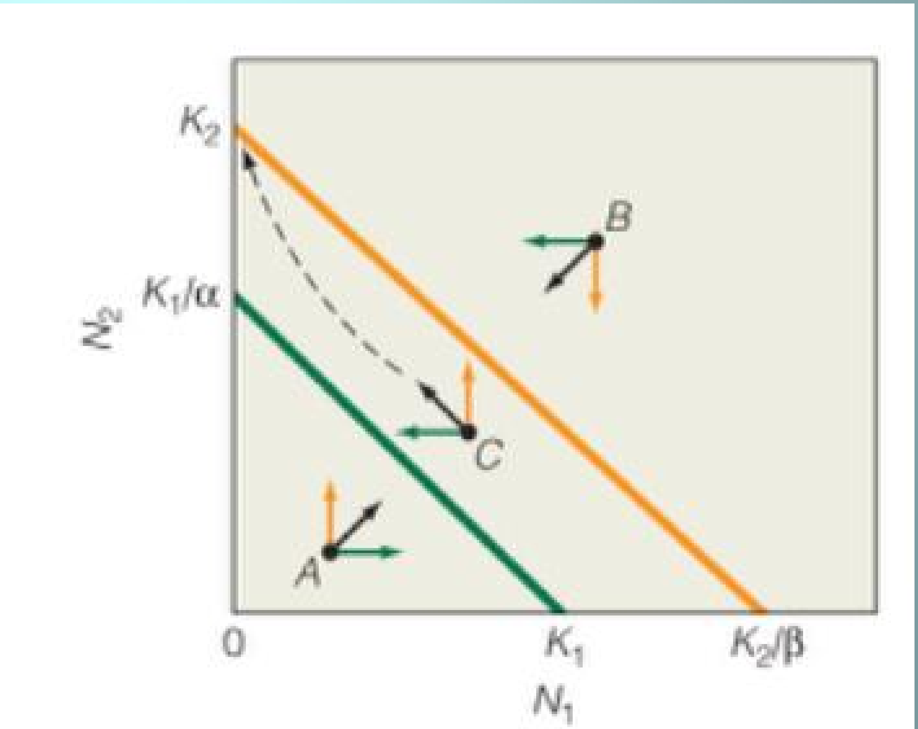
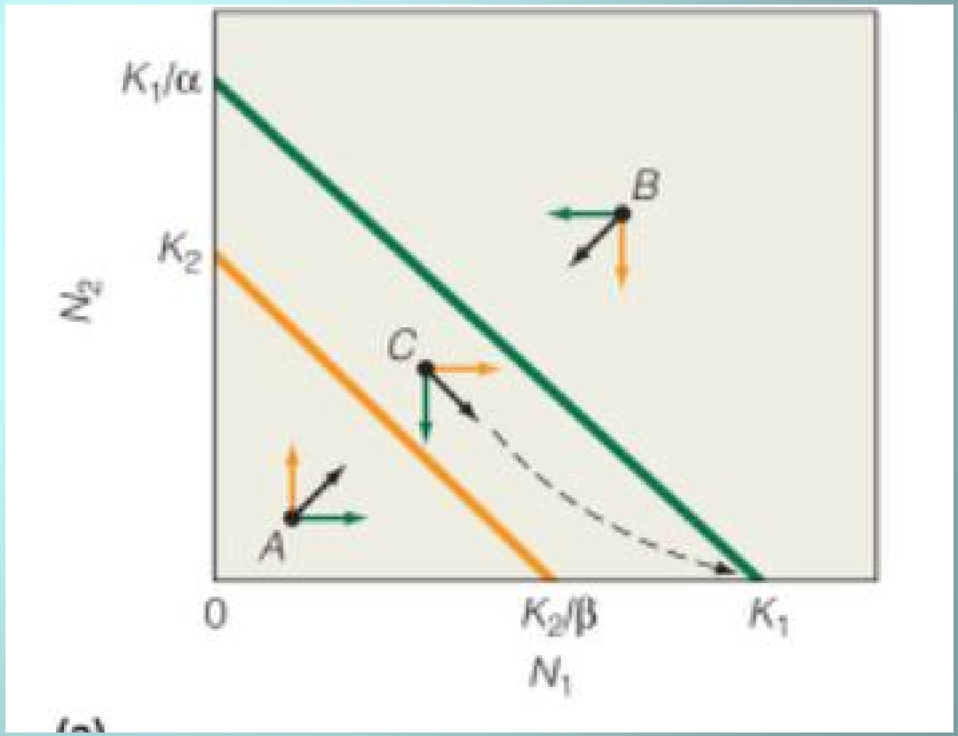


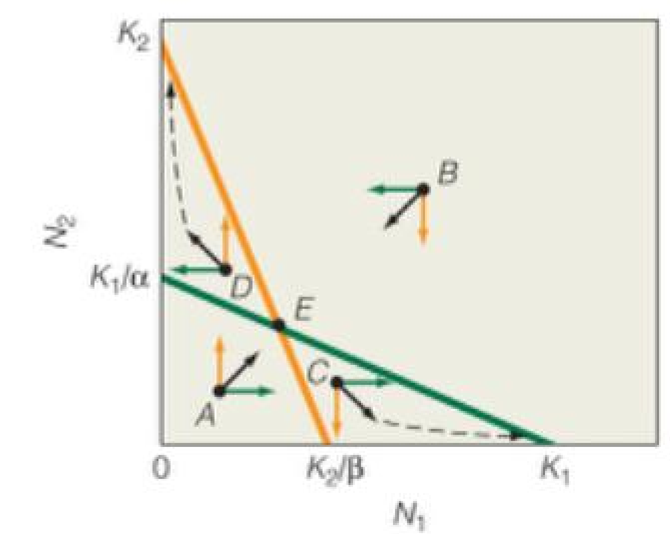
Figure d. The isocline of species 1 fall outside the isocline of species 2. Species 1 always wins, leading to the extinction of species 2

In figure d, the isocline of species 1 is parallel to, and lies outside, the Isocline of species 2. In this case,even when the population of species 2 is at its carrying capacity (K_2), its density cannot stop the population of Species 1 from increasing ($K_2 < K_1 /\alpha$). As species 1 continues to increase, Species 2 eventually extinct.



In the third outcome the diagonal equilibrium lines cross each other. The equilibrium point is resenting at their crossing, but it is unstable. The vectors are directed away from the equilibrium point, indicating that the true equilibrium points are K_1 and K_2. In this situation equilibrium between Competing species is unstable and either of the two Species can win. Above the line K_2, K_2/β species 2 is unable to increase; and above K_1, K_1/α species 1is unable to increase. If the mix of the species is such that the point N_1, N_2 falls within the triangle K_2, E/ K_1/α, species 1 is above its carrying capacity and species 2 is not. Species 2 will continue to increase and species 1 will decrease until it is gone. The reverse situation occurs in triangle K_1, E/ K_2/β. What happens in parts of the diagram outside the triangle depends upon whether the starting value of N_1 is larger or smaller than that of $N_{2.}$

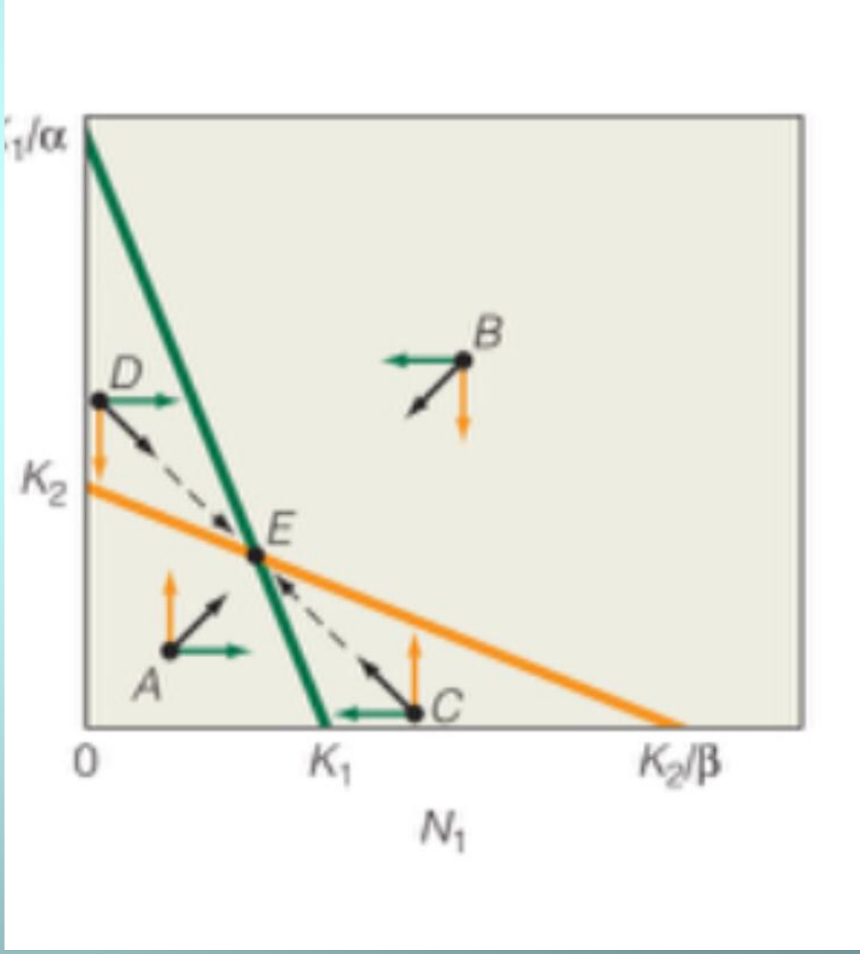
In figure e. each species inhibits the growth of the other species more than it inhibits its own growth. Which species win often depends upon the initial proportion of the two species



(d)

Finally the two species coexist with their populations in equilibrium. As species 1 increases, species 2 may decrease and vice versa. Each species inhibits the growth of its own population more than that of the other by intraspecific competition. In this case, K_1 is less than K_2/β, the population of species 1 can never reach a density sufficient to eliminate species 2. Likewise K_2 is less than K_1/α, the population of species 2 can never reach a density sufficient to eliminate species 1.

f. Coexistence



THANKS