Beta (β) - Decay

Course: MPHYCC-13 Nuclear and Particle Physics
(M.Sc. Sem-III)

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Important remark on Q-value

Now, we define the **Q value** (also known as disintegration energy) which is basically the difference between the initial and final nuclear mass energies. In other words, the Q value is calculated by multiplying the mass difference of parent nucleus and daughter products (daughter nucleus and decay particles) to c^2. This is the net energy released during a decay process.

To further explain the physical significance of Q-value, we consider a following decay process

$$X \quad \text{------->} \quad Y \quad + \quad d \text{ (decay particle)}$$

Now, note that every decay process should satisfy the conservation of energy, linear momentum, and angular momentum.

Let's first consider the conservation of energy in the a decay process. We assume the initial decaying nucleus X to be at rest.
Then the energy of the initial system is just the rest mass energy of X which is given by $m_n(X)c^2$.

The final state consists of daughter nucleus Y and decay particle d, each of which will be in motion (to conserve linear momentum). Thus, the final total energy is $m_n(Y)c^2 + T_Y + m_d c^2 + T_d$, where T represents the kinetic energy of the final particles.

Therefore, the conservation of energy gives

$$m_n(X)c^2 = m_n(Y)c^2 + T_Y + m_d c^2 + T_d$$

$$T_Y + T_d = [\, m_n(X) - m_n(Y) - m_d \,]c^2$$

We know that RHS of the above equation represents the Q-value of the decay process.

$$Q = [\, m_n(X) - m_n(Y) - m_d \,]c^2$$

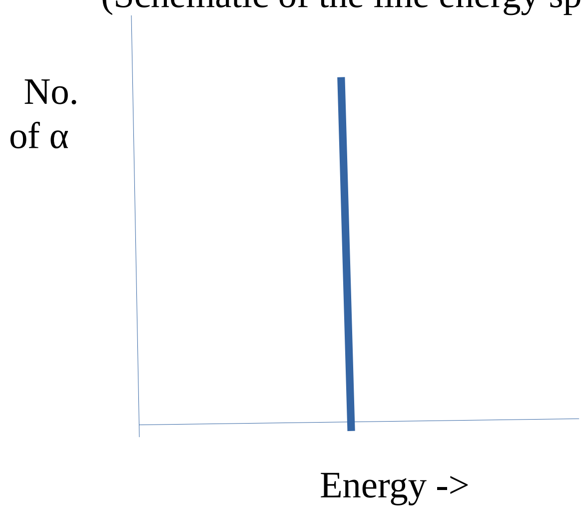
As a result, the Q value is also equal to the total kinetic energy achieved by the decay fragments (Y and d):

$$Q = T_Y + T_d$$

If the original nucleus X is at rest, then its linear momentum is zero, and conservation of linear momentum then requires that Y and d move with equal and opposite momenta in order that the final momentum also be zero.

The kinetic energy of duaghter nucleus Y (T_Y) is called the recoil energy. In general, the dughter nucleus is much heavier than the decay partilce and, therefore, $T_Y << T_d$ i.e., most of the decay energy appears in the form of kinetic energy of the decay particle. For example, $^{212}_{83}Bi$ ----- > $^{208}_{81}Tl + \alpha$ Q-value for above decay process is **6.20 MeV** out of which **6.08 MeV** appear as kinetic energy of the α-particle and **0.12 MeV** as kinetic energy of $^{208}_{81}Tl$. Also, it is important to note that the α-particle has a sharp line spectrum of energy i.e., all the α-particles are emitted with almost same kinetic energy.

(Schematic of the line energy spectrum)



Beta Decay

The beta (β) -decay is a spontaneous process in which an unstable nucleus transforms into an isobar nucleus by changing Z to Z+1 or Z-1 and N to N-1 and N+1 to ensure a constant A.

During the β-decay process, an electron or positron (known as beta particle) is emitted from the nucleus by transforming an neutron to proton or a proton to neutron. Along with the electron or positron, an antineutrino or a neutrino is also emitted.

In addition to these, sometimes, the capture of an electron from its atomic orbital by the nucleus is observed. All these process collectively known as β-decay process.

Therefore, the basic β-decay processes can be represented as:

(i) β^- - decay

$$^{A}_{z}X \longrightarrow {}^{A}_{z+1}Y + {}^{0}_{-1}e + \upsilon_{\text{anti-neutrino}}$$

$$(\ n \longrightarrow p + e^- + \upsilon_{\text{anti-neutrino}}\ )$$

(ii) β^+ - decay

$$^{A}_{z}X \longrightarrow {}^{A}_{z-1}Y + {}^{0}_{1}e + \upsilon_{\text{neutrino}}$$

$$(\ p \longrightarrow n + e^+ + \upsilon_{\text{neutrino}}\ )$$

(iii) Electron capture

$$^{A}_{z}X + e^- \longrightarrow {}^{A}_{z-1}Y + \upsilon_{\text{neutrino}}$$

$$(p + e^- \longrightarrow n + \upsilon_{neutrino})$$

Note: Neither the beta particle nor its associated (anti-) neutrino exist within the nucleus prior to beta decay, but are created in the decay process.

Energy release in β Decay

Initially, when the β-decay was discovered in 1900, the decay process was assumed to similar to α-decay i.e., only one decay particle is emitted along the daughter nuclei. People didn't know about the existance of neutrino at that time.

Here first we will also consider that in the β-decay process only beta particle is emitted and then will see what difficulty arises if we don't consider the neutrino.

To estimate the energy released during such a beta decay process in which presence of neutrino is not considered, first consider the case of β^- decay.

(i) Energetics of β^- decay

We consider the β^- decay given as:

$$^{A}_{z}X \longrightarrow {}^{A}_{z+1}Y + {}^{0}_{-1}e$$

(Once again note that we are not considering neutrino. Keep this in mind!)

The Q value for the above nuclear reaction is obtained as:

$$Q = [\, m_n(^{A}_{z}X) - m_n(^{A}_{z+1}Y) - m_e \,]\, c^2 \qquad \text{------ (1)}$$

where $m_n(^A_zX)$ -> nuclear mass of the parent nuclei
$m_n(^A_{z+1}Y)$ -> nuclear mass of the daughter nuclei
m_e -> mass of electron/positron (beta particle)

As instead of nuclear masses, atomic masses are easily available. Therefore, we re-write the above expression of the Q value in terms of atomic masses by replacing nuclear masses in atomic masses as:

$$m_n(^A_zX) = m(^A_zX) - Z\,m_e$$
$$m_n(^A_{z+1}Y) = m(^A_{z+1}Y) - (Z+1)\,m_e$$

where $m(^A_zX)$ and $m(^A_{z+1}Y)$ represent atomic masses of parent and daughter nuclei respectively.

Using these atomic masses in equation (1), we get:

$$Q = [\,\{m(^A_zX) - Z\,m_e\} - \{m(^A_{z+1}Y) - (Z+1)\,m_e\} - m_e\,]\,c^2$$

$$Q = [\,m(^A_zX) - m(^A_{z+1}Y)\,]\,c^2$$

Hence, the Q-value can be given as:

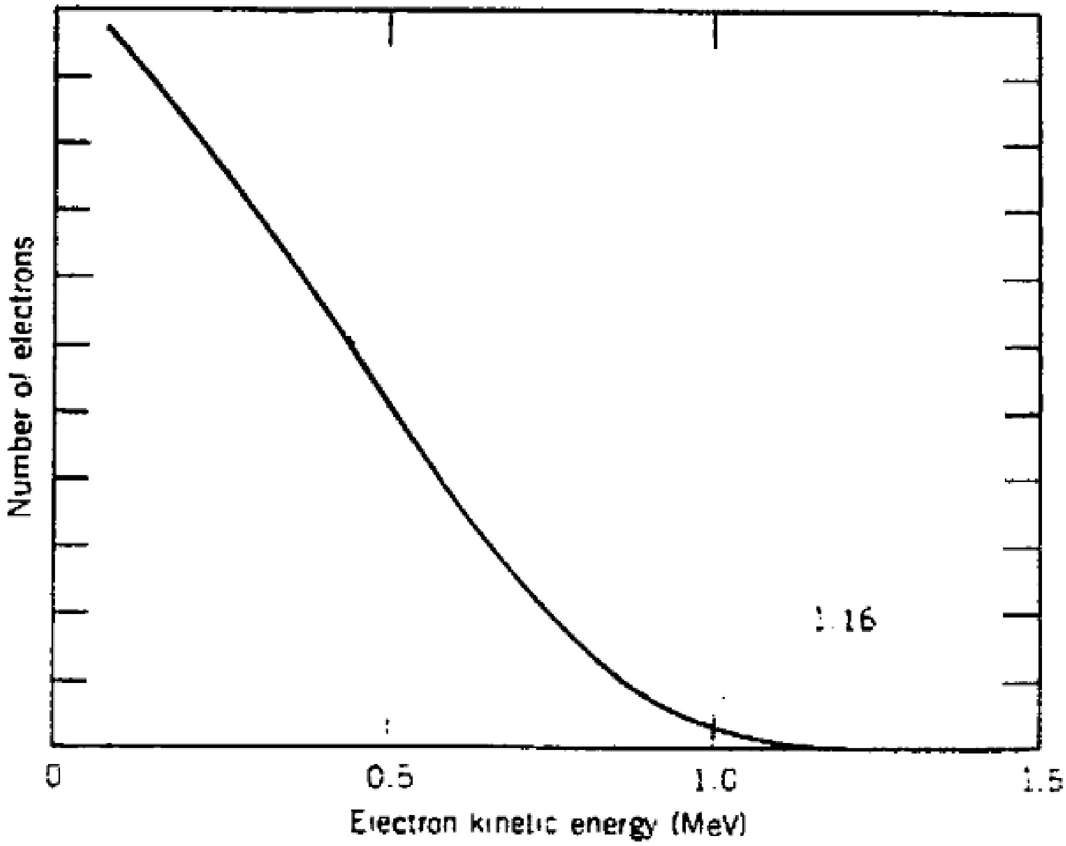
$$Q = [\,m(^A_zX) - m(^A_{z+1}Y)\,]\,c^2 \qquad \text{----- (2)}$$

This energy Q is released in the form of kinetic energy (recoil energy) of the duaghter nuclei $^A_{z+1}Y$ (T_Y) and kinetic energy of electron (T_e). And since duaghter nuclei is much heavier than the electron, $T_Y << T_e$ and we can say:

$$T_e \approx Q$$

For instance, we might expect on the basis of nuclear mass differences that the β particles from ^{210}Bi would be emitted with a kinetic energy of 1.16 MeV.

Yet experimentally we find a continuous distribution of beta particles energy from 0 up to 1.16 MeV.



In the absence of the discovery of neutrinos, the continuous energy distribution of the β-decay electrons was a puzzling experimental result in the 1920s. Alpha particles are emitted with sharp, well-defined energies equal to the difference in mass energv between the initial and final states (less the small recoil corrections). On the other hand, beta particles have a continuous distribution of energies, from zero up to an upper limit (known as the endpoint energy) which is equal to the energy difference between the initial and final states.

To explain this puzzling continuous energy distributation of beta particles, Pauli in 1931 proposed the presence of a new particle which is emitted as a second particle (first is electron) in the decay process. Later Fermi named the new particle as "neutrino". Neutrino is found to be an electrically neutral spin-1/2 particle.

Now with the presence of neutrino, the decay equation becomes:

$$ {}^{A}_{z}X \longrightarrow {}^{A}_{z+1}Y + {}^{0}_{-1}e + \upsilon_{\text{anti-neutrino}} $$

The Q value for the above nuclear reaction is obtained as:

$$ Q = [\, m_n({}^{A}_{z}X) - m_n({}^{A}_{z+1}Y) - m_e - m_\upsilon \,]\, c^2 $$

where m_υ -> mass of neutrino/anti-neutrino

In terms of atomic masses, we get:

$$ Q = [\, m({}^{A}_{z}X) - m({}^{A}_{z+1}Y) - m_\upsilon \,]\, c^2 $$

Mass of neutrino is very small and negligible in comparison to the atomic masses. Hence, the Q-value can be given as:

$$ Q \approx [\, m({}^{A}_{z}X) - m({}^{A}_{z+1}Y) \,]\, c^2 $$

Now note that this energy Q is released in the form of kinetic energy of the duaghter nuclei ${}^{A}_{z+1}Y$ (T_Y), kinetic energy of electron (T_e) and antineutrino (T_υ). Now, $T_Y << T_e$, $T_Y << T_\upsilon$. This implies that the disintegration energy is distributed as kinectic energy of beta particles (electrons) and kinetic energy of antineutrinos.

$$ Q \approx T_e + T_\upsilon $$

Hence, the antineutrino and electron will then share the decay energy, which accounts for the continuous electron spectrum. The maximum-energy electrons correspond to minimum-energy antineutrinos and when the antineutrinos have vanishingly small energies Q $\approx$ $(T_e)_{max}$ which is **the end-point energy**.

(ii) Energetics of β^+ decay

$$^{A}_{z}X \longrightarrow \, ^{A}_{z-1}Y + \, ^{0}_{1}e + \upsilon_{neutrino}$$

The Q value for the above nuclear reaction is obtained as:

$$Q = [\, m_n(^{A}_{z}X) - m_n(^{A}_{z-1}Y) - m_e - m_\upsilon \,]\, c^2$$

In terms of atomic masses, we get:

$$Q = [\, \{m(^{A}_{z}X) - Z\, m_e\} - \{m(^{A}_{z-1}Y) - (Z-1)\, m_e\} - m_e - m_\upsilon \,]\, c^2$$

$$Q = [\, m(^{A}_{z}X) - m(^{A}_{z-1}Y) - 2\, m_e - m_\upsilon \,]\, c^2$$

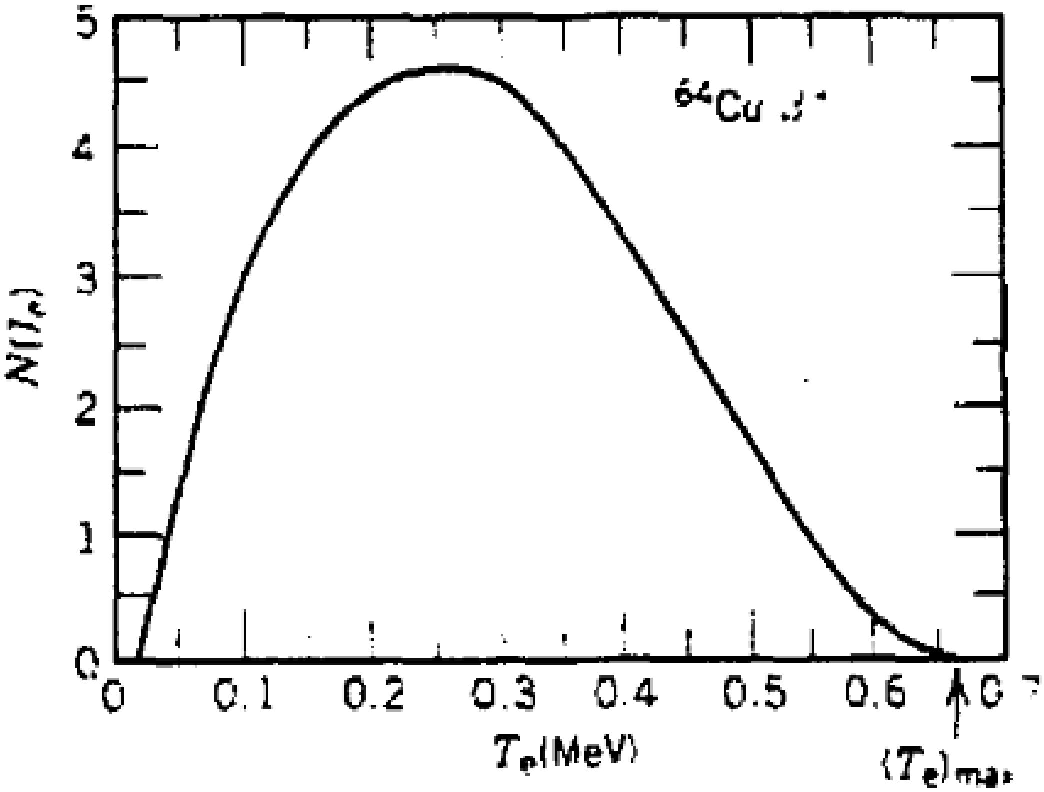
Mass of neutrino is very small and negligible in comparison to the atomic masses. Hence, the Q-value can be given as:

$$Q \approx [\, m(^{A}_{z}X) - m(^{A}_{z-1}Y) - 2\, m_e]\, c^2$$

This is the energy which is librated in the form of kinetic energy of the duaghter nuclei $^{A}_{z-1}Y$ (T_Y), kinetic energy of positron (T_e) and neutrino (T_υ). Now, $T_Y \ll T_e$, $T_Y \ll T_\upsilon$. Hence, the disintegration energy is distributed as kinectic energy of beta particles (positron) and kinetic energy of neutrinos.

$$Q \approx T_e + T_\upsilon$$

Like β^- decay, the positron and neutrino will then share the decay energy, which accounts for the continuous beta spectrum. The maximum-energy positron correspond to minimum-energy neutrinos and when the neutrinos have vanishingly small energies $Q \approx (T_e)_{max}$ which is **the end-point energy**.



Kinetic energy spectra of positrons emitted in the decay of ^{64}Cu.

(iii) Energetics of electron capture

$$ {}^{A}_{z}X + {}^{0}_{-1}e \longrightarrow {}^{A}_{z-1}Y + \; + \upsilon_{neutrino} $$

The Q value is:

$$ Q = [\, m_n({}^{A}_{z}X) + m_e - m_n({}^{A}_{z-1}Y) - m_\upsilon \,]\, c^2 $$

In terms of atomic masses:

$$ Q = [\, \{m({}^{A}_{z}X) - Z\, m_e\} + m_e - \{m({}^{A}_{z-1}Y) - (Z-1)\, m_e\} - m_\upsilon \,]\, c^2 $$

$$Q = [\, m(^{A}_{z}X) - m(^{A}_{z-1}Y) - m_{\upsilon} \,]\, c^2$$

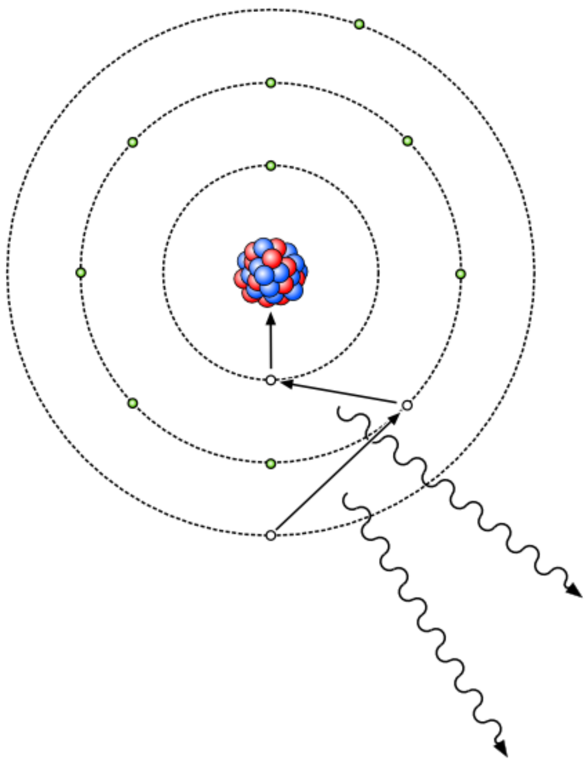
$$Q = [\, m(^{A}_{z}X) - m(^{A}_{z-1}Y) - m_{\upsilon} \,]\, c^2$$

Mass of neutrino is very small and negligible in comparison to the atomic masses. Hence, the Q-value can be given as:

$$Q \approx [\, m(^{A}_{z}X) - m(^{A}_{z-1}Y) \,]\, c^2$$

Note that in both β^+-decay and electron capture processes, a proton gets transformed into neutron but the Q-values of both the processes are different. In electron capture, we have the two-body final state results in unique values for the recoil energy (T_Y) and kinetic energy of neutrino T_υ. Neglecting the recoil, monoenergetic neutrinos with energy Q are emitted.

<u>Note:</u> If in the electron capture process, the capture of electron takes place from an inner shell of the atom, the K shell for instance, an electronic vacancy in that shell is created. The vacancy is quickly filled as electrons from higher shells make downward transitions and emit characteristic X rays.



Thanks for the attention!