Reliability Theory

Introduction of Reliability

Reliability: One can find many definitions for reliability engineering, according to E.E.Lewis, "Reliability is probability that a component, device, equipment or a system will perform its intended function adequately for a specific period of time under a given set of conditions".

There are four elements of this definition.

(1) Probability **(2)** Intended function **(3)** Time
(4) Operating Conditions

In the other words, Reliability is defined to be the probability that a component on system will perform a required for a given period of time when used under stated operating conditions. It is the probability of a non failure overtime.

Reliability Function: Reliability in its simple form means the probability that is failure may not occurs in a given time interval (0 to t). Here T is a non-negative continuous random variables that denotes the time to failure of the system. It is denoted by $S(t)$ or $R(t)$.

$$R(t) = P[T > t] = 1 - P[T \leq t],$$

The probability of failure can be defined as $F(t) = P[T \leq t]$, where T denotes the failure time. Then $F(t)$ is the probability that the system will fail by time t.

- $R(t) = P[T > t] = 1 - P[T \leq t] = 1 - F(t)$
- $0 \leq F(t) \leq 1, \quad \text{so,} \quad 0 \leq R(t) \leq 1$
- $R(0) = 1$ and $\mathrm{R}(\infty) = 0$
- $R(t)$ is a non-increasing function between 0 to 1.

- $R(t) = 1 - F(t) = 1 - \int_0^t f(x)\,dx = \int_t^\infty f(x)\,dx$
- $f(t) = \dfrac{dF(t)}{dt} = -\dfrac{dR(t)}{dt}$

Ex: If $T \sim exp(\theta)$, then find $R(t)$.

Sol: We know that $f(t) = \frac{1}{\theta}\, e^{-t/\theta} \quad ; t \geq 0,$

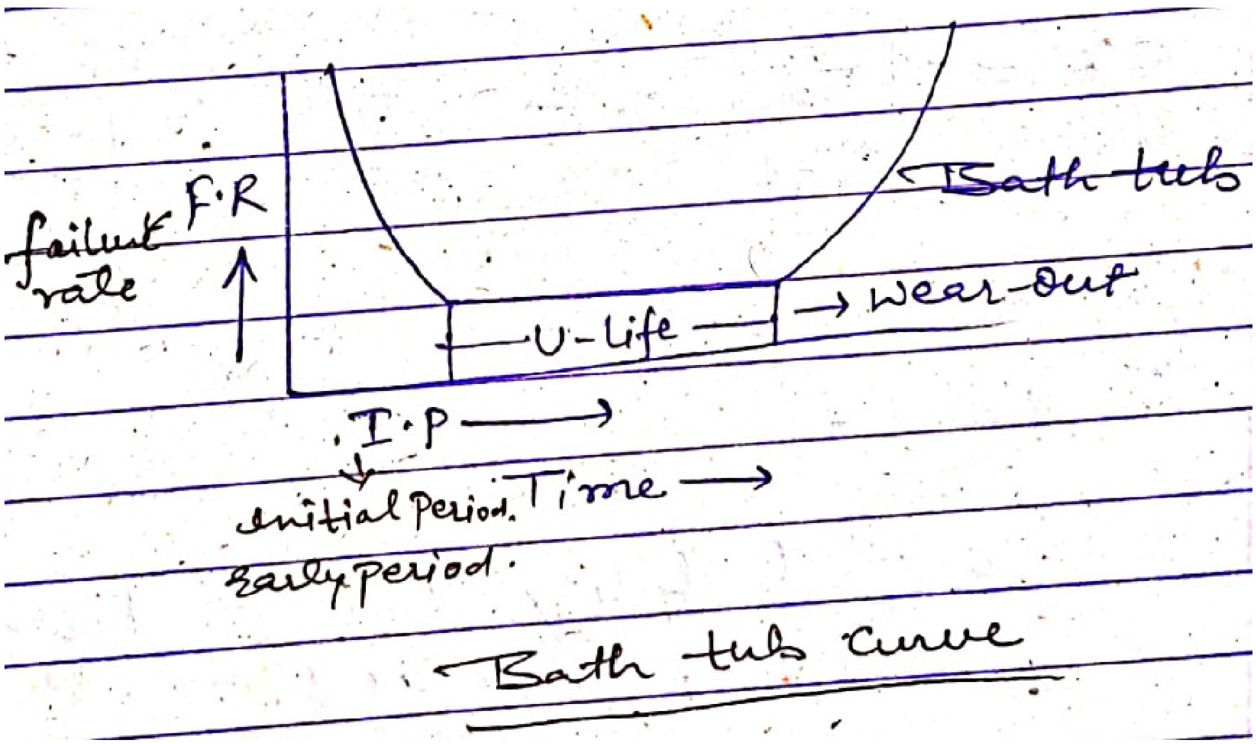
so $F(t) = 1 - e^{-t/\theta}$ then $R(t) = 1 - F(t) = e^{-t/\theta}$

For $t = 0, 1, 2, ...$

$R(0) = e^{-0/\theta} = 1$ and $R(1) = e^{-1/\theta}$

Failure and its types

Some components have well defined failures; others do not. In the beginning, when the item or component is installed, the item fails with high frequency, which is known as initial failure or infant mortality. These are generally due to manufacturing defects. They are very high at initial stages and gradually decreases and stabilize over a longer period of time. Stable or constant failures due to chance can be observed on an item for a longer period. These types of failures are known as random failures and characterized by constant number of failures per unit of time. Due to wear and tear with the usage, the item gradually deteriorates and frequency of failures again increases. These types of failures are called as wear-out failures. At this stage failure rate seems to be very high due to deterioration. Therefore the whole pattern of failures could be depicted by a bathtub curve.



Assignment

1. Find the Reliability function of the continuous random variable T that has the probability distribution

$$f_T(t) = \frac{\theta^3}{(\theta^2+2)}\,(1+t^2)\,e^{-\theta\,t}\ ,\ t>0,\ \theta>0,$$

2. Compute the Reliability function of the distribution defined by $f(t) = t\,e^{-t};\ t>0.$

3. Compute the Reliability function of the distribution defined by

$$f_T(t) = 2\,\alpha\,\beta^2\,t\,e^{-(t\,\beta)^2}\left\{1 - e^{-(t\,\beta)^2}\right\}^{(\alpha-1)}; \;\; t > 0.$$

4. Find the Reliability function of the continuous random variable T that has the probability distribution

$$f_T(t) = \frac{\theta^2}{(\theta^2+1)}\,(\theta + t)\,e^{-\theta\,t}\;,\; t > 0,\; \theta > 0$$

5. Compute the Reliability function of the distribution defined by

$$f_T(t) = \alpha\,\beta\,t^{-(\beta+1)}\,(1 + t^{-\beta})^{-(1+\alpha)};\; t > 0.$$

References

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