MPHYEC-1F Measurement and Instrumentation

Unit 3 Notes (2)

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Lock in Amplifier

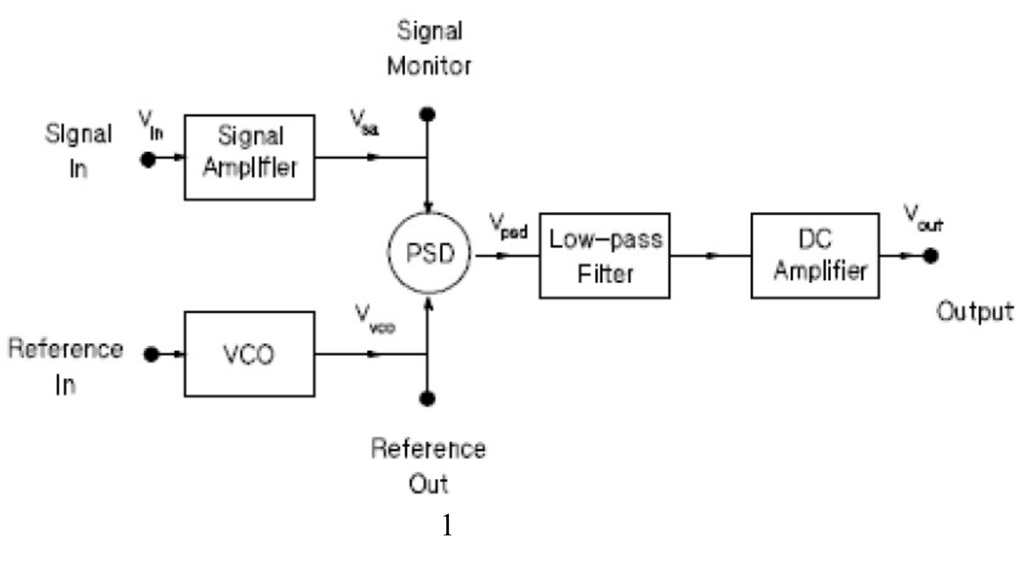
Lock-in amplifiers are a special type of amplifiers that are used for detecting and measuring very small amplitude AC signals, as small as a few nanovolts. Accurate measurement is possible even in the presence of relatively large noise - many thousands of times larger. The signal should be at a specific frequency. The Lock-in amplifier uses a technique known as phase-sensitive detection to single out the component of the signal at a specific reference frequency and phase. Noise signals, at frequencies other than the reference frequency, are rejected, so that they do not affect the measurement.

Let us see how the Lock in amplifier is able to do so. If we use a very good band pass filter with a Q=100 centered at 10 kHz, to filter out the noise, any signal in a 100 Hz bandwidth (10 kHz/Q) will be detected. The noise in the filter pass band will be 50 μV(5 nV/√Hz ×√100 H z × 1000), and the signal will still be 10 μV. The output noise is much greater than the signal, and an accurate measurement can not be made. Further gain will not help the signal-to-noise problem.

Let us now put a phase-sensitive detector (PSD) after the amplifier. The PSD can detect the signal at 10 kHz with a bandwidth as narrow as 0.01 Hz! In this case, the noise in the detection bandwidth will be 0.5 μV (5 nV/√Hz × √.01 Hz × 1000), while the signal is still 10 μV. The signal-to-noise ratio is now 20, and an accurate measurement of the signal is possible.

Block Diagram:

The block diagram of a lock-in amplifier is shown in Figure below.



The Lock-in amplifier consists of five stages:

1. AC amplifier, called the signal amplifier;
2. Voltage controlled oscillator (VCO);
3. Multiplier, called the phase sensitive detector (PSD);
4. Low-pass filter; and
5. DC amplifier.

The signal to be measured is fed into the input of the AC Amplifier. The output of the DC amplifier is a DC voltage proportional to V0. This voltage is displayed on the lock-in's own meter, and is also available at the output connector. The functions of the five stages are described below.

The AC amplifier is simply a voltage amplifier combined with variable filters. Some lock-in amplifiers let you change the filters as you wish, others do not. Some lock-in amplifiers have the output of the AC amplifier stage available at the signal monitor output. Many do not.

The Phase sensitive detector, the voltage controlled oscillator and the low pass filter are components of a Phase Lock Loop circuit that is an integralpart of the Lock in Amplifier.

The voltage controlled oscillator is just an oscillator, except that it can synchronize with an external reference signal (i.e., trigger) both in phase and frequency. Some lock-in amplifiers contain a complete oscillator and need no external reference. In this case they operate at the frequency and amplitude that you set, and you must use their oscillator output in your experiment to derive the signal that you ultimately wish to measure. Virtually all lock-in amplifiers are able to synchronize with an external reference signal.

The VCO also contains a phase-shifting circuit that allows the user to shift its signal from 0-360 degrees with respect to the reference.

The phase sensitive detector is a circuit which takes in two voltages as inputs V1 and V2 and produces an output which is the product V1*V2. That is, the PSD is just a multiplier circuit.

The low pass filter is an RC filter whose time constant may be selected. In many cases you may choose to have one RC filter stage (single pole filter) or two RC filter stages in series (2-pole filter). In newer lock-in amplifiers, this might be a digital filter with the attenuation of a "many pole" filter.

The DC amplifier is just a low-frequency amplifier similar to those frequently assembled with op-amps. It differs from the AC amplifier in that it works all the way down to zero frequency (DC) and is not intended to work well at very high frequencies, say above 10 Khz.

Phase sensitive detection:

Lock-in amplifier uses a frequency reference for its measurement. The signal should be at a fixed frequency (generated by an oscillator or a function generator), and the lock-in amplifier detects the response from the experiment at that reference frequency. In the diagram shown below, the reference signal is a square wave at frequency ω_r. This might be a synchronous square wave output from the function generator used for providing a sine wave output for the experiment. The response might be the signal waveform shown below. The signal is $V_{sig}\sin(\omega_r t + \theta_{sig})$ where V_{sig} is the signal amplitude, ω_r is the signal frequency, and θ_{sig} is the signal's phase.

A Lock-in amplifier generates its own internal reference signal usually with help of a phase-locked-loop locked to the external reference. In the diagram, the external reference, the lock-in amplifier's reference, and the signal are all shown. The internal reference is $V_L\sin(\omega_L t + \theta_{ref})$.

The lock-in amplifier amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive detector or multiplier. The output of the PSD is simply the product of two sine waves.

$$V_{psd} = V_{sig}V_L\sin(\omega_r t + \theta_{sig})\sin(\omega_L t + \theta_{ref})$$

$$= \tfrac{1}{2}V_{sig}V_L\cos([\omega_r - \omega_L]t + \theta_{sig} - \theta_{ref}) - \tfrac{1}{2}V_{sig}V_L\cos([\omega_r + \omega_L]t + \theta_{sig} + \theta_{ref})$$

The PSD output contains two AC signals, one at the difference frequency $(\omega_r - \omega_L)$ and the other at the sum frequency $(\omega_r + \omega_L)$. If the PSD output is passed through a low pass filter, the AC signals are removed. What will be left? In the general case, nothing. However, if ω_r equals ω_L, the difference frequency component will be a DC signal. In this case, the filtered PSD output will be:

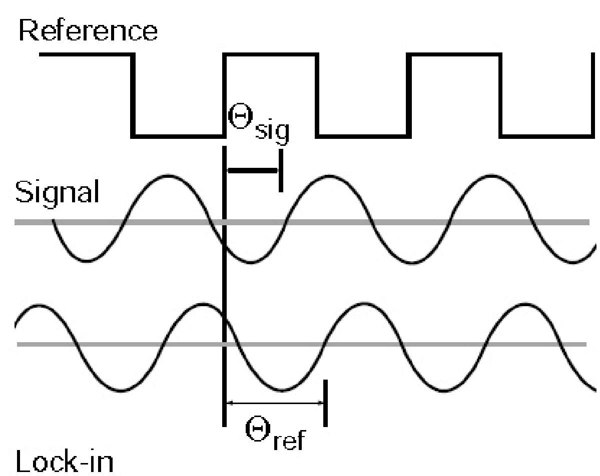
$$V_{psd} = \tfrac{1}{2}V_{sig}V_L\cos(\theta_{sig} - \theta_{ref})$$

This is an interesting signal. It is a DC signal proportional to the signal amplitude.

There are two approaches for carrying out this operation. The traditional way is to use analog circuits. Nowadays, the same operation is also done digitally. In traditional analog lock-in amplifiers, the signal and reference are both analog voltage signals. The signal and reference are multiplied using an analog multiplier, and the result is filtered using one or more stages of RC filters. In a digital lock-in amplifiers, the signal and reference are both represented by sequences of numbers. Multiplication and filtering are performed mathematically using a digital signal processing (DSP) chip.

Narrow Band Detection:

Suppose that instead of a pure sine wave, the input signal contains a signal plus noise. The PSD and low pass filter only detect signals whose frequencies are very close to the lock-in reference frequency. Noise signals, at frequencies far from the reference, are attenuated at the PSD output by the low pass



filter (neither $\omega_{noise} - \omega_{ref}$ nor $\omega_{noise} + \omega_{ref}$ are close to DC). Noise at frequencies very close to the reference frequency will result in very low frequency AC outputs from the PSD ($|\omega_{noise} - \omega_{ref}|$ is small). Their attenuation depends upon the bandwidth of the low pass filter and its rolloff (rate of change of attenuation with frequency, i.e. 20 db per decade, etc.). A narrower bandwidth will remove noise sources very close to the reference frequency; a wider bandwidth allows these signals to pass. The low pass filter bandwidth determines the bandwidth of detection. Only the signal at the reference frequency will result in a true DC output and be unaffected by the low pass filter. This is the signal we want to measure.

Generation of Lock-In Reference:

We need to make the lock-in reference at the same frequency as the signal, i.e. $\omega_r = \omega_L$. Not only do the frequencies have to be the same, the phase between the signals should not change with time. This will happen only when signal frequency and reference frequency will exactly match. Otherwise, $\cos(\theta_{sig} - \theta_{ref})$ will change and V_{psd} will not be a DC signal. In other words, the lock-in reference needs to be phase-locked to the signal reference.

A Lock-in amplifier uses a phase-locked loop (PLL) to generate the reference signal. An external reference signal (in this case, the reference square wave) is provided to the lock-in amplifier. The PLL in the lock-in amplifier locks the internal reference oscillator to this external reference, resulting in a reference sine wave at ω_r with a fixed phase shift of θ_{ref}. Since the PLL actively tracks the external reference, changes in the external reference frequency do not affect the measurement.

Internal reference sources:

In the case that we have discussed, the reference signal is provided by the excitation source (the function generator). This is called an external reference source. In many situations the internal oscillator of the lock-in amplifier is used for providing the reference signal. The internal oscillator is just like a function generator (with variable sine wave output and a TTL compatible synchronizing signal) which is always phase-locked to the reference oscillator.

Magnitude and Phase:

If we recall that the PSD output is proportional to $V_{sig}\cos\theta$, where $\theta = (\theta_{sig} - \theta_{ref})$. Here, θ is the phase difference between the signal and the lock-in reference oscillator. By adjusting θ_{ref} we can make θ equal to zero. In such a case, we can measure V_{sig} for ($\cos\theta = 1$). Conversely, if θ is 90°, there will be no output at all. A lock-in amplifier with a single PSD is called a single-phase lock-in and its output is $V_{sig}\cos\theta$. Now, for a given value of $\cos\theta$, there are two possible values of θ, i.e. θ and $-\theta$ as $\cos(-\theta)=\cos(\theta)$. This phase dependence of the output can be eliminated by adding a second PSD. If the second PSD multiplies the signal with the reference oscillator shifted by 90°, i.e. $V_L\sin(\omega_L t + \theta_{ref} +$

90°), its low pass filtered output will be:

$$V_{psd2} = \frac{1}{2} V_{sig} V_L \sin(\theta_{sig} - \theta_{ref})$$

$$V_{psd2} \sim V_{sig} \sin\theta$$

Now we have two outputs: one proportional to cosθ and the other proportional to sinθ. If we call the first output X and the second Y, then

$$X = V_{sig}\cos\theta \quad Y = V_{sig}\sin\theta$$

These two quantities represent the signal as a vector relative to the lock-in reference oscillator. X is called the 'in-phase' component and Y the 'quadrature' component. This is because when $\theta = 0$, X measures the signal while Y is zero.

By computing the magnitude (R) of the signal vector, the phase dependency is removed.

$$R = (X_2 + Y_2)^{1/2} = V_{sig}$$

R measures the signal amplitude and does not depend upon the phase between the signal and lock-in reference.

A dual-phase lock-in has two PSDs with reference oscillators 90° apart, and can measure X, Y and R directly. In addition, the phase difference (θ) between the signal and lock-in is defined as:

$$\theta = \tan^{-1}(Y/X).$$

Digital PSD versus Analog PSD:

It was mentioned earlier that the implementation of a PSD is different for analog and digital lock-in amplifiers. A digital lock-in amplifier, such as the SR830 from Stanford Research system, multiplies the signal with the reference sine waves digitally. The amplified signal is converted to digital form using a 16-bit A/D converter sampling at 256 kHz. The A/D converter is preceeded by a 102 kHz anti-aliasing filter to prevent higher frequency inputs from generating spurius signals below 102 kHz.

This input data stream is multiplied, one point at a time, with the computed reference sine waves described previously. Every 4 μs the input signal is sampled, and the result is multiplied by both reference sine waves (which are 90° apart). The phase sensitive detectors (PSDs) in the digital lock-in act as linear multipliers; that is, they multiply the signal with a reference sine wave. Analog PSDs (both square wave and linear) have many problems associated with them. The main problems are harmonic rejection, output offsets, limited dynamic reserve, and gain error.

The digital PSD multiplies the digitized signal with a digitally computed reference sine wave.

Because the reference sine waves are computed to 20 bits of accuracy, they have very low harmonic content. In fact, the harmonics are at the -120 dB level! This means that the signal is multiplied by a single reference sine wave (instead of a reference and its many harmonics), and only the signal at this single reference frequency is detected. The digital lock-in amplifiers are completely insensitive to signals at harmonics of the reference. In contrast, a square wave multiplying lock-in amplifier will detect at all of the odd harmonics of the reference (a square wave contains many large odd harmonics). Output offset is a problem because the signal of interest is a DC output from the PSD, and an output offset contributes to error and zero drift. The offset problems of analog PSDs are eliminated using the digital multiplier. There are no erroneous DC output offsets from the digital multiplication of the signal and reference. In fact, the actual multiplication is virtually error free.

The dynamic reserve of an analog PSD is limited to about 60 dB. When there is a large noise signal present, 1000 times (or 60 dB) greater than the full-scale signal, the analog PSD measures the signal with an error. The error is caused by nonlinearity in the multiplication (the error at the output depends upon the amplitude of the input). This error can be quite large (10 % of full scale) and depends upon the noise amplitude, frequency and waveform. Since noise generally varies quite a bit in these parameters, the PSD error causes a lot of output uncertainty.

In the digital lock-in amplifier, the dynamic reserve is limited by the quality of the A/D conversion. Once the input signal is digitized, no further errors are introduced. Certainly, the accuracy of the multiplication does not depend on the size of the numbers. The A/D converter used in the SR810 is extremely linear, i.e. the presence of large noise signals does not impair its ability to correctly digitize a small signal. In fact, the dynamic reserve of these lock-in amplifiers can exceed 100 dB without any problems. We'll talk more about dynamic reserve a little later.

A linear analog PSD multiplies the signal by an analog reference sine wave. Any amplitude variation in the reference amplitude shows up directly as a variation in the overall gain. Analog sine-wave generators are susceptible to amplitude drift: especially as a function of temperature. The digital reference sine wave has a precise amplitude and never changes. This avoids a major source of gain error common to analog lock-in amplifiers.

The overall performance of a lock-in amplifier is largely determined by the performance of its phase sensitive detectors. In virtually all respects, the digital PSD outperforms its analog counterparts.

What exactly does the lock-in amplifier measure?

Fourier's theorem states that any input signal can be represented as the sum of many sine waves of differing amplitudes, frequencies and phases. This is generally considered as representing the signal in the "frequency domain". Normal oscilloscopes display the signal in the "time domain".

Except in the case of clean sine waves, the time domain representation does not convey very much information about the various frequencies which make up the signal.

A lock-in amplifier multiplies the signal by a pure sine wave at the reference frequency. All components of the input signal are multiplied by the reference signal simultaneously. Mathematically speaking, sine waves of differing frequencies are orthogonal, i.e. the average of the product of two sine waves is zero unless the frequencies are EXACTLY the same. The product of this multiplication yields a DC output signal proportional to the component of the signal whose frequency is exactly locked to the reference frequency. The low pass filter (which follows the multiplier) provides the averaging which removes the products of the reference with components at all other frequencies.

A lock-in amplifier, because it multiplies the signal with a pure sine wave, measures the single Fourier (sine) component of the signal at the reference frequency. Let's take a look at an example. Suppose the input signal is a simple square wave at frequency f. The square wave is actually composed of many sine waves at multiples of f with carefully related amplitudes and phases. A 2 Vpp square wave can be expressed as:

$$S(t) = 1.273\sin(\omega t) + 0.4244\sin(3\omega t) + 0.2546\sin(5\omega t) + \ldots \qquad \text{where } \omega = 2\pi f.$$

The lock-in, locked to f, will single out the first component. The measured signal will be $1.273\sin(\omega t)$, not the 2 Vpp that you'd measure on a scope. In the general case, the input consists of signal plus noise. Noise is represented as varying signals at all frequencies. The ideal lock-in amplifier only responds to noise at the reference frequency. Noise at other frequencies is removed by the low pass filter following the multiplier. This "bandwidth narrowing" is the primary advantage that a lock-in amplifier provides. Only inputs with frequencies at the reference frequency result in an output.

Dynamic Reserve:

The term "dynamic reserve" comes up frequently in discussions about lock-in amplifiers. Let us discuss this term in a little more detail. Assume the lock-in input consists of a full-scale signal at fref plus noise at some other frequency. The traditional definition of dynamic reserve is the ratio of the largest tolerable noise signal to the full-scale signal, expressed in dB. For example, if full scale is 1 μV, then a dynamic reserve of 60 dB means noise as large as 1 mV (60 dB greater than full scale) can be tolerated at the input without overload.

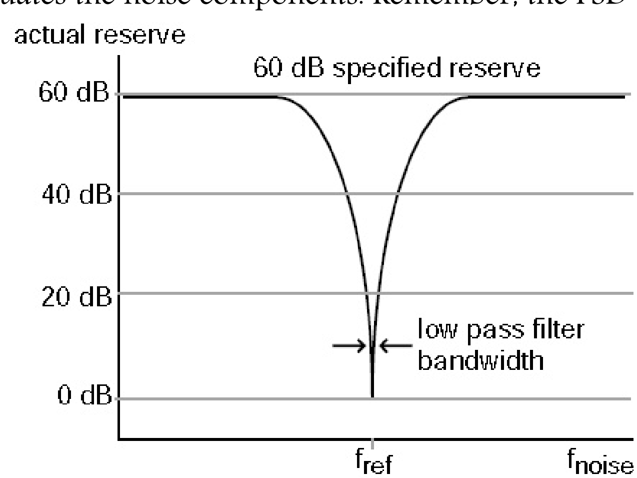
The problem with this definition is the word "tolerable". Clearly, the noise at the dynamic reserve limit should not cause an overload anywhere in the instrument - not in the input signal amplifier, PSD, low pass filter or DC amplifier. This is accomplished by adjusting the distribution of the gain. To achieve high reserve, the input signal gain is set very low so the noise is not likely to overload. This means that the signal at the PSD is also very small. The low pass filter removes the large noise components from the PSD output which allows the remaining DC component to be amplified (a lot) to reach 10 V full scale. There is no problem running the input amplifier at low gain. However, as we have discussed previously, analog lock-ins have a problem with high reserve because of the linearity of the PSD and the DC offsets of the PSD and DC amplifier. In an analog lock-in, large noise signals almost always disturb the measurement in some way.

The most common problem is a DC output error caused by the noise signal. This can appear as an offset or as a gain error. Since both effects are dependent upon the noise amplitude and frequency, they can not be offset to zero in all cases and will limit the measurement accuracy. Because the errors are DC in nature, increasing the time constant does not help. Most lock-in amplifiers define tolerable noise as levels which do not affect the output more than a few percent of full scale. This is more severe than simply not overloading.

Another effect of high dynamic reserve is to generate noise and drift at the output. This comes about because the DC output amplifier is running at very high gain, and low frequency noise and offset drift at the PSD output or the DC amplifier input will be amplified and appear large at the output. The noise is more tolerable than the DC drift errors since increasing the time constant will attenuate the noise. The DC drift in an analog lock-in is usually on the order of 1000 ppm/°C when using 60 dB of dynamic reserve. This means that the zero point moves 1 % of full scale over 10 °C temperature change. This is generally considered the limit of tolerable.

Lastly, dynamic reserve depends on the noise frequency. Clearly noise at the reference frequency will make its way to the output without attenuation. So the dynamic reserve at f_{ref} is 0 dB. As the noise frequency moves away from the reference frequency, the dynamic reserve increases. Why?

Because, the low pass filter after the PSD attenuates the noise components. Remember, the PSD outputs are at a frequency of $|f_{noise} - f_{ref}|$. The rate at which the reserve increases depends upon the low pass filter time constant and rolloff. The reserve increases at the rate at which the filter rolls off. This is why 24 dB/octave filters are better than 6 or 12 dB/octave filters. When the noise frequency is far away, the reserve is limited by the gain distribution and overload level of each gain element. This reserve level is the dynamic reserve referred to in the specifications.



The above graph shows the actual reserve vs. the frequency of the noise. In some instruments, the signal input attenuates frequencies far outside the lock-in's operating range (f_{noise}>>100 kHz). In these cases, the reserve can be higher at these frequencies than within the operating range. While this creates a nice specification, removing noise at frequencies very far from the reference does not require a lock-in amplifier. Lock-ins are used when there is noise at frequencies near the signal. Thus, the dynamic reserve for noise within the operating range is more important.

Applications:

References:

1. About Lock in amplifiers, *Application Note 3,* Stanford Research Systems, downloaded from https://www.thinksrs.com/downloads/pdfs/applicationnotes/AboutLIAs.pdf
2. Lock in Amplifiers and Applications, Lehigh University Class Notes Physics 262. https://www.lehigh.edu/~jph7/website/Physics262/LockInAmplifierAndApplications.pdf