PROBABILITY

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INTRODUCTION

- Probability is defined as “ Measure of the relative chance of occurrence of an event from a set of alternatives”.
- According to Laplace “ Probability is the ratio of the number of favorable cases to the total number of equally like cases.”
- Definition of probability says that probability is the chance that an even will occur.
- It varies from 0 to 1.

Statistical Explanation of Probability

If an event can happen in ‘a’ ways and fails to happen in ‘b’ ways in a trial, then the probability (p) of happening of ‘a’ can be written as

p(a) = $\frac{a}{a+b}$ = no. of events occurring\total no. of trials

Likewise

p(b) = $\frac{b}{a+b}$ = no. of events not occurring\ total no. of trials

p(a) + p(b) = $\frac{a}{a+b} + \frac{b}{a+b} = \frac{a+b}{a+b}$ = 1

p(a) = 1 –p(b) , p(b) = 1- p(a)

Examples:

1. If the twins are born once in 80 different pregnancies,

Then p for births of twins = $\frac{1}{80}$

2. Probability of being Rh^+ = $\frac{1}{10}$

Terms Used in Probability

- Experiment : Experiment refers to describe an act which can be repeated under some given conditions.
- Random experiment: An act which can be repeated under some conditions but the results (outcomes) can not be predicted.
- Simple Event: Only one outcome of each trial (happening or not happening of single event)
- Sure event: An event whose occurrence is evitable ,when a random experiment is performed.
- Random event: An event which may or may not occur performing a certain experiment
- Impossible event (Null event): Event which is not possible
- Sample space: All possible outcomes of experiment

Terms Used in Probability

- Mutually Exclusive Events: When both (among two events) can not happen simultaneously in a single trial ex. Birth of either ♂ or ♀ baby (twin birth is exception) or both two events can not occur simultaneously.
- Mixed \Compound \Joint Events: Occurrence of two or more simple events simultaneously. It may be independent or dependent or equally like events
- Independent Events: Outcome of one event does not affect another in one trial
- Dependent Events: One event affects another in second trial
- Equally like Events: One event does not occur often more than the others. Ex. In general ♂ birth 50% , ♀ birth 50%
- Exhaustive Events: When event totally includes all the possible outcomes of a random experiment. Ex. If two dice are thrown once , the possible outcomes = 6^2 = 36 (R^n , R = no. of options, n= no. of events)

Rules or Theorems of Probability

There are two rules of probability which is the basis of test of significance (1) Addition Rule (2) Multiplication Rule .

1. ADDITION RULE OF PROBABILITY:

This rule is applied when events are mutually exclusive ex. birth of either male or female child

Theorem states that if two events A and B are mutually exclusive , the probability of occurrence of either A or B is the sum of the individual probability of A and B .

Symbolically, P(A or B) = P(A) + P (B)

If there are three events A, B and C, then

P(A or B or C) = P(A) + P(B) + P (C)

Examples of Addition Rule of Probability

Example

In a basket , one fish of each species of *Catla* (C), Rohu (R), Mrigal (M) , *Clarias* (Cl), Singhi (S) and *Puntius* (P) are kept. What is the probability of taking out either catla or singhi in one trial?

$$P(C) = \frac{1}{6},\ P(S) = \frac{1}{6}$$

$$P(C \text{ or } S) = P(C) + P(S) = \frac{1}{6} + \frac{1}{6} = \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3}$$

Hence $P(C \text{ or } S) = \frac{1}{3}$

Examples of Addition Rule of Probability

When Events are not mutually Exclusive:

Example:

The managing committee of Vaishali Association formed a sub committee of 5 persons to look into child trafficking. Profiles of 5 persons are (1) Male aged 40 yrs. (2) Male aged 43 yrs.© Female aged 38 yrs. (d) Female 27 yrs. (e) Male aged 65 yrs.

If a chairperson has to be selected from this , what is the probability that the person would be either female or over 30 yrs.

P (Female or over 30 yrs.) = P (Female) + P(over 30 yrs.) –P(female and over 30 yrs.) = $\frac{2}{5} + \frac{4}{5} - \frac{1}{5} = \frac{2+4+1}{5} = \frac{5}{5} = 1$

Hence, P(Female or over 30 yrs.) = 1

Multiplication Rule of Probability

Theorem states that if two events A and B are independent , the probability that both A and B will occur is equal to he product of individual probability of A and B . If A and B events are independent, then

Symbolically, P(A and B) = P(A) x P(B)

The theorem can be extended to three or more independent events

If there are three events A, B and C, then

P(A , B and C) = P(A) x P(B) x P (C)

Example 1. : Sex of birth child and Rh-factor are independent events. What will be the probability of a single child being male and Rh^+

$P(♂) = \frac{1}{2} = P_1$, $P(Rh^{+}) = \frac{9}{10} = P_2$, $P(\text{Single birth}) = \frac{79}{80} = P_3$

Hence, probability of child being male and $Rh^+ = P_1 \times P_2 \times P_3$

$$= \frac{1}{2} \times \frac{9}{10} \times \frac{79}{80} = \frac{711}{1600}$$

Examples of Multiplication Rule of Probability

Example 2: A man wants to marry an Indian girl having following qualities (1) White complexion, probability of getting such a girl is one in twenty (2) employed with handsome salary, probability of getting this is one in fifty (3) westernized manners and etiquettes, probability of getting such a girl is one in hundred. Find out the probability of getting to such a girl when the possession of these three attributes is independent.

Soln.

Probability of a girl with white complexion $= \frac{1}{20}$

Probability of a girl with handsome salary $= \frac{1}{50}$

Probability of a girl with westernized manner $= \frac{1}{1oo}$

Since the events are independent , the probability of simultaneous occurrence of all these qualities: $\frac{1}{20}$ x $\frac{1}{50}$ x $\frac{1}{1oo}$ $= \frac{1}{100000}$ = 0.00001

Examples of Multiplication Rule of Probability

Example 3: When two children are born one after the other, the possible sequences will be any of the following:

S.N.	Sequence	Probability
1.	M and M	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
2.	M and F	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
3.	F and M	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
4.	F and F	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Chance of getting one of either sex will be total of probability of 2^{nd} and 3^{rd}

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ = 50 %

Examples of Multiplication Rule of Probability

Conditional Probability:

Multiplication theorem explained is not applicable in case of dependent events.

Two events A and B are said to be dependent when B can occur only when A is known to have occurred or vice-versa. Here conditional probability is applied.

If two events A and B are dependent , then the conditional probability of B given A is

P(B\A) = $\frac{P\ (AB)}{P(A)}$

or P(AB) = P(B) x P(B\A)

Or P(AB) = P(A) x P(A\B)

Examples of Multiplication Rule of Probability

Example of Conditional Probability: In a class of elective course of Post Graduate class , there are 5 girls and 3 boys. Two students were called by HoD at random one after the other without replacement . Find the probability that both students are boys.

Sol^n: Probability of boy student in the first call

P(A) = $\frac{3}{5+3} = \frac{3}{8}$

Probability of calling the second boy student given that the first student was boy P(B\A) = $\frac{2}{5+2} = \frac{2}{7}$

Probability that both boy students were called

P(AB) = P(A) x P(B\A) = $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$

Examples of Mixed Rules of Probability

Example: There are three groups of children having 3 girls and 1 boy;2 girls and 2 boys and 1 girl and 3 boys . One child is selected from each group. Find the probability that the three children include 1 girl and 2 boys.

Sol^n: There may be 3 mutually exclusive events A, B and C.

Event A = Girl from 1st group , boys from 2nd & 3rd groups

Event B = Girl from 2nd group and boys from 1st and 3rd groups

Event C = Girl from 3rd group and boys from 1st and 2nd groups

$$P(A) = \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{9}{32}$$

$$P(B) = \frac{2}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$$

$$P(C) = \frac{1}{4} \times \frac{1}{4} \times \frac{2}{4} = \frac{1}{32}$$

All these events are mutually exclusive , therefore, the probability that any one of them happens is given below:

$$P(A \text{ or } B \text{ or } C) = \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$$

Importance of Probability

1. It is the foundation of the classical decision procedures of estimation and testing i.e. test of significance.
2. Probability models, particularly construction of economic models are very useful for making predictions.
3. It has become an indispensable tool for all the types of formal studies that involve uncertainty ex. Observation of phenotypes of the off springs, discussion about sex of unborn baby etc.
4. According to Ya-lun-Chou, a method of decision making under uncertainty , is found on probability theory.

Thanks