Magnetic Reconnection

Course: MPHYEC-01 Plasma Physics
(M.Sc. Sem-IV)

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Magnetic Reconnection

Magnetic topology

Before going into the discussion of magnetic reconnection, we need to understand the concept of magnetic topology and its preservation. For this, we note that, in the previous lectures, we have discussed the magnetohydrodynamics (MHD) model of plasma. Moreover, for a plasma having zero electrical resistivity (or infinity magnetic Reynolds number R_M), the corresponding dynamics is explained by ideal MHD. Under ideal MHD, Alfven's theorem of flux-freezing is satisfied and according to that the magnetic field is completely frozen in the plasma. In previous lecture, we have also mentioned one important consequence of Alfven's theorem. If two fluid elements lie on a magnetic field line, then they would always lie on the same field line. We may have two far-away fluid elements in the ideal plasma connected by a magnetic field line. No matter what happens to the plasma or how it evolves in time, this connectivity between the two far-away fluid elements remains preserved if the resistivity is zero. The preservation of such connectivities may introduce some constraints on the dynamics of the system. Relevant to the discussion, it is worthy to mention that topology (which is a branch of mathematics) deals with the different transformations that preserve certain connectivities.

To further explain the concept of magnetic topology, lets consider two magnetic field lines denoted by A and B. Figure 1 shows three configurations of the field lines in different panels. The field line B can be wrapped around A as shown in panel (b) of the figure. It should be noted that no cutting or pasting of field lines was necessary in order to deform the configuration of panel (a) to the configuration of panel (b). We show another possible configuration of these two field lines in panel (c) of the figure. It must be clear that one has to cut and rejoin at least one field line in order to arrive at this configuration. We say that the configurations of panel (a) and panel (b) are topologically equivalent, whereas the configuration of panel (c) is topologically different from the other two. After giving the general idea, let us now give the mathematical definition. If two magnetic configurations $\mathbf{B_1(r)}$ and $\mathbf{B_2(r)}$ are such that one of them can be deformed into the other by continuous displacements without cutting or pasting field lines anywhere, then the two magnetic configurations are said to have the same magnetic topology. If this is not possible, then the magnetic topologies are different.

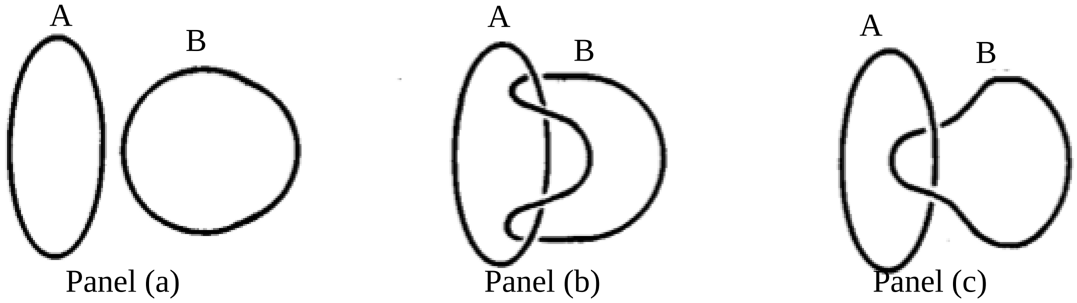

Figure (1)

If a plasma is ideal (i.e. has zero resistivity or infinite magnetic Reynolds number), then its magnetic topology can never change. So, as a result of any dynamics, it has to evolve through successive configurations which are all topologically equivalent. However, in presence of small but non-zero resistivity, the magnetic topology can change locally by the process of magnetic reconnection which we discuss in the following section.

Magnetic Reconnection

We discussed in the previous section that magnetic topology is exactly preserved in a plasma with zero electrical resistivity. Now, we aim to understand the important question: What happens if the plasma has a very small, but finite electrical resistivity or magnetic diffusivity (recall from the previous lectures, if η represents the resistivity then $\lambda=\eta/\mu_0$ is magnetic diffusivity) or large magnetic Reynolds number R_M? In an attempt to answer this question, we know that the resistivity or magnetic diffusivity appears in the induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times\boldsymbol{v}\times\boldsymbol{B}+\lambda\nabla^2\boldsymbol{B} \qquad \text{-------} \quad (1)$$

Moreover, we note that the magnetic diffusivity appears in the induction equation as a coefficient in front of the second derivative magnetic field **B** (the diffusion term). Because of this, even if the magnetic diffusivity is small and R_M is large, its effect can become important in a localized regions where the gradient in the magnetic field **B** is large. Large gradient in **B** leads to an enhanced value of $\nabla^2\mathbf{B}$ in the region and, therefore, producing a significant value of the diffusion term ($\lambda\nabla^2\mathbf{B}$). At this region, magnetic energy associated with the magnetic field can decay with time because of the effective diffusion. Since large gradients of magnetic field are associated with large current densities (as $\mathbf{J}=(1/\mu_0)(\nabla\times\mathbf{B})$), such regions are often called current sheets as they have intense values of current density. The region shown in color red in figure 2 represents a current sheet. From the figure, it is clear that magnetic field lines in across the current sheet are oppositely directed (hence magnetic field are in opposite direction across the current sheet). Therefore, gradient in magnetic field is very high at the current sheet. In a low-resistivity plasma, cutting and pasting of field lines can take place within current sheets due to the diffusion term and hence magnetic topology can be changed locally. But the magnetic fields may be taken to be frozen in the plasma outside the current sheets and magnetic topologies are preserved everywhere except in the current sheets. At the current sheets, the diffusion term ($\lambda\nabla^2\mathbf{B}$) plays an important role and converts magnetic energy stored in the magnetic into kinetic energy of the plasma and heat by locally changing the magnetic topology. *This process is known as magnetic reconnection.*

Sweet-Parker Model of magnetic reconnection:

One of the traditional reconnection models, known as the Sweet-Parker model, was proposed by Sweet and Parker. The Sweet-Parker model describes the steady state magnetic reconnection across a current sheet by approximately solving MHD equations for a steady state in a two-dimensional Cartesian geometry for an incompressible plasma.

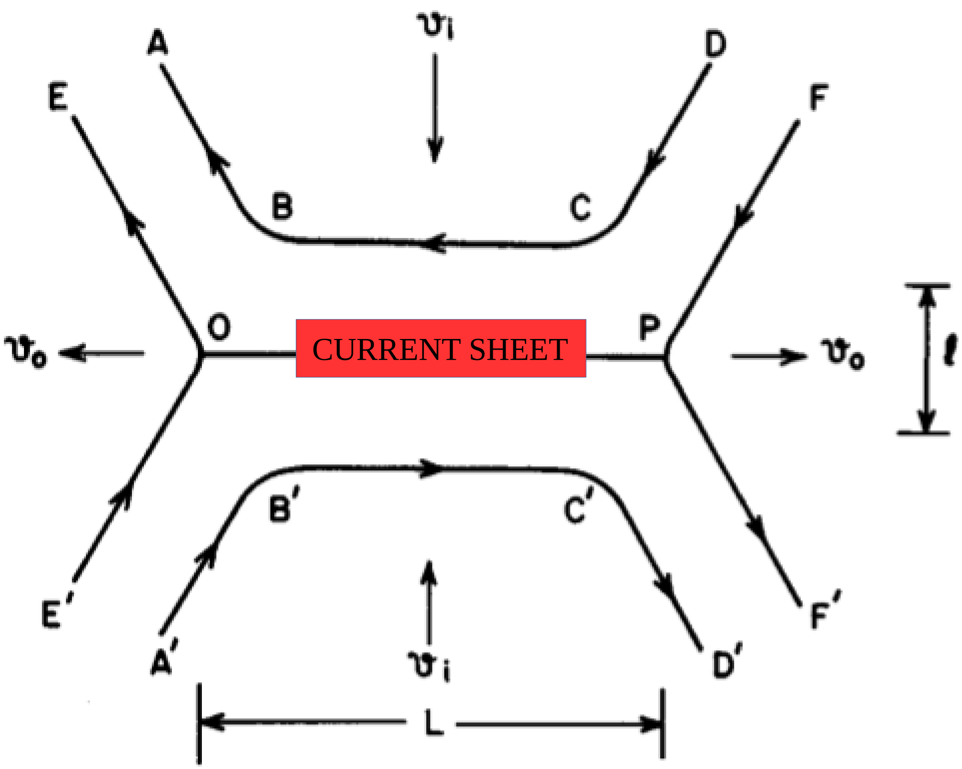

Figure 2

Since the plasma is incompressible, therefore the mass density ρ is constant and, as a result, the mass continuity equation for a steady state ($\frac{\partial}{\partial t}\rightarrow 0$) modifies as:

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\boldsymbol{v}\rho)=0 \qquad \left(\text{as } \frac{\partial}{\partial t}\rightarrow 0 \text{ for a steady state}\right)$$

$$\rho\nabla\cdot\boldsymbol{v}=0 \qquad (\text{as } \rho \text{ is constant})$$

Therefore, the continuity equation for incompressible steady state plasma is

$$\nabla \cdot \boldsymbol{v} = 0 \qquad \text{--------- (2)}$$

The induction equation for the steady state is obtained by using $\frac{\partial}{\partial t} \rightarrow 0$ in equation (1) and it is:

$$\nabla \times \boldsymbol{v} \times \boldsymbol{B} + \lambda \nabla^2 \boldsymbol{B} = 0 \qquad \text{--------- (3)}$$

To understand the physics of magnetic reconnection, let us look at figure 2 more carefully. The field lines ABCD and A'B'C'D' are moving with inward velocity v_i towards the central region. Eventually the central parts BC and B'C' of these field lines decay away. The part AB is moved to EO and the part A'B' f to E'O. These parts originally belonging to different field lines now make up one field line EOE'. Similarly the parts CD and C'D' eventually make up the field line FPF'. We thus see that the cutting and pasting of field lines take place in the current sheet. Since plasmas from the top and the bottom in figure 2 push against the current sheet, the plasma in the central region is eventually squeezed out sideways through the points 0 and P. Let v_0 be the outward velocity with which reconnected field lines EOE' and FPF' move away from the reconnection region. Our aim now is to estimate the incoming velocity v_i which essentially gives the rate at which the reconnection proceeds.

To calculate v_i, in the Sweet-Parker model, the differential equations (2) and (3) are approximated by algebraic equations and showed that rough estimates of various quantities can be obtained. Let L be the width of the current sheet over which the magnetic field decays, as indicated in figure 2. After the magnetic field has decayed, the field-free plasma is squeezed through the points 0 and P. We consider the speed of outflowing plasma to be v_0. If l be the thickness of the outflowing plasma, then the continuity equation (2) can be replaced by the approximate mass conservation condition

$$v_i L \approx v_0 l \qquad \text{-------- (4)}$$

From the momentum transport equation, the fluid parcels are pushed by Lorentz force and, hence, the force is responsible for the generation of the outflow. Then we expect that the kinetic energy associated with the outflow should be comparable to themagnetic energy stored in the current sheet. Therefore, we have

$$\frac{1}{2} \rho {v_0}^2 \approx \frac{B^2}{2\mu_0} \qquad \text{------- (5)}$$

$$v_0 \approx \frac{B}{\sqrt{\mu_0 \rho}} = v_a \qquad \text{------- (6)}$$

revealing that the outflow speed v_0 is almost equal to the Alfvén speed v_a —the speed at which magnetic disturbances propagate in plasma.

Further with the steady state approximation, the induction equation (2) is stating that the magnetic field is dissipated (via the $\lambda\nabla^2\mathbf{B}$ term) as rapidly as it is convected towards the current sheet at a speed v_i (via the $\nabla \times \mathbf{v} \times \mathbf{B}$ term). The diffusion term, which is of order $\lambda B/l^2$ (where l also represent the length scale over which magnetic field varies), has to be balanced by the other term corresponding to the supply of fresh magnetic flux at velocity v_i. Since this term $\nabla \times \mathbf{v} \times \mathbf{B}$ should be of order $v_i B/l$. Now, from equation (3), we get

$$\frac{\lambda B}{l^2} \approx \frac{v_i B}{l} \qquad \text{-------- (7)}$$

$$v_i \approx \frac{\lambda}{l} \qquad \text{-------- (8)}$$

From equation (4)

$$l \approx \frac{v_i L}{v_0}$$

Now use equation (6), we can get

$$l \approx \frac{v_i L}{v_a} \qquad \text{--------- (9)}$$

Put this value of l into equation (8) and we get,

$$v_i \approx \frac{\lambda v_a}{v_i L}$$

$$v_i^2 \approx \frac{\lambda v_a^2}{v_a L}$$

$$v_i^2 \approx \frac{v_a^2}{(v_a L/\lambda)}$$

$$v_i \approx \frac{v_a}{\sqrt{R_M}} \qquad \text{------- (10).}$$

where magnetic Reynolds number $R_M = (v_a L/\lambda)$ is calculated with Alfven speed (v_a). Equation (10) is the reconnection rate in the Sweet-Parker reconnection rate. From our previous lecture, we know that R_M is typically a very large number in astrophysical situations. Therefore equation (10) implies that the reconnection proceeds at a rate which is a tiny fraction of the Alfven speed.

For the completeness of the discussion, we would like to mention here that observations reveal the occurrence of sudden energy release in the form of solar and stellar flares in astrophysical plasmas. For example, in a large solar flare, an energy of the order of 10^{32} ergs is released within a few minutes. In a magnetically dominated solar corona, it is believed that magnetic reconnection is the process responsible for such eruptions. With the typical solar coronal parameters: Alfvén speed v a ≈ 10 6 m/s, the characteristic length scale L ≈ 10^7 m (typical height of magnetic loops) and magnetic diffusivity (calculated using Spitzer resistivity) $\lambda \approx 1 m^2$ /s; the magnetic Reynolds number R_M are 10^{13} and 10s. Following Sweet-Parker model, the reconnection rate v_i and the characteristic reconnection time τ_d are approximately 0.3m/s and 3×10^6 s respectively. The τ_d is clearly much larger than the observed flaring time—inferring the model to be inadequate to account for solar flares. In an attempt to increase the reconnection rate, Petschek assumed a two-dimensional current sheet where L ≈ l. Further physical arguments (which are beyond the scope of this course) lead to a reconnection rate given by

$$v_i \approx \frac{v_a}{\ln R_M}$$

which is known as the Petschek reconnection rate. Note that, for a given $R_{M,}$ the Petschek reconnection rate is much faster compared to the Sweet-Parker reconnection rate. Furthermore, Priest and Forbes proposed an unified model where the Sweet-Parker and Petschek rates appear as special cases. In addition to these steady state models, various time-dependent models of reconnections are proposed. However, magnetic reconnection in three-dimensional conditions is still not fully understood phenomena and it is an open area of research.

Reference: Book "Solar Magnetohydrodynamics" by Eric Priest.

Thanks for the attention!