

M.Sc. I year, Sem II, CC-08, Distribution Theory
Question Bank (Model Questions)

Objective Questions

- (1) Out of Geometric, Gamma and Exponential distribution which do not have reproduction property?
(a) Geometric only (b) Geometric and Exponential (c) Gamma and Exponential (d) Exponential only
- (2) If $F(x, y)$ is the joint distribution function of X and Y then which one of the following is not correct?
(a) $F(\infty, y) = F(y)$ (b) $F(\infty, -\infty) = 0$ (c) $F(\infty, \infty) = 0$ (d) $F(\infty, \infty) = P(X \leq x)$
- (3) For large m and n , the mean of the number of runs under H_0 is
(a) $\frac{mn}{m+n}$ (b) $\frac{mn}{m+n} - 1$ (c) $\frac{m+n}{mn} + 1$ (d) $\frac{m+n+2mn}{m+n}$
- (4) The ratio of two independent standard normal variates is
(a) Normal (b) Gamma (c) Beta of Second Kind (d) Cauchy
- (5) If $X \sim \text{Gamma}(a, n)$ then the distribution of $2X$ is
(a) $\text{Gamma}(2a, n)$ (b) $\text{Gamma}(a/2, n)$ (c) χ_{2n}^2 (d) χ_n^2
- (6) If X and Y two independent random variable having their densities $f(x) = \frac{1}{6} x^3 e^{-x}$, $0 < x < \infty$ and $f(y) = \frac{1}{\sqrt{\pi y}} e^{-y}$, $y > 0$. The distribution of $X + Y$ is
(a) $\text{Gamma}(7/2)$ (b) $\text{Gamma}(9/2)$ (c) χ_4^2 (d) None of these
- (7) If X and Y are two Poisson variates such $X \sim P(1)$ and $Y \sim P(2)$, the probability $P(X + Y < 3)$ is
(a) e^{-3} (b) $3e^{-3}$ (c) $4e^{-3}$ (d) $8.5e^{-3}$
- (8) The distribution in which the probability at each successive draw varies is
(a) Hypergeometric distribution (b) Geometric distribution (c) Binomial distribution (d) Discrete uniform distribution
- (9) The joint pdf of X and Y is $f(x, y) = Ax^3y^3$, $0 \leq x, y \leq 2$. The value of A is
(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$
- (10) The inequality $f(E(X)) \leq E(f(X))$ where f is convex is known asinequality
(a) Jensen (b) Markov (c) holder (d) Liepnouff

(11) $t' = \frac{X}{\sqrt{Y/n}}$ is non-central t statistic if for independent X and Y

(a) $X \sim N(\mu, 1)$ and $Y \sim \text{central } \chi^2$ with n df (b) $X \sim N(\mu, \sigma^2)$ and $Y \sim \text{central } \chi^2$ with n df (c) $X \sim N(\mu, 1)$ and $Y \sim \text{non central } \chi^2$ with n df (d) $X \sim N(0, 1)$ and $Y \sim \text{non central } \chi^2$ with n df

(12) The mean of $X_{(r)}$ for sample of size n from $U(0, 1)$ is

(a) $\frac{1}{n}$ (b) $\frac{1}{n+1}$ (c) $\frac{r}{n}$ (d) $\frac{r}{n+1}$

Short Questions

- (1) If $X \sim N(0, 1)$. Find the distribution of $Y = X^2$.
- (2) If $(X_1, X_2, \dots, X_n) \sim U(0, \theta)$. Find the distribution of $X_{(r)}$.
- (3) Show that if X and Y are two independent Random variable the $V(aX + bY) = a^2V(X) + b^2V(Y)$.
- (4) Describe the method of transformation for obtaining the distribution of a function of random variable.
- (5) If X_1 and X_2 are independent $N(0, 1)$ variates then show that $X_1 + X_2$ and $X_2 - X_1$ are independently distributed.
- (6) A random variable X has the following probability distribution.

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

- (i) Find k . (ii) Evaluate $P(X < 6), P(X \geq 6), P(0 < X < 5)$
- (7) Determine the value of the constant k such that $f(x)$ as defined below is a pdf and also find Mean and Variance of X . $f(x) = kx(1-x); 0 < x < 1$
- (8) For the joint probability density $f(x, y) = 4xy e^{-(x^2+y^2)}$ for $x \geq 0, y \geq 0$. Show that X and Y are independent.
- (9) Show that negative Binomial distribution is a compound distribution of Poisson and Gamma distribution.
- (10) Find the mean and variance of a zero-truncated binomial distribution.

Long Questions

- (1) If X_1 and X_2 are independent $N(0, 1)$ variates, show that $X_1 + X_2$ and $X_2 - X_1$ are independently distributed and also identify their distribution.
- (2) If X_1 and X_2 are independent Gamma variates, then show that $(X_1 + X_2)$ and $\frac{X_1}{(X_1 + X_2)}$ are independently distributed.
- (3) Define order statistics and find the joint distribution of r^{th} and s^{th} order statistics. Hence find the distribution of range.
- (4) Define non-central Chi-square distribution and obtain its distribution. Also find its mean and variance.
- (5) Define non-central F statistic and obtain its distribution. Also find its mean and variance.
- (6) Obtain the asymptotic distribution of sample median from a random sample from $U(0, 1)$ population.