What is Radiation?

Course: MPHYCC-06 Electrodynamics and Plasma Physics
(M.Sc. Sem-II)

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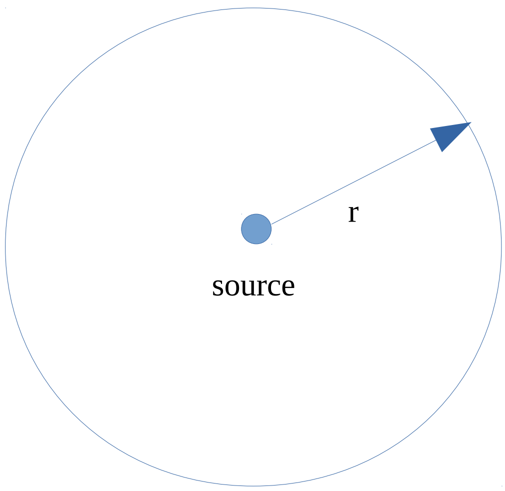
What do we understand by “Radiation” ?

In simple words, “radiation” is basically related to the electromagnetic (EM) waves and energy transported by EM waves. Unit-I of this course discusses about the EM wave and its properties. But what we didn’t discuss is that “what causes the EM wave at the first place?”. We know that the electric and magnetic fields are generated by electric charges and electric currents. Similarly, the sources of EM waves are also the moving charges and time-varying currents (*in the following, we will establish that stationary charges and steady current can’t produce EM waves*). Once generated, the EM wave can travel to infinity in vacuum from its originating point. Or, in other words, the EM waves can trasport energy to infinity from their sources and, hence, there is an irreversible flow of the energy from the source point to infinity.

To further eloborate, at some moment of time, lets assume the source of the EM waves is sitauted at the origin and we are interested to calculate the total power (energy per unit time) passing through a spherical surface of radius r as shown in the figure.



To calculate the power, we note that the Poynting vector **S** (the EM energy flowing through per unit area per unit time) is given as:

$$\boldsymbol{S} = \frac{\boldsymbol{E} \times \boldsymbol{B}}{\mu_0}$$

Therefore, the power P (energy passing through the spherical surface per unit time) is given by:

$$P(r)=\oint \boldsymbol{S}.d\boldsymbol{a}=\oint \left(\frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0}\right).d\boldsymbol{a} \qquad \text{----- (1)}$$

Then, the power radiated by the source (P_{rad}) is the value of P obtained in the limit of r tends to infinity (*imagine the sphere with a very very large radius and we are interested to know energy passing through its surface per unit time*) i.e.,

$$P_{rad}=\lim_{r\to\infty} P(r) \qquad \text{----- (2)}$$

This basically represents the EM energy (per unit time) trasported by the EM waves (generated at the source region) at infinity and this energy never return back to the source. We can also say that this much energy is radiated by the source situated at the origin.

To show that electrostatic (static charges) and magnetostatic (steady currents) fields don't lead to the radiation, we recall the expressions of electric (**E)** and magnetic (**B)** field generated by a point charge moving with an arbitrary velocity (**v**):

$$\boxed{\mathbf{E}(\mathbf{r},t)=\frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\boldsymbol{\mathscr{r}}\cdot\mathbf{u})^3}[(c^2-v^2)\mathbf{u}+\boldsymbol{\mathscr{r}}\times(\mathbf{u}\times\mathbf{a})].} \qquad \text{---- (3)}$$

$$\boxed{\mathbf{B}(\mathbf{r},t)=\frac{1}{c}\hat{\boldsymbol{\mathscr{r}}}\times\mathbf{E}(\mathbf{r},t).} \qquad \text{----- (4)}$$

where

$$\mathbf{u}\equiv c\hat{\boldsymbol{\mathscr{r}}}-\mathbf{v}$$

Now for a static charge **v**=0 and **a**=0. As a result, from equation (3), the electric field **E** by the static charge located at origin is

$$\boldsymbol{E}(\boldsymbol{r},\boldsymbol{t})=\frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

This is exactly the same expression which we get from the Coloumb's law. Here magnitude of **E** decreases like ($1/r^2$). Similarly, for a steady current (i.e., **v**=constant and **a**=0), from equation (4), we get the expression of magnetic field **B** which we also get from the Bio-Savart law. The magnetic field strength (B) also falls off like $(1/r^2)$. So, for the electrostatic and the magnetostatic condition, the Poynting vector magnitude (S), being product of E and B, decreases like $(1/r^4)$. Moreover, we know that the area of spherical surface is $4\Pi r^2$. From equation (1), we can infer that for a given radius r, the power is multiplication of the S and the area. As a result, power P is proportional to $(1/r^2)$. Now for calculating P_{rad}, we need to apply the limit of r tends to infinity at P. Since as r tends to infinity, P being proportional to $(1/r^2)$ vanishes. Hence P_{rad} also vanishes. In other words, *no power is radiated by static charge and current sources.*

Further to illustrate that it is the acceleration (**a**) which is responsible for the radiation, we again focus on equations (3) and (4). First lets closely inspect the expression of electric field (equation (3)). The electric field is made of two kind of terms: $\mathbf{E}=\mathbf{E_c}+\mathbf{E_r}$. The first term (denoted by $\mathbf{E}_c$) is:

$$\boldsymbol{E_c}=\frac{qr}{4\pi\varepsilon_0(\boldsymbol{r}.\boldsymbol{u})^3}(c^2-v^2)\boldsymbol{u} \qquad \text{----- (5)}$$

and the second term (denoted by $\mathbf{E}_r$) is:

$$\boldsymbol{E_r}=\frac{qr}{4\pi\varepsilon_0(\boldsymbol{r}.\boldsymbol{u})^3}(\boldsymbol{r}\times\boldsymbol{u}\times\boldsymbol{a}) \qquad \text{----- (6)}$$

Note that, in equation (5), the expression of $\mathbf{E_c}$ has r in numerator and r^3 in denominator. As a result, we can infer that the magnitude of the first term (E_c) is proportional to $(1/r^2)$. This proportionality is identical to the Coulomb field. That's why the first term is known as "the coulomb field". From equation (6), the expression of $\mathbf{E_r}$ has r^2 in numerator and r^3 in denominator. Thus, the magnitude of the second term (E_r) is proportional to (1/r). In a moment, we will show that the second term is responsible for giving rise the radiation. That's why the second term is known as "the radiation field". Similarly, the magnetic field (equation (4)) can also be decomposed into two terms: $\mathbf{B}=\mathbf{B_c}+\mathbf{B_r}$. Where first term magnitude (B_c) is proportional to $(1/r^2)$ and known as "the Coulomb field". While second term magnitude (B_r) is proportional to (1/r) and known as "the radiation field". Now, the Poynting vector **S** is

$$\mathbf{S} = (1/\mu_0)\,(\mathbf{E_c}+\mathbf{E_r}) \times (\mathbf{B_c}+\mathbf{B_r})$$

$$=(1/\mu_0)\,(\mathbf{E_c}\times\mathbf{B_c} + \mathbf{E_c}\times\mathbf{B_r} + \mathbf{E_r}\times\mathbf{B_c} + \mathbf{E_r}\times\mathbf{B_r})$$

↓ ↓ ↓ ↓

$(1/r^4)$ $(1/r^3)$ $(1/r^3)$ $(1/r^2)$

For calculating the power, we refer to equation (1) and infer that for a given radius r, the power is multiplication of S and area ($4\Pi r^2$). Since we have four terms in **S**, we also get four term in P. First, second, third and fourth terms of power (P) obtained from product of $\mathbf{E_c} \times \mathbf{B_c}$, $\mathbf{E_c} \times \mathbf{B_r}$, $\mathbf{E_r} \times \mathbf{B_c}$ and $\mathbf{E_r} \times \mathbf{B_r}$ with area are proportional to ($1/r^2$), (1/r), (1/r) and 1 as depicted below:

$$P(r)=\frac{1}{\mu_0}\oint\left((\boldsymbol{E_c}\times\boldsymbol{B_c}).d\boldsymbol{a}+(\boldsymbol{E_c}\times\boldsymbol{B_r}).d\boldsymbol{a}+(\boldsymbol{E_r}\times\boldsymbol{B_c}).d\boldsymbol{a}+(\boldsymbol{E_r}\times\boldsymbol{B_r}).d\boldsymbol{a}\right)$$

($1/r^2$) (1/r) (1/r) (1)

Now for calculating P_{rad}, we need to apply the limit of r tends to infinity at P. Since as r tends to infinity, the first three terms (being proportional to $1/r^2$ and 1/r) vanish. Only the last term which corresponds to $\mathbf{E_r} \times \mathbf{B_r}$ of the Poyting vector (**S**) survives. As a result, we get non-vanishing P_{rad} . In other words, the radiation fields (which depends on the acceleration (**a)** of the charged particle) are responsible for the radiation or we can say that "*the accelerating charged particle emits radiation*."

In the next lecture, we will calculate the expression of power radiated by an accelerated charged particle.

Referece:

"Introduction to Electrodynamics" by David J. Grifitths.

Thanks for the attention!