Redioactive Decay-II

Course: MPHYCC-13 Nuclear and Particle Physics
(M.Sc. Sem-III)

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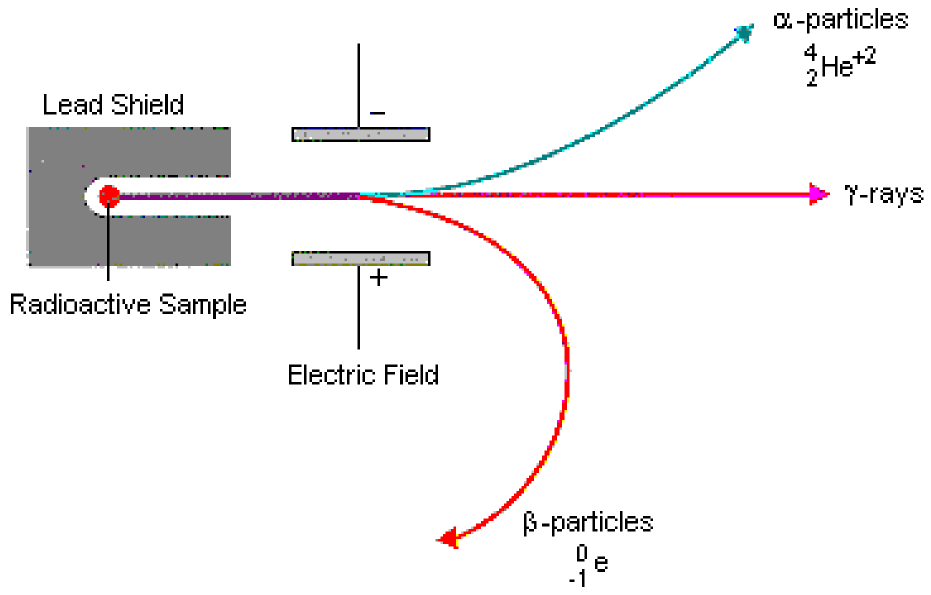
Radioactive Decay

- Effect of electric field on the radiation (alpha, beta and gamma particles) emitted by radioactive material:



- The process of radioactive decay, being spotaneous, is statistical in nature, and we can only tell about the evolution of the expectation values of quantities of interest (as we used to do in quantum mechanics), for example the number of atoms that decay per unit time. Hence, we can say that these radioactive decays are describing quantum processes, i.e. transitions among two quantum states (we will know more about this when we study each type of decay in details).
- If we observe a single unstable nucleus (X), we cannot know beforehand when it will decay to its daughter nuclide (Y).

X (parent nucleus) --> Y (daughter nucleus) + decay particle (radiation)

The time at which the decay happens (i.e. X convets to Y) is random, thus at each instant we can have the parent nuclide with some probability p and the daughter with probability $1 - p$ i.e., we only say that, at time t, the probability of the nucleus to be X is p and the probability of the nucleus to be Y is *p-1*. Qautum mechanically, we consider X as one quantum state and Y as another quantum state than we can calculate *p* by calculating the transition probability from X to Y by using time-dependent perturbation theory (this we are going to do in upcoming lectures).

- However, if we experimentally observe a collection of radioactive nuclei, we can predict at each instant the average number of parent and daughter nuclides.
- If we take a sample of radioactive material and have the number of radioactive nuclei N at given instant of time and the number of decaying atoms per unit time is dN/dt. Experimetally, it is found that this rate remains constant in time and it is proportional to the number of nuclei (N) themselves:

$$\frac{dN}{dt} \alpha N$$

$$\frac{dN}{dt} = -\lambda N(t) \qquad \text{-------------- (1)}$$

The constant of proportionality λ is called the **decay constant**. If we re-write the above equation as:

$$\lambda = -\frac{dN/dt}{N(t)}$$

From this equation, we can infer that the RHS can be related to the probability per unit time for one atom to decay. Note that this probability per time is a constant and, hence, it is a characteristic of all radioactive decay.

- From equation (1), we can derive the *exponential law of radioactive decay:*

$$N(t) = N_0 e^{-\lambda t} \qquad \text{-------------- (2)}$$

where N_0 represents the number of parent nuclei at t=0.

- **Mean lifetime**:

  Lets assume N_1 nuclei has lifetime t_1, N_2 nuclei has lifetime t_2, N_3 nuclei has lifetime t_3 ..... and N_n nuclei has lifetime t_n where total number of nuclei $N = N_1 + N_2 + ...+N_n$. Then the average lifetime or Mean lifetime can be given as:

$$\tau = \frac{t_1 N_1 + t_2 N_2 + t_3 N_3 + ... + t_n N_n}{N_1 + N_2 + N_3 + .. N_n}$$

$$\tau = \frac{\sum t_i N_i}{\sum N_i}$$

with large n, we may re-write the above equation as:

$$\tau = \frac{\int t\, dN}{\int dN} \qquad \text{--------------(3)}$$

where dN represents the number of nuclei decaying between t to t+dt time. From equation (2)

$$dN = -N_0 \lambda\, e^{-\lambda t}\, dt$$

Now equation (3) becomes:

$$\tau = \frac{\int_0^\infty t\, e^{-\lambda t}\, dt}{\int_0^\infty e^{-\lambda t}\, dt} = \frac{(1/\lambda^2)}{(1/\lambda)}$$

Therefore, **mean lifetime** $\tau = \frac{1}{\lambda}$

- **Half-life:** Half-life is the time at which half of the initial nuclei (N_0) undergo radioactive decay. It is generally denoted by $t_{1/2}$. So if at t=0, we have N_0 nuclei that will decay. Then, from equation (2), at $t=t_{1/2}$, $N(t)= N_0/2$. As a result at $t=t_{1/2}$, we have

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

Half-life $$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- Relation between mean-life and half-life:

$$\tau = \frac{t_{1/2}}{\ln 2} = \frac{t_{1/2}}{0.693}$$

- **Activity (A)** : The product of the decay constant λ and the number of unstable nuclei at any instant of time (N) is called "Activity".

$$A(t) = \lambda\, N(t) = \lambda\, N_0\, e^{-\lambda t} \quad \text{----------- (4)}$$

Also from equation (1): $\frac{dN}{dt} = -\lambda N(t) = -A$ the activity determines the rate of decay, i.e., number of nuclei decaying per unit time.

Unit of activity of 1gm sample material is:

1 Curie (1Cu) = 3.7 x 10^{10} decays/ sec

Sometimes unit "Rutherford" is also used which is given as:

1 Rd = 10^6 decays/sec

- **Radioactive Decay Chain**: So far we have considered the decay of a radioactive nuclide in which the product of the decay, the daughter, is stable. But, what happens if the daughter itself is also radioactive?

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$$

Let's assume that N_A (t) is the number of nuclei of the original radioactive nuclide (the parent) A and its decay constant λ_A.

Similarly, consider that that $N_B(t)$ is the number of nuclei of the radioactive nuclide (daughter of first radioactive decay process) B and its decay constant λ_B.

Now, lets suppose that at t=0 there are N_0 nuclei of A. ***Our aim is to find the numbers of nuclei of A and B at some later time t.***

[Remark: We can easily answer this problem if B would have been stable i.e.,

$$A \xrightarrow{\lambda_A} B \ \text{(stable)}$$

We would have used equation (1)

$$\frac{dN_A}{dt} = -\lambda_A N_A(t)$$

to get $N_A(t) = N_0 e^{-\lambda_A t}$ and $N_B(t) = N_0 - N_A(t)$.]

Note that the considered radioactive decay chain, for the original nuclide A, as usual we have

$$\frac{dN_A}{dt} = -\lambda_A N_A(t)$$

and

$$N_A(t) = N_0 e^{-\lambda_A t}$$

But for the unstable daughter nuclide B, the decay equation modifies as:

$$\frac{dN_B}{dt} = -\lambda_B N_B(t) + \lambda_A N_A(t)$$

Rate at which number of nuclei of B are changing ($\frac{dN_B}{dt}$)

Rate at which number of nuclei of B are decaying ($-\lambda_B N_B(t)$)

Rate at which number of nuclei of B are forming due decay of nuclei of A ($\lambda_A N_A(t)$)

Now to calculate $N_B(t)$, we need to solve above equation. For this we put the $N_A(t)$ in the above equation to get:

$$\frac{dN_B}{dt} = -\lambda_B N_B(t) + \lambda_A N_0 e^{-\lambda_A t}$$

To solve this we can re-arrange the equation as:

$$\frac{dN_B}{dt} + \lambda_B N_B(t) = \lambda_A N_0 e^{-\lambda_A t}$$

Note that the equation has a form similar to

$$\frac{dy}{dt} + P\, y = Q(t)$$

Solution is then obtained as:

$$Iy = \int IQ(t)\, dt$$

where I is known as integrating factor and given as

$$I = e^{\int P\, dt}$$

For the equation of N_B, integrating factor

$$I = e^{\int \lambda_B dt} = e^{\lambda_B t}$$

Then, we have $$I\, N_B = \int I\, \lambda_A N_0 e^{-\lambda_A t}\, dt$$

$$e^{\lambda_B t} N_B = \lambda_A N_0 \int e^{\lambda_B t} e^{-\lambda_A t}\, dt$$

$$e^{\lambda_B t} N_B = \frac{\lambda_A N_0}{\lambda_B - \lambda_A} e^{(\lambda_B - \lambda_A) t} + C_1$$

where C_1 is constant of integration

Now,

$$N_B(t)=\frac{\lambda_A N_0}{\lambda_B-\lambda_A}e^{-(\lambda_A)t}+C_1 e^{-\lambda_B t}$$

Now, to determine C_1, we utilize the initial condition that at t=0, number of nuclei of B is zero i.e., N_B (t=0) = 0. As a result,

$$C_1=-\frac{\lambda_A N_0}{\lambda_B-\lambda_A}$$

Finally, we get

$$N_B(t)=\frac{\lambda_A N_0}{\lambda_B-\lambda_A}e^{-\lambda_A t}-\frac{\lambda_A N_0}{\lambda_B-\lambda_A}e^{-\lambda_B t}$$

$$N_B(t)=\frac{\lambda_A N_0}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)$$

If we want to calcuate the activity, then we use the usual expression.

Activity for A = λ_A N_A = $\lambda_A N_0 e^{-\lambda_A t}$

Activity for B = λ_B N_B

$$Activity\ for\ B=\frac{\lambda_A \lambda_B N_0}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)$$

$$\frac{Activity\ for\ B}{Activity\ of\ A}=\frac{\lambda_B}{\lambda_B-\lambda_A}\left(1-e^{-(\lambda_B-\lambda_A)t}\right)$$

If the activity of A is equal to the activity of B, then this situation is called “ideal equilibrium”. Lets assume at time t=t_c, the activity of A becomes equal to the activity of B, then from the above equation, we can get:

$$\frac{\lambda_B}{\lambda_B-\lambda_A}\left(1-e^{-(\lambda_B-\lambda_A)t_c}\right)=1$$

$$\left(1-e^{-(\lambda_B-\lambda_A)t_c}\right)=\frac{\lambda_B-\lambda_A}{\lambda_B}$$

$$e^{-(\lambda_B-\lambda_A)t_c}=1-\frac{\lambda_B-\lambda_A}{\lambda_B}$$

$$t_c=\frac{\ln\left(\frac{\lambda_B}{\lambda_A}\right)}{(\lambda_B-\lambda_A)}$$

Read about: Radioactivity dating which is used to determine the ages of the rocks, the age of Earth and the age of mineral and organic material.
In particular, read about RadioCarbon dating to get an idea of: how radioactivity dating works.

Few problems related to radioactive decay (home work!):

Q. 1 In the uranium radioactive series, the initial nucleus is $_{92}U^{238}$ and the final nucleus is $_{82}Pb^{206}$. When the uranium nucleus decays to lead, the number of α-particle emitted will be

(A) 1 (B) 2

(C) 4 (D) 8

(Hint: Focus on mass number)

Q. 2 The activity of a radioactive sample is 1.6 curie, and its half-life is 2.5 days. Its activity after 10 days will be

(A) 0.8 curie (B)0.4 curie

(C) 0.1 curie (D) 0.16 curie

(Hint: expression the formula of activity in terms of half-life and then use it)

Q. 3 In a mean life of a radioactive sample:

(A) About 1/3 of substance disintegrates

(B) About 2/3 of the substance disintegrates

(C) About 90% of the substance disintegrates

(D) Almost all the substance disintegrates.

Q. 4 Half-life of an element is 30 days. How much part will remain after 90 days?

(A) 1/4th part (B) 1/16 part

(C) 1/8th part (D) 1/3rd part

Q. 5 If half-life of a substance is 3.8 days & its initial quantity is 10.38 gm, then quantity of substance reaming after 19 days will be:

(A) 0.151 gm (B) 0.32 gm

(C)1.51 gm (D) 0.16 gm

Thanks for the attention!