Plasma Physics

MSc. Physics Semester 2

Paper - MPHY CC6

Unit 5

Dr. Surabhi Prasad

Prof.& Head

Department of Physics

Patna University

Dr. Surabhi
Deptt of Physics
P.U.

Plasma Physics

Momentum Equation from Boltzmann equation

or, Fluid equation of motion from Boltzmann eqn.

Q.
1st order momentum in Plasma.

The lowest moment by using $\frac{\partial f}{\partial t}$ = (moments of the Boltzmann equation ~~the lowest moment is obtained by~~ integrating $\boxed{1}$

$$\left(\frac{\partial f}{\partial t}\right) + (\vec{v}\cdot\nabla)f + \frac{q}{m}\left(\vec{E} + \vec{u}\times\vec{B}\right)\cdot\frac{\partial f}{\partial \vec{v}}\, d\vec{v} = \left(\frac{\partial f}{\partial t}\right)_c \quad \text{— } \boxed{1}$$

By integrating $\boxed{1}$ the lowest moment is

$$\int\left(\frac{\partial f}{\partial t}\right)d\bar{v} + \int \bar{v}\cdot\vec{\nabla} f\, dv + \frac{q}{m}\int\left[\vec{E} + \vec{v}\times\vec{B}\right]\cdot\frac{\partial f}{\partial v}\, dv = \int\left(\frac{\partial f}{\partial t}\right)_c d\vec{u}$$

multipling $m\vec{v}$ and integrating whole equation over velocity space

$$\underbrace{m\int \bar{v}\,\frac{\partial f}{\partial t}\, d^3v}_{\text{1st term}} + \underbrace{m\int \vec{v}\,(\vec{v}\cdot\vec{\nabla})f\, d^3v}_{\text{2nd term}} + \underbrace{m\frac{q}{m}\int \vec{v}\left(\vec{E} + \vec{u}\times\vec{B}\right)\frac{\partial f}{\partial \bar{v}}\, d\bar{v}}_{\text{3rd term}} = \underbrace{\int m\bar{v}\left(\frac{\partial f}{\partial t}\right)_c d^3v}_{\text{4th term}}$$

1. Solution

$$\underbrace{m\int \vec{v}\frac{\partial f}{\partial t}d^3\vec{v}}_{\text{1st term}} + \underbrace{m\int \vec{v}.(\vec{v}.\vec{\nabla})d^3v}_{\text{2nd term}} + \underbrace{\frac{mq}{m}\int \vec{v}(\vec{E}+\vec{u}\times\vec{B})\frac{\partial f}{\partial v}d^3v}_{\text{third term}}$$

$$= \underbrace{\int mv\left(\frac{\partial f}{\partial t}\right)_c d^3v}_{\text{4th term}}$$

$$\underbrace{m\frac{\partial}{\partial t}(n\vec{u})}_{\text{1st term}} + \underbrace{m\left[\vec{u}\vec{\nabla}.(n\bar{u}) + n(\vec{u}.\vec{\nabla})\vec{u}\right]}_{\text{2nd term}} + \nabla p$$

$$- qn\left[\vec{E} + \vec{u}\times\vec{B}\right] = P_{ij}$$

$$mn\frac{\partial \bar{u}}{\partial t} + \cancel{m\bar{u}\frac{\partial n}{\partial t}} + m\left[\cancel{-u\frac{\partial n}{\partial t}} + n(\vec{u}.\vec{\nabla})\vec{u}\right]$$

$$= nq\left[E + \vec{u}\times\vec{B}\right] - \nabla p + P_{ij}$$

Substituting 1st, 2nd, 3rd & 4th term (solution)

$$mn\left[\frac{\partial u}{\partial t} + (\vec{u}.\vec{\nabla})\vec{u}\right] = nq(\vec{E}+\vec{u}\times\vec{B}) - \vec{\nabla}p + P_{ij}$$

$$\boxed{mn\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\vec{\nabla})\vec{u}\right] = nq(\vec{E}+\vec{u}\times\vec{B}) - \vec{\nabla}p + P_{ij}}$$

This is the Plasma momentum equation or equation of motion of Plasma fluid

[From Continuity equation

$$\frac{\partial n}{\partial t} + \vec{\nabla}(n\vec{u}) = 0$$

$$\therefore \vec{\nabla}(n\vec{u}) = -\frac{\partial n}{\partial t}\Big]$$

Solution of 1st, 2nd, 3rd & 4th term.

① Solving 1st term

$$m\int \vec{v}\frac{\partial}{\partial t} d^3v$$

$$= m\frac{\partial}{\partial t}\int \vec{v} f d^3v$$

$$= m\frac{\partial}{\partial t}(n\vec{u})$$ — 1st term solution

$\vec{u}$ is the average velocity

② Solving 2nd term.

$$m\int \vec{v}(\vec{v}.\vec{\nabla}) f d^3v$$

$$= m\vec{\nabla}.\int \vec{v}.\vec{v} f d^3v)$$

$$= m\vec{\nabla}.(n\bar{v}\bar{v})$$

$$= m\vec{\nabla}.(n\bar{u}\bar{u}) + \vec{\nabla}.(mn(\bar{\omega}\bar{\omega})$$

$$= m\bar{\nabla}.(n\bar{u}\bar{u}) + \nabla\vec{p} = m[\bar{u}\vec{\nabla}(n\bar{u}) + n(\bar{u}.\bar{\nabla})\bar{u}] + \vec{\nabla}p$$

$\bar{v} = \bar{u} + \bar{\omega}$ → thermal velocity

particle velocity ↓ $\bar{v}$; average velocity per unit volume ↓ $\bar{u}$

$$\bar{v} = \bar{u} + \bar{\omega}$$ u is already average velocity

$$\bar{v}\bar{v} = \bar{u}\bar{u} + \bar{\omega}\bar{\omega}$$

$$\vec{\nabla}n\bar{v}\bar{v} = \bar{\nabla}n\bar{u}\bar{u} + \bar{\nabla}n\bar{\omega}\bar{\omega}$$

$$\bar{\nabla}.n\bar{v}\bar{v} = \bar{\nabla}.n\bar{u}\bar{u} + \bar{\nabla}.n\bar{\omega}\bar{\omega}$$

$$\vec{\nabla}.mn\bar{v}\bar{v} = \vec{\nabla}.mn\bar{u}\bar{u} + \vec{\nabla}.mn\bar{\omega}\bar{\omega}$$

Sunalini R.

$m\nabla\cdot(n\bar{v}\bar{v}) = m\nabla\cdot(nuu) + \nabla\cdot m n \bar{\omega}\bar{\omega}$

Now $p = \frac{1}{3} m n \bar{v}^2$ & $p = nkT$

$\bar{v}^2 = \frac{3K_bT}{m}$

$\therefore p = \frac{1}{\cancel{3}} \cancel{m} n \frac{\cancel{3}K_bT}{\cancel{m}}$

$\therefore p = nkT$

$m n \bar{\omega}\cdot\bar{\omega}$ is the dimension of velocity

$m n \omega^2$

3rd term

$$q\int \vec{v}(\vec{E}+\vec{v}\times\vec{B})\cdot\frac{\partial f}{\partial v}\, d^3v \rightarrow$$

$$= \int\frac{\partial}{\partial v}\cdot f v(\bar{E}+\bar{v}\times\bar{B})\, d^3v$$

$$= \int \bar{v}(\bar{E}+\bar{v}\times\bar{B})\cdot\frac{\partial f}{\partial v}\, d^3v + \int(\bar{E}+\bar{v}\times\bar{B}) f\, d^3v + \int v f \frac{\partial}{\partial v}(\bar{E}+\bar{v}\times\bar{B})\, d^3v$$

$$\oint_S f v(\bar{E}+\bar{v}\times\bar{B})\, d^2v = \int\vec{v}(\bar{E}+\bar{v}\times\vec{B})\frac{\partial f}{\partial v}\, d^3v + \int(\bar{E}+\bar{v}\times\vec{B}) f\, d^3v + \int\frac{\partial}{\partial v}\underbrace{(vf)}_{=0}(\vec{E}+\vec{v}\times\vec{B})\, d^3v$$

$v=\infty$

$f=0$ when $v=\infty$

(4)

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$\phi(uf) = \phi$ is the function of ~~ϕ~~ uf

$$\therefore \int \vec{v}\,(\vec{E}+\bar{u}\times\vec{B})\frac{\partial f}{\partial v}\,d^3v = -\int (\vec{E}+\bar{u}\times\vec{B})\,f\,d^3v.$$

we know $\int uf\,d^3v = nu$

$\rightarrow \Rightarrow \underline{-n(\vec{E}+\vec{u}\times\vec{B})}$ → sol^n: Third term

4^{th} term

$\underline{\int mv\left(\frac{\partial f}{\partial t}\right)_c d^3v} = P_{ij}$ – sol^n: fourth term

P_{ij} represent momentum change due to collision.

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This notes is from
· Plasma Physics
— F. F. Chen

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