[Estd: 1917]

Basics of Quark Model and Chromodynamics

M.Sc. Semester 4

Advanced Quantum Mechanics (EC-01)

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Quarks

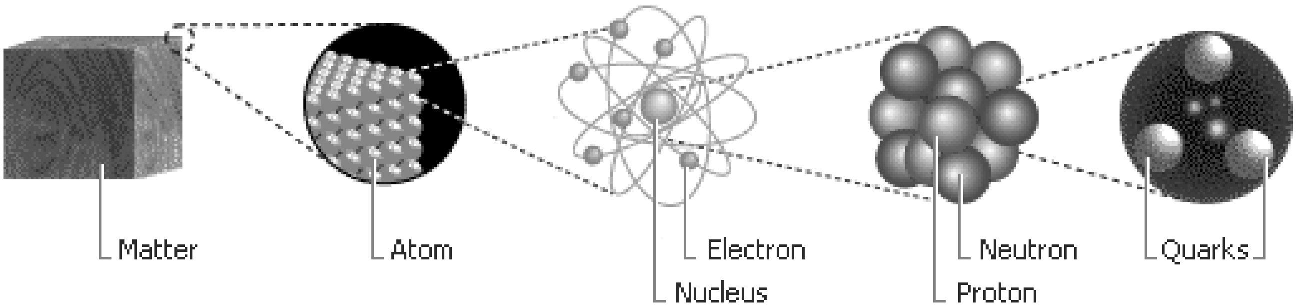
Introduction

Quarks are the smallest building blocks of matter as of today. They are the fundamental constituents of all the hadrons. They have fractional electronic charge. Quarks never exist in isolation in nature. They are always found in combination with other quarks or antiquark in matter. By studying particles, scientists have determined the properties of quarks. Protons and neutrons, the particles that make up the nuclei of the atoms consist of quarks. Without quarks there would be no atoms, and without atoms, matter would not exist. Quarks form triplets called baryons such as proton and neutron or doublets called mesons such as Kaons and pi mesons.

Quarks exist in six components:

up (u), down(d), charm (c), strange (s), bottom (b), and top (t)

which are known as quark flavors. Each quark has an antimatter counterpart called antiquark (designated by a line over the latter symbol) of opposite charge, baryon number, strangeness etc. The six flavors of quarks together with the six leptons (the electron, muon, tau, and their neutrinos) can be called truly elementary in nature.



Quarks Are Building Blocks Of Matter

Characteristics and Behaviour

The six flavors of quarks are divided into three different categories called generations. The up and down quarks belong to the first generation, the charm and strange belong to the second generation, and the top and bottom belong to the third generation. Unlike other elementary particles, quarks have electric charge which is a fraction of standard charge i.e. the charge (e) of one proton. The different flavors of quarks have different charges. The up (u), charm (c) and top (t) quarks have electric charge +2e/3 and the down (d), strange (s) and bottom (b) quarks have charge -e/3; -e is the charge of an electron. The masses of these quarks vary greatly, and of the six, only the up and down quarks, which are by far the lightest, appear to play a direct role in normal matter. There are four fundamental forces that act between the quarks. They are strong force, electromagnetic force, weak force and gravitational force. The quantum of strong force is gluon. Gluons bind quarks or quark and antiquark together to form hadrons. The electromagnetic force has photon as quantum that couples the quarks charge. The weak force causes the beta decay and allows a quark of one type to change into another. The gravitational force couple’s quark mass.

Table -1 Quark and their properties

S. N.	Properties Quarks	Electric Charge (Q)	Spin (J)	Baryon Number (N)	Strangeness (S)	Charmness (C)	Bottomness (B)	Topness (T)	Mass in MeV/c^2	Anti-quarks
1	Up (u)	+2e/3	1/2	1/3	0	0	0	0	3-9	$\bar{u}$
2	Down (d)	-e/3	1/2	1/3	0	0	0	0	1-5	$\bar{d}$
3	Strange (s)	-e/3	1/2	1/3	-1	0	0	0	125	$\bar{s}$
4	Charm (c)	+2e/3	1/2	1/3	0	+1	0	0	1,200	$\bar{c}$
5	Bottom (b)	-e/3	1/2	1/3	0	0	-1	0	4,200	$\bar{b}$
6	Top (t)	+2e/3	1/2	1/3	0	0	0	+1	175000	$\bar{t}$

Quarks Composition of Hadrons

Quarks bind together into states that can be observed directly in the laboratory as hadrons, particles with the strong force. The best-known examples of hadrons are the nucleons, the proton and neutron, from which all atomic nuclei are formed. The idea of quarks arose to explain the regularities of hadrons states, their charges and spins could be readily explained (and even predicted) by simply combining the then known u, d and s quarks. The quark model consists of all these features. The confinement of quarks in hadrons is perhaps the most extraordinary feature of quarks A free quark, one that is separated from a nucleon, would be readily detectable because its charge would be -2/3 or 1/3 of the charge of an electron. No convincing evidence for such a particle has been found, and it is now believed that in quarks confinement is an unavoidable consequence. There are three quarks to make a proton or a neutron. The proton and neutron are different because the proton is a combination of two u quarks and one d quark and hence it has a total charge of 2(2e/3) + (-e/3) = +e. The neutron is made up of one u and two d quarks and hence it has total charge (2e/3) + 2(-e/3) = 0. Figure below shows quark composition of proton and neutron.

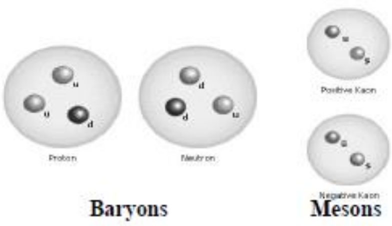


Fig-2 Quarks in baryons and mesons

In addition to nucleons, other combinations of three quarks have been observed, and all are known collectively as baryons. For example, from the u, d, and s quarks, it is possible to make ten distinct combinations, and all have been seen (notice that they all have charges that are integer multiples of e). In addition, baryons with c and b quarks have also been observed. All the baryons except for the proton and neutron decay quite rapidly because the weak interactions make all the quarks unstable. In addition to baryons, quarks can combine with their antiparticles, the antiquark, to form mesons. For example, the combinations us and us can form the K- and K+, the Kaons, of electric charge-e and +e, respectively as shown in the figure. All the mesons are bosons with integer spins. The table below shows quark composition of some hadrons.

Quarks Composition of Some Baryons and Mesons

Baryons	Quarks Composition
Proton (p)	uud
Neutron (n)	udd
Σ^+	uus
Sigma Σ^-	dds
Σ^0	uds
Xi Ξ^0	uss
Ξ^-	dss
Omega (Ω^-)	sss
Delta Δ^-	ddd
Δ^{++}	uuu

Mesons	Quarks Composition
π^+	$u\bar{d}$
Pion π^0	$u\bar{u}$
π^-	$\bar{u}d$
K^+	$u\bar{s}$
Kaons K^0	$d\bar{s}$
K^-	$\bar{u}s$
J/ψ particle	$c\bar{c}$
Upsilon (ϒ)	$b\bar{b}$

Color of Quarks

It is well known that particles are of two types: fermions, whose spins are odd multiples of ½, and bosons, which have zero or integral spin. Fermions obey the Pauli's Exclusion Principle that states that two of them cannot occupy the same physical state simultaneously. The Ω- (sss) was found to have a spin of 3/2, and the simplest configuration of the ground state to give this result would require all three quarks to have spins up. This violates the exclusion principle as the quarks are supposed to be fermions. This problem was resolved by the introduction of the concept of color, as formulated in quantum chromodynamics (QCD). In this theory of strong interactions color has nothing to do with the colors of the everyday world but rather represents a property of quarks that is the source of the strong force. The colors red, green, and blue are ascribed to quarks, and their opposites, antired, antigreen, and antiblue, are ascribed to antiquarks. According to QCD, all combinations of quarks must contain mixtures of these imaginary colors that cancel out one another, with the resulting particle having no net color. A baryon, for example, always consists of a combination of one red, one green, and one blue quark

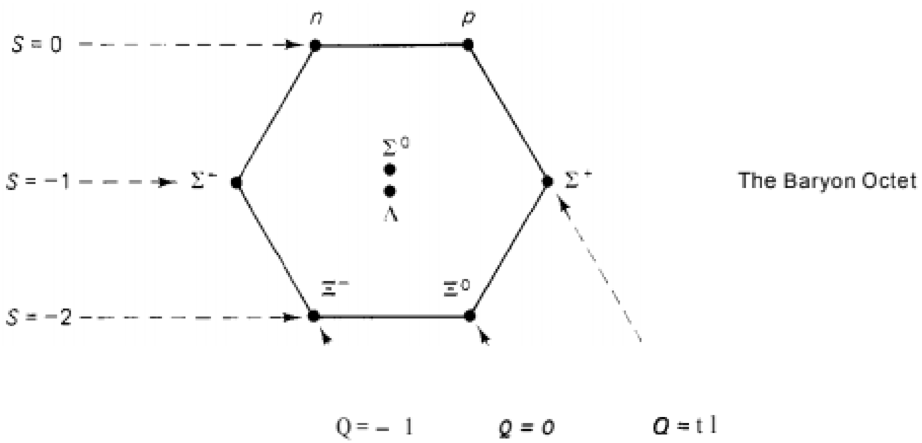
and so never violates the exclusion principle. Thus, the three quarks in the Ω- have their spins parallel and the same spatial wave function, but these are of three different colors. Since all the three colors are equally present, it is said that the state is color-neutral, and since this additional quantum number is not seen in real hadrons, one knows that these states must all be colorless. The baryons accomplish colorlessness by being composed of three different-colored quarks; the mesons are color-neutral as they are composed of quarks and antiquarks. All hadrons have zero net r, b and g colors; they are all colorless. The property of color in the strong force plays a role analogous to that of electric charge in the electromagnetic force, and just as charge implies the exchange of photons between charged particles, so does color involve the exchange of massless particles called gluons among quarks. Just as photons carry electromagnetic force, gluons transmit the forces that bind quarks together. Quarks change their color as they emit and absorb gluons, and the exchange of gluons maintains proper quark color distribution. The binding forces carried by the gluons tend to be weak when quarks are close together. Within a proton (or other hadrons), at distances of less than 10−15 meter, quarks behave as though they were nearly free.

When one begins to draw the quarks apart, however, as when attempting to knock them out of a proton, the effect of the force grows stronger. This is because gluons have the ability to create other gluons as they move between quarks. Thus, if a quark starts to speed away from its companions after being struck by an accelerated particle, the gluons utilize energy that they draw from the quark's motion to produce more gluons. The larger the number of gluons

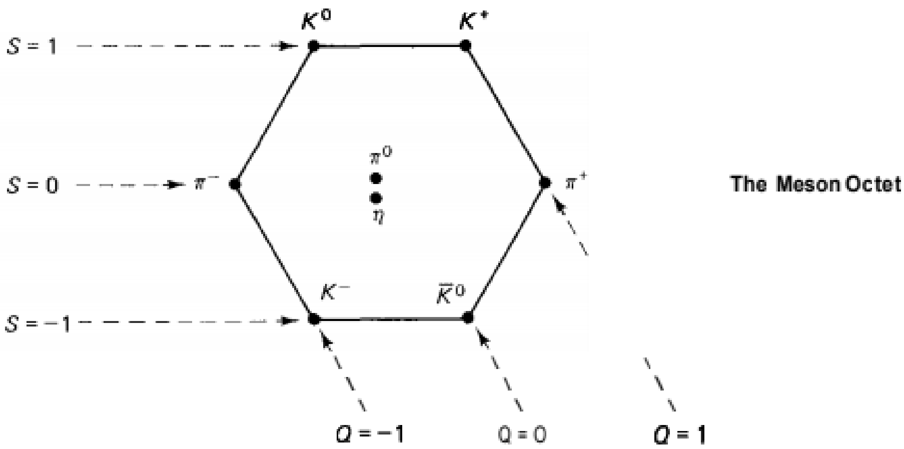
exchanged among quarks, the stronger the effective binding forces become. Supplying additional energy to extract the quark only results in the conversion of that energy into new quarks and antiquarks with which the first quark combines. This phenomenon is observed at high-energy particle accelerators in the production of "jets" of new particles that can be associated with a single quark.

THE EIGHTFOLD WAY (1961 -1 964)

The Mendeleev of elementary particle physics was Murray Gell-Mann, who introduced the so-called Eightfold Way in 196 1 .21 (Essentially the same scheme was proposed independently by Ne'eman.) The Eightfold Way arranged the baryons and mesons into weird geometrical patterns, according to their charge and strangeness. The eight lightest baryons fit into a hexagonal array, with two particles at the center:

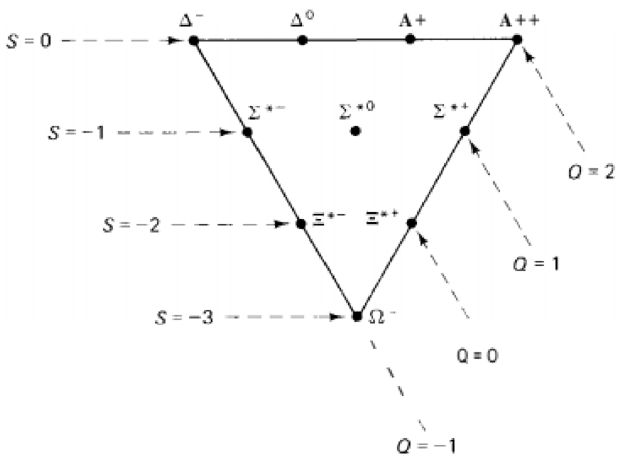


This group is known as the baryon octet. Notice that particles of like charge lie along the downward-sloping diagonal lines: $Q = +1$ (in units of the proton charge) for the proton and the $\sum^{+}$; $Q = 0$ for the neutron, the lambda, the $\sum^{0}$, and the Ξ^{0} ; $Q = -1$ for the $\sum^{-}$ and the Ξ^{-}. Horizontal lines associate particles of like strangeness: $S = 0$ for the proton and neutron, $S = -1$ for the middle line and $S = -2$ for the two Ξ's. The eight lightest mesons fill a similar hexagonal pattern, forming the (pseudo-scalar) meson octet:



Once again, diagonal lines determine charge, and horizontals determine strangeness; but this time the top line has S = 1, the middle line S = 0, and the bottom line S = - I. (This discrepancy is a historical accident; Gell-Mann could just as well have assigned S = 1 to the proton and neutron, S = 0 to the $\sum$'s and the Λ, and S = -1 to the $\sum$'s. In 1953 he had no reason to prefer that choice, and it seemed most natural to give the familiar particles-proton, neutron, and pion- a strangeness of zero. After 196 1 a new term-hypercharge-was introduced, which was equal to S for the mesons and to S + 1 for the baryons. But later developments showed that strangeness was the better quantity after all, and the word “hypercharge” has now been taken over for a quite different purpose.) Hexagons were not the only figures allowed by the Eightfold Way; there was also, for example, a triangular array, incorporating 10 heavier baryons-

The Baryon decuplet:



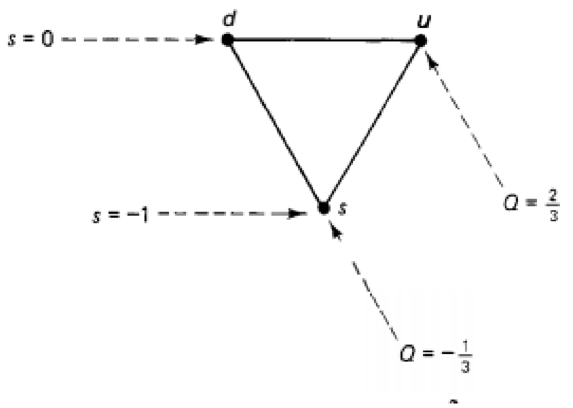
The Baryon Decuplet

THE QUARK MODEL (1964)

But the very success of the Eightfold Way begs the question: Why do the hadrons fit into these curious patterns?

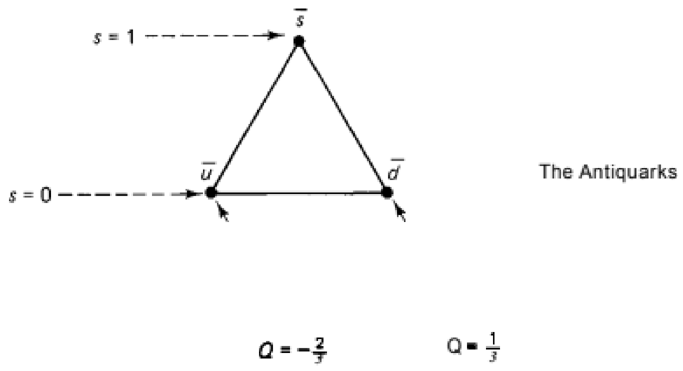
In particle physics, the quark model is a classification scheme for hadrons in terms of their valence quarks—the quarks and antiquarks which give rise to the quantum numbers of the hadrons. The quark model underlies "flavor SU(3)",or the Eightfold Way, the successful classification scheme organizing the large number of lighter hadrons that were being discovered starting in the 1950s and continuing through the 1960s. It received experimental verification beginning in the late 1960s and is a valid effective classification of them to date. The model was independently proposed by physicists Murray Gell-Mann, who dubbed them "quarks" in a concise paper, and George Zweig, who suggested "aces" in a longer manuscript. André Peterman also touched upon the central ideas from 1963 to 1965, without as much quantitative substantiation. Today, the model has essentially been absorbed as a component of the established quantum field theory of strong and electroweak particle interactions, dubbed the Standard Model. Hadrons are not really "elementary", and can be regarded as bound states of their "valence quarks" and antiquarks, which give rise to the quantum numbers of the hadrons. These quantum numbers are labels identifying the hadrons, and are of two kinds. One set comes from the Poincare symmetry—JPC, where J, P and C stand for the total angular momentum-symmetry, and C-symmetry, respectively. The remaining are flavor quantum numbers such as the isospin, strangeness, charm, and so on. The strong interactions binding the quarks together are insensitive to these quantum numbers, so variation of them leads to systematic mass and coupling relationships among the hadrons in the same flavor multiplate. All quarks are assigned a baryon number of 1⁄3. Up, charm and top quarks have an electric charge of +2⁄3, while the down, strange, and bottom quarks have an electric charge of −1⁄3. Antiquarks have the opposite quantum numbers. Quarks are spin-1⁄2 particles, and thus fermions. Each quark or antiquark obeys the Gell-Mann−Nishijima formula individually, so any additive assembly of them will as well. Mesons are made of a valence quark−antiquark pair (thus have a baryon number of 0), while baryons are made of three quarks (thus have a baryon number of 1). This article discusses the quark model for the up, down, and strange flavors of quark (which form an approximate flavor SU (3) symmetry). There are generalizations to larger number of flavors.

The Periodic Table had to wait many years for quantum mechanics and the Pauli exclusion principle to provide its explanation. An understanding of the Eightfold Way, however, came already in 1964, when Gell-Mann and Zweig independently proposed that all hadrons are in fact composed of even more elementary constituents, which Gell-Mann called quarks. The quarks come in three types (or “flavors”), forming a triangular **<u>“Eightfold-Way” pattern:</u>**



The Quarks

The u (for "up") quark carries a charge of $\frac{2}{3}$ and a strangeness of zero; the d("down") quark carries a charge of - $\frac{1}{3}$and S = 0; the s (originally "sideways", but now more commonly "strange") quark has Q = - $\frac{1}{3}$ and S = - 1. To each quark (q) there corresponds an antiquark ($\overline{q}$), with the opposite charge and strangeness:



The quark model asserts that

1. Every baryon is composed of three quarks (and every antibaryon is composed of three antiquarks).

2. Every meson is composed of a quark and an antiquark.

With these two rules it is a matter of elementary arithmetic to construct the baryon decuplet and the meson octet. All we need to do is list the combinations of three quarks (or quark-antiquark pairs), and add up their charge and strangeness:

THE BARYON DECUPLET

444	Q	S	Baryon
uuu	2	0	Λ++
uud	1	0	Λ+
udd	0	0	Δ^0
ddd	−1	0	Δ^-
uus	1	−1	Z*+
uds	0	−1	Σ^{*0}
dds	−1	−1	Σ^{*-}
uss	0	−2	Ξ^{*0}
dss	−1	−2	Ξ^{*-}
sss	−1	−3	R−

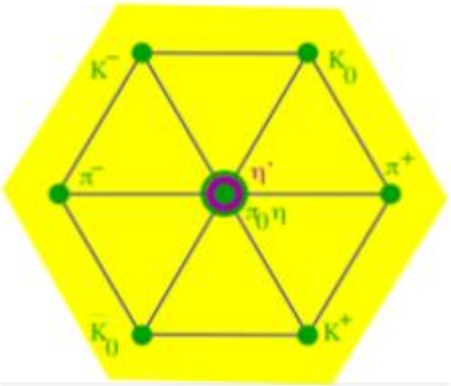
Notice that there are 10 combinations of three quarks. Three u's, for instance, at $Q = \frac{2}{3}$ each, yield a total charge of +2, and a strangeness of zero. This is the Δ^{++} particle. Continuing down the table, we find all the members of the decuple ending with the Ω^-, which is evidently made of three s quarks.

A similar enumeration of the quark-antiquark combinations yields the meson table:

THE MESON NONET

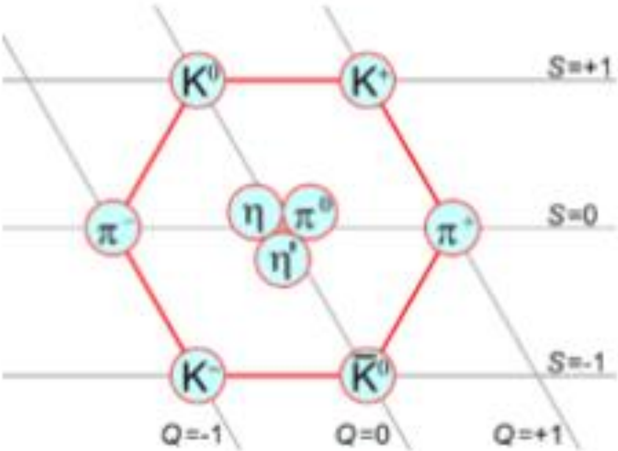
$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	7r+
$d\bar{u}$	−1	0	π^-
$d\bar{d}$	0	0	η
US	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	− 1	−1	K^-
$s\bar{d}$	0	− 1	$\bar{K}^0$
$s\bar{s}$	0	0	??

But wait! There are nine combinations here, and only eight particles in the meson octet. The quark model requires that there be a third meson (in addition to the to π^0 and the η) with Q = 0 and S = 0. As it turns out, just such a particle had already been found experimentally-the η'. In the Eightfold Way the η' had been classified as a singlet, all by itself. According to the quark model it properly belongs with the other eight mesons to form a meson nonet. (Actually, since $u\bar{u}$,$d\bar{d}$, and $s\bar{s}$ all have Q = 0 and S = 0, it is not possible to say, on the basis of anything we have done so far, which is theπ^0 , which the η, and which the η'. But never mind, the point is that there are three mesons with Q = S = 0.) By the way, the antimensions automatically fall in the same supermultiple as the mesons: $u\bar{d}$ is the antiparticle of $d\bar{u}$, and vice versa.

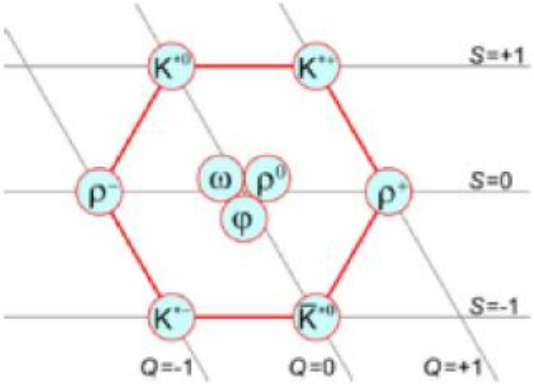


The pseudo scalar meson nonet. Members of the original meson "octet" are shown in green, the singlet in magenta. Although these mesons are now grouped into a nonet, the Eightfold Way name derives from the patterns of eight for the mesons and baryons in the original classification scheme.

Mesons



Pseudo scalar Mesons of spin 0 form a nonet



Mesons Of spin 1 form a nonet

The Eightfold Way classification is named after the following fact. If we take three flavors of quarks, then the quarks Figure 2: Pseudo scalar mesons of spin 0 form a nonet . Mesons of spin 1 form a nonet lie in the fundamental representation, 3 (called the triplet) of flavor SU(3). The antiquarks lie in the complex conjugate representation 3. The nine states (nonet) made out of a pair can be decomposed into the trivial representation, 1 (called the singlet), and the adjoint representation, 8 (called the octet). The notation for this decomposition is,

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$

shows the application of this decomposition to the mesons. If the flavor symmetry were exact (as in the limit that only the strong interactions operate, but the electroweak interactions are notionally switched off), then all nine mesons would have the same mass. However, the physical content of the full theory includes consideration of the symmetry breaking induced by the quark mass differences, and considerations of mixing between various multiples (such as the octet and the singlet). N.B. Nevertheless, the mass splitting between the η and the η′ is larger than the quark model can accommodate, and this "η–η′ puzzle" has its origin in topological peculiarities of the strong interaction vacuum, such as instanton configurations. Mesons are hadrons with zero baryon number. If the quark antiquark pair are in an orbital angular momentum L state, and have spin S, then

* $|L - S| \leq J \leq L + S$, where S = 0 or 1,

* $P = (-1)^{L+1}$, where the 1 in the exponent arises from the intrinsic parity of the quark–antiquark pair.

* $C = (-1)^{L+S}$ for mesons which have no flavor. Flavored mesons have indefinite value of C.

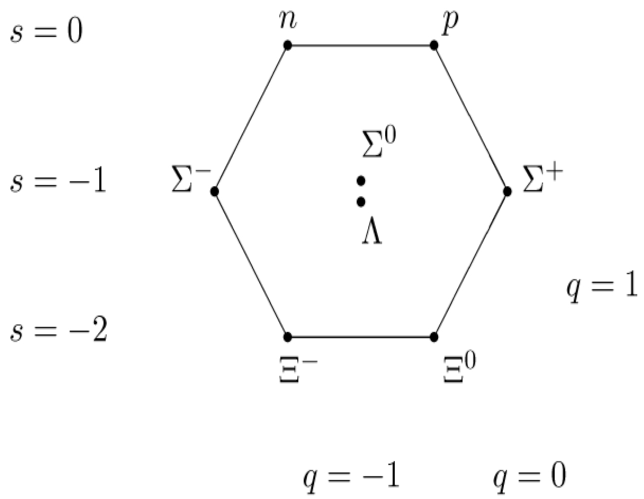
*For isospin I = 1 and 0 states, one can define a new multiplicative quantum number called the G-parity such that

$G = (-1)^{I+L+S}$.

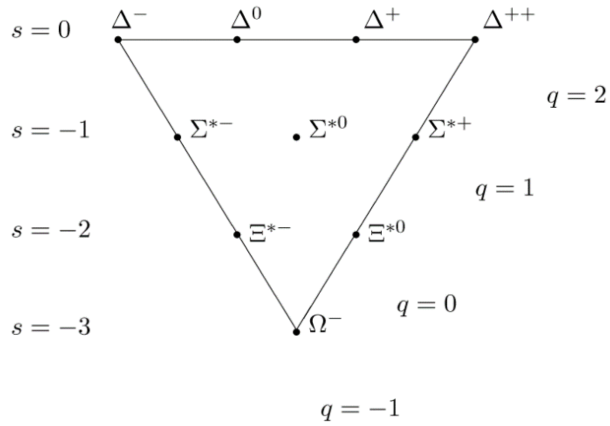
If $P = (-1)^{J}$, then it follows that S = 1, thus

PC= 1. States with these quantum numbers are called natural parity states; while all other quantum numbers are thus called exotic.

Baryons



The $S = \frac{1}{2}$ ground state baryon octet



The S = $\frac{3}{2}$ baryon decuplet

Since quarks are fermions, the spin– statistics theorem implies that the wavefunction of a baryon must be antisymmetric under exchange of any

two quarks. This antisymmetric wavefunction is obtained by making it fully antisymmetric in color, discussed below, and symmetric in flavor, spin and

space put together. With three flavors, the decomposition in flavor is

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A$$

The decuplet is symmetric in flavor, the singlet antisymmetric and the two octets have mixed symmetry. The space and spin parts of the states are thereby fixed once the orbital angular momentum is given.

It is sometimes useful to think of the basis states of quarks as the six states of three flavors and two spins per flavor. This approximate symmetry is called spin-flavor SU(6). In terms of this, the decomposition is,

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A$$

The 56 states with symmetric combination of spin and flavour decompose under flavor SU(3) into

$$\mathbf{56} = \mathbf{10}^{\frac{3}{2}} \oplus \mathbf{8}^{\frac{1}{2}}$$

where the superscript denotes the spin, S, of the baryon. Since these states are

symmetric in spin and flavor, they should also be symmetric in space—a condition

that is easily satisfied by making the orbital angular momentum L = 0. These are the ground state baryons. The S = 1⁄2 octet baryons are the two

nucleons(p^+,n^0) and Sigmas($\Sigma^+, \Sigma^0, \Sigma^-$), Xis ($\Xi,^0\ \Xi^-$), and the Lambda ($\Lambda^0$).

The S = 3⁄2 decuplet baryons Deltas ($\Delta^{++}, \Delta^+, \Delta^{0,}, \Delta$-), Sigmas($\Sigma^+, \Sigma^0, \Sigma^-$), Xis ($\Xi,^0\ \Xi^-$),

And the Omega(Ω^-).

For example, the constituent quark model wavefunction for the proton is

$$|p_\uparrow\rangle = \frac{1}{\sqrt{18}}[2|u_\uparrow d_\downarrow u_\uparrow\rangle + 2|u_\uparrow u_\uparrow d_\downarrow\rangle + 2|d_\downarrow u_\uparrow u_\uparrow\rangle - |u_\uparrow u_\downarrow d_\uparrow\rangle - |u_\uparrow d_\uparrow u_\downarrow\rangle - |u_\downarrow d_\uparrow u_\uparrow\rangle - |d_\uparrow u_\downarrow u_\uparrow\rangle - |d_\uparrow u_\uparrow u_\downarrow\rangle - |u_\downarrow u_\uparrow d_\uparrow\rangle]$$

Mixing of baryons, mass splitting within and between multiplets, and magnetic moments are some of the other quantities that the model predicts successfully.

Flavour symmetry

In the early days of nuclear physics, it was realized that the proton and neutron have very similar masses and that the nuclear force is approximately charge independent. In other words, the strong force potential is the same for two protons, two neutrons or a neutron and a proton

$$V_{pp} \approx V_{np} \approx V_{nn}.$$

Heisenberg suggested that if you could switch off the electric charge of the proton, there would be no way to distinguish between a proton and a neutron. To reflect this observed symmetry of the nuclear force, it was proposed that the neutron and proton could be considered as two states of a single entity, the nucleon, analogous to the spin-up and spin-down states of a spin-half particle,

$$P = \binom{1}{0} \text{ and } n = \binom{0}{1}.$$

This led to the introduction of the idea of isospin, where the proton and neutron form an isospin doublet with total isospin $I = \frac{1}{2}$ and third component of isospin $I_3 = \pm\frac{1}{2}$. The charge independence of the strong nuclear force is then expressed in terms of invariance under unitary transformations in this isospin space. One such transformation would correspond to replacing all protons with neutrons and vice versa. Physically, isospin has nothing to do with spin. Nevertheless, it will be shown in the following section that isospin satisfies the same SU(2) algebra as spin.

Flavour symmetry of the strong interaction

The idea of proton/neutron isospin symmetry can be extended to the quarks. Since the QCD interaction treats all quark flavours equally, the strong interaction possesses a flavour symmetry analogous to isospin symmetry of the nuclear force. For a system of quarks, the Hamiltonian can be broken down into three components

$$\hat{H} = \hat{H}^0 + \hat{H}_{strong} + \hat{H}_{em},$$

Where $\hat{H}_0$ is the kinetic and rest mass energy of the quarks, and $\hat{H}_{strong}$ and $\hat{H}_{em}$ are respectively the strong and electromagnetic interaction terms. If the (effective) masses of the up- and down-quarks are the same, and $\hat{H}_{em}$ is small compared to $\hat{H}_{strong}$, then to a good approximation the Hamiltonian possesses an up–down (ud) flavour symmetry; nothing would change if all the up-quarks were replaced by down-quarks and vice versa. One simple consequence of an exact ud flavour symmetry is that the existence of a (uud) bound quark state implies that there will a corresponding state (ddu) with the same mass.

The above idea can be developed mathematically by writing the up- and down- quarks as states in an abstract flavour space

$$u = \binom{1}{0} \text{ and } d = \binom{0}{1}.$$

If the up- and down-quarks were indistinguishable, the flavour independence of the QCD interaction could be expressed as an invariance under a general unitary transformation in this abstract space

$$\binom{u'}{d'} = \hat{U}\binom{u}{d} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}\binom{u}{d}.$$

Since a general 2×2 matrix depends on four complex numbers, it can be described by eight real parameters. The condition $\hat{U}\hat{U}^{\dagger} = I$, imposes four constraints; therefore a 2×2 unitary matrix can be expressed in terms of four real parameters or, equivalently, four linearly independent 2×2 matrices representing the generators of the transformation

$$\hat{U} = \exp\left(i\alpha_i \hat{G}_i\right).$$

One of the generators can be identified as

$$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\Phi}$$

This U(1) transformation corresponds to multiplication by a complex phase and is therefore not relevant to the discussion of transformations between different flavour states. The remaining three unitary matrices form a special unitary SU(2) group with the property[1] det U = 1. The three matrices representing the Hermitian generators of the SU(2) group are linearly independent from the identity and are there- fore traceless. A suitable choice[2] for three Hermitian traceless generators of the ud flavour symmetry are the Pauli spin-matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{and}\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The ud flavour symmetry corresponds to invariance under SU(2) transformations leading to three conserved observable quantities defined by the eigenvalues of Pauli spin-matrices. The algebra of the ud flavour symmetry is therefore identical to that of spin for a spin-half particle. In analogy with the quantum-mechanical treatment of spin-half particles, isospin $\hat{\mathbf{T}}$ is defined in terms of the Pauli spin-matrices

$$\hat{T} = \frac{1}{2}\sigma.$$

Any finite transformation in the up–down quark flavour space can be written in terms of a unitary transformation

$$\hat{U} = e^{i\alpha\cdot\hat{\mathbf{T}}},$$

Such that

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = e^{i\alpha\cdot\hat{T}}\begin{pmatrix} u \\ d \end{pmatrix}$$

where $\alpha \cdot \hat{\mathbf{T}} = \alpha_1\hat{T}1 + \alpha_2\hat{T}2 + \alpha_3\hat{T}3$. Hence, the general flavour transformation is a "rotation" in flavour space, not just the simple interchange of up and down quarks. A general unitary transformation in this isospin space would amount to relabelling the up-quark as a linear combination of the up-quark and the down-quark. If the flavour symmetry were exact, and the up- and down-quarks were genuinely indistinguishable, this would be perfectly acceptable. However, because the up- and down-quarks have different charges, it does not make sense to form states which are linear combinations of the two, as this would lead to violations of electric charge conservation. Consequently, the only physical meaningful isospin transformation is that which corresponds to relabelling the states, u ↔ d.

SU(3) flavour symmetry

The SU(2) flavour symmetry described above is almost exact because the difference in the masses of the up- and down-quarks is small and the Coulomb interaction represents a relatively small contribution to the overall Hamiltonian compared to the strong interaction. It is possible to extend the flavour symmetry to include the strange quark. The strong interaction part of the Hamiltonian treats all quarks equally and therefore possesses an exact uds flavour symmetry. However, since the mass of the strange quark is different from the masses of the up- and down-quarks, the overall Hamiltonian is not flavour symmetric. Nevertheless, the difference between m_s and $m_{u/d}$, which is of the order 100 MeV, is relatively small compared to the typical binding energies of baryons, which are of order 1 GeV. It is therefore possible to proceed as if the overall Hamiltonian possessed a uds flavour symmetry. However, the results based on this assumption should be treated with care as, in reality, the symmetry is only approximate.

The assumed uds flavour symmetry can be expressed by a unitary transformation in flavour space

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

In general, a 3×3 matrix can be written in terms of nine complex numbers, or equivalently 18 real parameters. There are nine constraints from requirement of unitarity, $\hat{U}\hat{U}^{\dagger} = I$. Therefore $\hat{U}$ can be expressed in terms of nine linearly independent 3×3 matrices. As before, one of these matrices is the identity matrix multiplied by a complex phase and is not relevant to the discussion of transformations between different flavour states. The remaining eight matrices form an SU(3) group and can be expressed in terms of the eight independent Hermitian generators $\hat{T}_i$ such that the general SU(3) flavour transformation can be expressed as

$$\hat{U} = e^{i\alpha \cdot \hat{\mathbf{T}}}.$$

The eight generators are written in terms of eight λ-matrices with

$$\hat{T} = \frac{1}{2}\lambda,$$

where the matrices act on the SU(3) representations of the u, d and s quarks

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ \text{ and } \ s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The SU(3) uds flavour symmetry contains the subgroup of SU(2) $u \leftrightarrow d$ flavour symmetry. Hence, three of the λ-matrices correspond to the SU(2) ud isospin symmetry and have the Pauli spin-matrices in the top left 2×2 block of the 3×3 matrix with all other entries zero,

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ \text{ and}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The third component of isospin is now written in terms of the operator

$$\hat{T}_3 = \frac{1}{2}\lambda_3,$$

Such that

$$\hat{T}^3 u = +\frac{1}{2}u, \quad \hat{T}^3 d = -\frac{1}{2}d \ \text{ and } \hat{T}^3 s = 0.$$

As before, isospin lowering and raising operators are defined as $T_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$. The remaining λ-matrices can be identified by realising that the SU(3) uds flavour symmetry also contains the subgroups of SU(2) $u \leftrightarrow s$ and SU(2) $d \leftrightarrow s$ flavour symmetries, both of which can also be expressed in terms of the Pauli spin-matrices. The corresponding 3×3 λ-matrixes for the $u \leftrightarrow s$ symmetry are

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \ \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \text{ and } \lambda_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

And for the $d \leftrightarrow s$ symmetry they are

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \text{ and } \ \lambda_Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Of the nine λ-matrices identified above, only eight are independent; one of the three diagonal matrices, λ_3, λ_X and λ_Y, can be expressed in terms of the other two. Because the u $\leftrightarrow$ d symmetry is nearly exact, it is natural to retain λ_3 as one of the eight generators of the SU(3) flavour symmetry. The final generator is chosen as the linear combination of λ_X and λ_Y that treats u and d quarks symmetrically

$$\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The eight matrices used to represent the generators of the SU(3) symmetry, known as the Gell-Mann matrices, are therefore

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

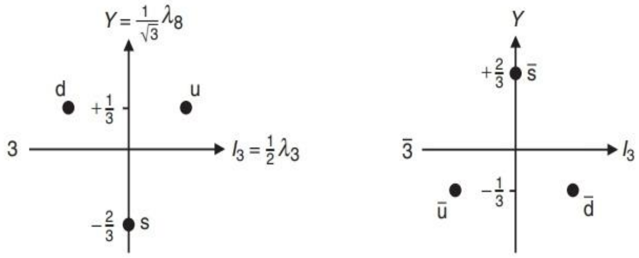
SU(3) flavour states

For the case of SU(2) flavour symmetry there are three Hermitian generators, each of which corresponds to an observable quantity. However, since the generators do not commute, they correspond to a set of incompatible variables. Consequently SU(2) states were defined in terms of the eigenstates of the third component of isospin $\hat{T}_3$ and the total isospin $\hat{T}^2 = \hat{T}^2 + \hat{T}^2 + \hat{T}^2$. In SU(3) there is an analogue of total isospin, which for the fundamental representation of the quarks can be written

$$\hat{T}^2 = \sum_{i=1}^{8} \hat{T}_i^2 = \frac{1}{4}\sum_{i=1}^{8} \lambda_i^2 = \frac{4}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Of the eight SU(3) generators, only $T_3 = \frac{1}{2}\lambda_3$ and $T_8 = \frac{1}{2}\lambda_8$ commute and therefore describe compatible observable quantities. Hence, in addition to the analogue of the total isospin, SU(3) states are described in terms of the eigenstates of the λ_3 and λ_8 matrices. The corresponding quantum numbers are the third component of isospin and the flavour hypercharge defined by the operators

$$\hat{T}^3 = \frac{1}{2}\lambda_3 \text{ and } \hat{Y} = \frac{1}{\sqrt{3}}\lambda_8.$$



Isospin and hypercharge in SU(3) flavour symmetry for the quarks and antiquarks.

The quarks are the fundamental "3" representation of the SU(3) flavour symmetry. Using the definitions of the quark states it is easy to verify that the isospin and hypercharge assignments of the u, d and s quarks are

$$\hat{T}_3 u = +\frac{1}{2} u \text{ and } \hat{Y} u = +\frac{1}{3} u,$$
$$\hat{T}_3 d = -\frac{1}{2} d \text{ and } \hat{Y} d = +\frac{1}{3} d,$$
$$\hat{T}_3 s = 0 \text{ and } \hat{Y} s = -\frac{2}{3} s.$$

The flavour content of a state is uniquely identified by $I_3 = n_u - n_d$ and $Y=\frac{1}{3}(n_u + n_d - 2n_s)$ where n_u, n_d and n_s are the respective numbers of up-, down- and strange quarks. The I_3 and Y quantum numbers of the antiquarks have the opposite signs compared to the quarks and they form a 3 multiplet, Whilst the Gell-Mann λ_3 and λ_8 matrices label the SU(3) states, the six remaining λ-matrices can be used to define ladder operators,

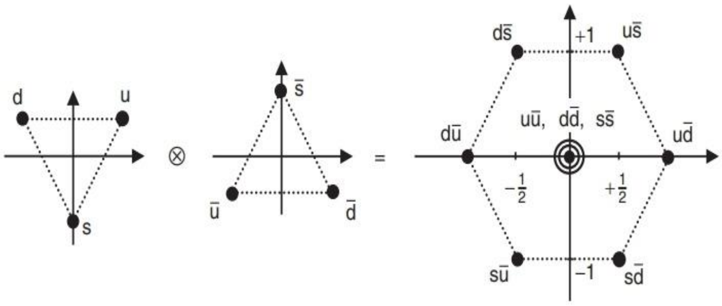
$$\hat{T}_\pm = 1/2\,(\lambda_1 \pm i\lambda_2)$$
$$\hat{V}_\pm = 1/2\,(\lambda_4 \pm i\lambda_5)$$
$$\hat{U}_\pm = 1/2\,(\lambda_6 \pm i\lambda_7)$$

which respectively step along the matrix representations of these ladder operators it is straightforward to verify that

$$\hat{V}_+ s = +u, \hat{V}_- u = +s, \quad \hat{U}_+ s = +d,$$
$$\hat{U}_- d = +s, \quad \hat{T}_+ d = +u \text{ and } \hat{T}_- u = +d$$

with all other combinations giving zero. In SU(3) flavour symmetry it is not possible to express the antiquarks as a triplet which transforms in the same way as the quark triplet. Nevertheless, following the arguments, the effect of a single ladder operator on an antiquark state must reproduce that from the corresponding SU(2) subgroup, such that the states can be obtained from

$$\hat{V}_+ \bar{u} = -\bar{s}, \hat{V}_- \bar{s} = -\bar{u}, \quad \hat{U}_+ \bar{d} = -\bar{s}, \quad \hat{U}_- \bar{s} = -\bar{d}, \quad \hat{T}_+ \bar{u} = -\bar{d} \text{ and } \hat{T}_- \bar{d} = -\bar{u}$$



SU(3) isospin and hypercharge assignments of the nine possible $q\bar{q}$ combinations.

Colour and QCD

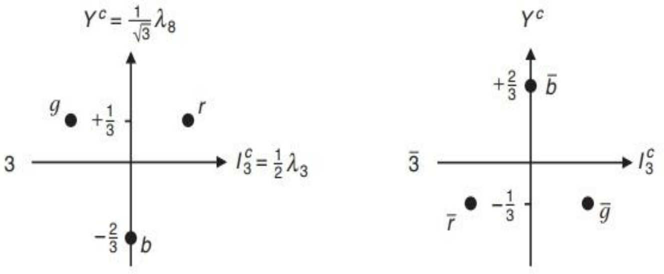
The underlying theory of quantum chromodynamics appears to be very similar to that of QED. The QED interaction is mediated by a massless photon corresponding to the single generator of the U(1) local gauge symmetry, whereas QCD is mediated by eight massless gluons corresponding to the eight generators of the SU(3) local gauge symmetry. The single charge of QED is replaced by three conserved "colour" charges, r, b and g (where colour is simply a label for the orthogonal states in the SU(3) colour space). Only particles that have non-zero colour charge couple to gluons. For this reason the leptons, which are colour neutral, do not feel the strong force. The quarks, which carry colour charge, exist in three orthogonal colour states. Unlike the approximate SU(3) flavour symmetry, the SU(3) colour symmetry is exact and QCD is invariant under unitary transformations in colour space. Consequently, the strength of QCD interaction is independent of the colour charge of the quark. In QED the antiparticles have the opposite electric charge to the particles. Similarly, in QCD the antiquarks carry the opposite colour charge to the quarks, $\vec{r}$, $\vec{g}$ and $\vec{b}$.

The three colour states of QCD can be represented by colour wavefunctions,

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

the colour states of quarks and antiquarks can be labelled by two additive quantum numbers, the third component of colour isospin I_3^c

and colour hypercharge Y^c as indicated in figure.



The representations of the colour of quarks and the anticolour of antiquarks.

The quark- gluon vertex

The SU(3) local gauge symmetry of QCD implies a conserved colour charge and an interaction between quarks and gluons of the form . By comparing the QCD interaction term to that for QED

$$-iq\gamma^{\mu}A_{\mu}\Psi \rightarrow -ig_s\frac{1}{2}\lambda^a\gamma^{\mu}G_{\mu}^a\Psi,$$

the QCD vertex factor can be identified as

$$-iq\gamma^{\mu} \rightarrow -ig_s\gamma^{\mu}\frac{1}{2}\lambda^a.$$

Apart from the different coupling constant, the quark–gluon interaction only differs from the QED interaction in the appearance of the 3×3 Gell-Mann matrices that only act on the colour part of the quark wavefunction. The quark wavefunctions therefore need to include this colour degree of freedom. This can be achieved by writing

$$u(p) \rightarrow c_i u(p)$$

where u(p) is a Dirac spinor and c_i represents one of the possible colour states

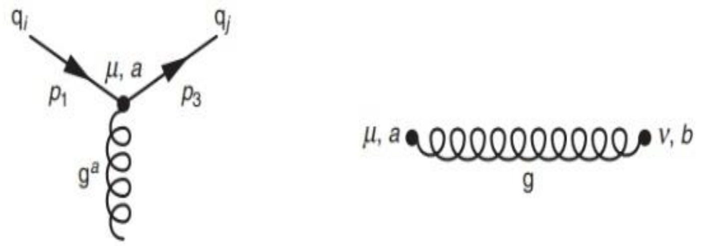
$$c_1 = r = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad c_2 = g = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \text{and} \quad c_3 = b = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

Consequently, the quark current associated with the QCD vertex, shown in figure, can be written

$$j_q^{\mu} = \bar{u}(p_3)c_j^{\dagger}\left\{-\left(\frac{1}{2}\right)ig_s\lambda^a\gamma^{\mu}\right\}c_i u(p_1),$$

Where the c_i and c_j are the colour wavefunctions of the quarks and the index a refers to gluon corresponding to the SU(3) generator T^a. (In other textbooks you may see the colour index appended to the spinor

$c_i u(p) \rightarrow u_i(p)$.)



The QCD quark–gluon vertex representing the interaction of quarks with colours i and j with a gluon of type a and the gluon propagator.

In the quark current of (10.12), the 3 × 3 Gell-Mann matrix λ^a acts on the three component colour wavefunction, whereas the 4 ×4 γ-matrices act on the four components of the Dirac spinor. Therefore the colour part of the current factorises,

$$\bar{u}(p_3)c_j^{\dagger}\{-(1/2)ig_s\lambda^a\gamma^\mu\}c_i u(p_1) = -(1/2)ig_s\left[c_j^{\dagger}\lambda^a c_i\right] \times [\bar{u}(p_3)\gamma^\mu u(p_1)].$$

Hence the qqg vertex can be written as

$$-(1/2)ig_s\lambda^a_{ji}[\bar{u}(p_3)\gamma^\mu u(p_1)],$$

Where λ^a_{ji} is just a number, namely the jith element of λ^a. therefore, the Feymann rule associated with the QCD vertex is

$$-(1/2)ig_s\lambda^a_{ji}\gamma^\mu.$$

For lowest-order diagrams, the Feymann rule for the gluon propagator of fig. is

$$-i\, g_{\mu\nu}/q^2\, \delta^{ab},$$

Where the delta-function ensures that the gluon of type a emitted at the vertex labelled μ is the same as that which is absorbed at vertex ν.

Gluons

The QCD interaction vertex includes a factor λ^a_{ji}, where i and j label the colours of the quarks. Consequently, gluons corresponding to the non-diagonal Gell-Mann matrices connect quark states of different colour. In order for colour to be con- served at the interaction vertex, the gluons must carry colour charge. For example, the gluon corresponding to λ_4, which has non-zero entries in the 13 and 31 positions, contributes to interactions involving the changes of colour r → b and b → r. This is illustrated in figure , which shows the QCD process of qq → qq scattering where the colour flow corresponds to br → rb, illustrated both in terms the colour flow in the Feynman diagram and as the two corresponding time-ordered diagrams. Because colour is a conserved charge, the interaction involves the exchange of a $b\bar{r}$ gluon in the first time-ordering and a rb gluon in the second time-ordering. From this discussion, it is clear that gluons must carry simultaneously both colour charge and anticolour charge.

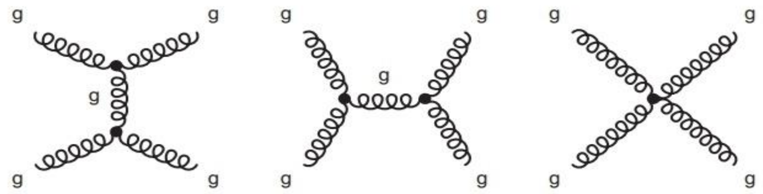
Since gluons carry a combination of colour and anticolour, there are six gluons with different colour and anticolour, $r\bar{g}$, $g\bar{r}$, $r\bar{b}$, $b\bar{r}$, $g\bar{b}$ and $b\bar{g}$. Naively one might expect three gluons corresponding to $r\bar{r}$, $g\bar{g}$, and $b\bar{b}$. However, the physical gluons correspond to the fields associated with the generators $\lambda_{1,..,8}$ of the SU(3) gauge symmetry. The gluons are therefore an octet of coloured states, analogous to the qq meson SU(3) flavour states. The colour assignments of the eight physical gluons can be written

$$r\bar{g}, g\bar{r}, r\bar{b}, b\bar{r}, g\bar{b}, b\bar{g}, 1/\sqrt{2}\,(r\bar{r} - g\bar{g}) \quad \text{and} \quad 1/\sqrt{6}\,(r\bar{r} + g\bar{g} - 2b\bar{b}).$$

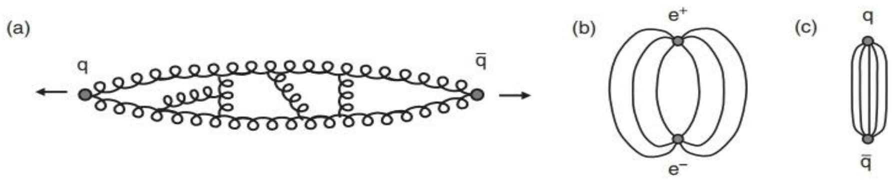
Even though two of these gluon states have $I_3^c = Y^c = 0$, they are part of a colour octet and therefore still carry colour charge (unlike the colourless singlet state).

Colour Confinement

There is a wealth of experimental evidence for the existence of quarks. However, despite many experimental attempts to detect free quarks, which would be observed as fractionally charged particles, they have never been seen directly. The non- observation of free quarks is explained by the hypothesis of colour confinement, which states that coloured objects are always confined to colour singlet states and that no objects with non-zero colour charge can propagate as free particles. Colour confinement is believed to originate from the gluon–gluon self-interactions that arise because the gluons carry colour charge, allowing gluons to interact with other gluons through diagrams such as those shown in figure



Lowest-order Feynman diagrams for the process gg $\rightarrow$ gg, formed from the triple and quartic gluon vertices



Qualitative picture of the effect of gluon–gluon interactions on the long-range QCD force.

There is currently no analytic proof of the concept of colour confinement, although there has been recent progress using the techniques of lattice QCD. Nevertheless, a qualitative understanding of the likely origin can be obtained by considering what happens when two free quarks are pulled apart. The interaction between the quarks can be thought of in terms of the exchange of virtual gluons. Because they carry colour charge, there are attractive interactions between these exchanged virtual gluons, as indicated in figure a. The effect of these interactions is to squeeze the colour field between the quarks into a tube. Rather than the field lines spreading out as in QED (figure b), they are confined to a tube between the quarks, as indicated in figure c, At relatively large distances, the energy density in the tube between the quarks containing the gluon field is constant. Therefore the energy stored in the field is proportional the separation of the quarks, giving a term in the potential of the form

$$\mathrm{V(r)} \sim \kappa\tau$$

Where experimentally $\kappa \sim \frac{1\mathrm{GeV}}{\mathrm{fm}}$. This experimentally determined value for κ corresponds to a very large force of $O(10^5)$N between any two unconfined quarks, regardless of separation! Because the energy stored in the colour field increases linearly with distance, it would require an infinite amount of energy to separate two quarks to infinity. Put another way, if there are two free colour charges in the Universe, separated by macroscopic distances, the energy stored in the resulting gluon field between them would be vast. As a result, coloured objects arrange themselves into bound hadronic states that are colourless combinations with no colour field between them. Consequently quarks are always confined to colourless hadrons.

Another consequence of the colour confinement hypothesis is that gluons, being coloured, are also confined to colourless objects. Therefore, unlike photons (the force carriers of QED), gluons do not propagate over macroscopic distances. It is interesting to note that had nature chosen a U(3) local gauge symmetry, rather than SU(3), there would be a ninth gluon corresponding to the additional U(1) generator. This gluon would be the colour singlet state,

$$\mathrm{G}_9 = 1/\sqrt{3}\left(\mathrm{r\bar{r}} + \mathrm{g\bar{g}} + \mathrm{b\bar{b}}\right).$$

Because this gluon state is colourless, it would be unconfined and would behave like a strongly interacting photon, resulting in an infinite range strong force; the Universe would be a very different (and not very hospitable) place with long-range strong interactions between all quarks.

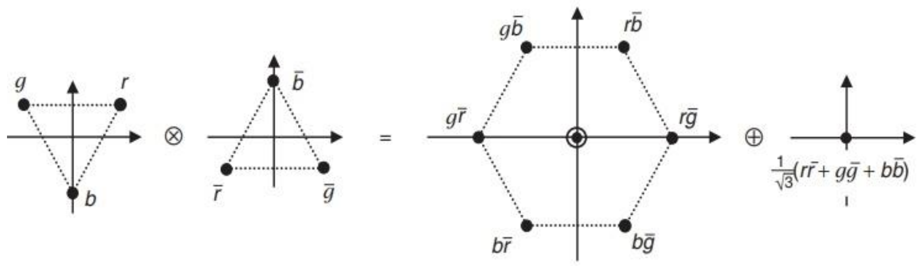
Colour confinement and hadronic states

Colour confinement implies that quarks are always observed to be confined to bound colourless states. To understand exactly what is meant by “colourless”, it is worth recalling the states formed from the combination of spin for two spin-half particles. The four possible spin combinations give rise to a triplet of spin-1 states and a spin-0 singlet ($2 \otimes 2 = 3 \oplus 1$):

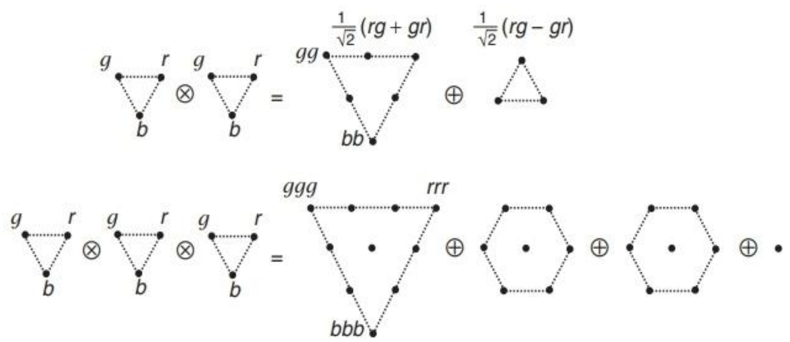
$|1,+1\rangle = \uparrow\uparrow, |1,0\rangle = 1/\sqrt{2}\ (\uparrow\downarrow + \downarrow\uparrow), |1,-1\rangle = \downarrow\downarrow$ and $|0,0\rangle = 1/\sqrt{2}\ (\uparrow\downarrow - \downarrow\uparrow)$.

The singlet state is “spinless” in the sense that it carries no angular momentum. In a similar way, SU(3) colour singlet states are colourless combinations which have zero colour quantum numbers, $I_3^c = Y^c = 0$. It should be remembered that $I_3^c = Y^c = 0$ is a necessary but not sufficient condition for a state to be colourless. The action of any of the SU(3) colour ladder operators on a colour singlet state must yield zero, in which case the state is analogous to the spinless $|0, 0\rangle$ singlet state.

The colour confinement hypothesis implies that only colour singlet states can exist as free particles. Consequently, all bound states of quarks and antiquarks must occur in colour singlets. This places a strong restriction on the structure of possible hadronic states; the allowed combinations of quarks and antiquarks are those where a colour singlet state can be formed. The algebra of the exact SU(3) colour symmetry was described in the context of SU(3) flavour symmetry and the results can be directly applied to colour with the replacements, u →r, d → g and s → b.



The colour combination of a quark and an antiquark, $3 \otimes \bar{3} = 8 \oplus 1$.



The multiplets from the colour combinations of two quarks, $3 \otimes 3 = 6 \oplus 3$, and three quarks, $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$.

First consider the possible colour wavefunctions for a bound qq state. The com- bination of a colour with an anticolour is mathematically identical to the construction of meson flavour wavefunctions in SU(3) flavour symmetry. The resulting colour multiplets, are a coloured octet and a colourless singlet. The colour confinement hypothesis implies that all hadrons must be colour singlets, and hence the colour wavefunction for mesons is

$$\Psi^{c}(q\bar{q}) = \left(1/\sqrt{3}\right)\left(r\bar{r} + g\bar{g} + b\bar{b}\right).$$

The addition of another quark (or antiquark) to either the octet or singlet state in fig. will not yield a state with $I_3^c = Y^c = 0$. Therefore, it can be concluded that bound states of $q\,q\bar{q}$ or $q\bar{q}\bar{q}$ do not exist in nature.

These arguments can be extended to the combinations of two and three quarks. the combination of two colour triplets yields a colour sextet and a colour triplet $(\bar{3})$. the absence of a colour singlet state for the qq system, implies that bound states of two quarks are always coloured objects and therefore do not exist in nature. However, the combination of three colours yields a single singlet state with the colour wavefunction

$$\Psi^{c}(qqq) = \left(1/\sqrt{6}\right)(rgb - rbg + gbr - grb + brg - bgr), \qquad \ldots\ldots (14)$$

analogous to the SU(3) flavour singlet wavefunction . This state clearly satisfies the requirement that $I^{c} = Y^{c} = 0$. The colour ladder operators can be used to confirm it is a colour singlet. For example, the action of the colour isospin raising operator T_+^c for which $T_+^c\, g = r$, gives

$$T_+^c\Psi^{c}(qqq) = (1/\sqrt{6})(rrb - rbr + rbr - rrb + brr - brr) = 0,$$

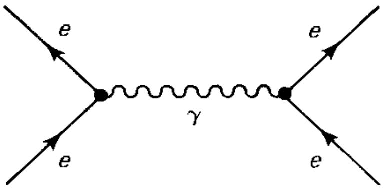
as required. Hence a SU(3) colour singlet state can be formed from the combination of three quarks and colourless bound states of qqq are observed in nature. Since the colour singlet wavefunction of (14) is totally antisymmetric, and it is the only colour singlet state for three quarks, the colour wavefunction for baryons is always antisymmetric. This justifies the assumption to construct the baryon wavefunctions.

Colour confinement places strong restrictions on the possible combinations of quarks and antiquarks that can form bound hadronic states. To date, all confirmed observed hadronic states correspond to colour singlets either in the form of mesons $(q\bar{q})$, baryons (qqq) or antibaryons $(\bar{q}\bar{q}\bar{q})$. In principle, combinations of $(q\bar{q})$ and (qqq) such as pentaquark states (qqq q̄q) could exist, either as bound states in their own right or as hadronic molecules such as (qq)-(qqq). In recent years there have been a number of claims for the existence of pentaquark states, but the evidence is (at best) far from convincing.

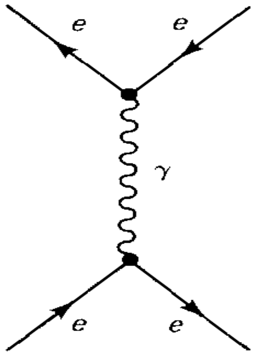
QUANTUM ELECTRODYNAMICS (QED)

Quantum electrodynamics is the oldest, the simplest, and the most successful of the dynamical theories; the others are self-consciously modelled on it. **All** electromagnetic phenomena are ultimately reducible to the following elementary process:

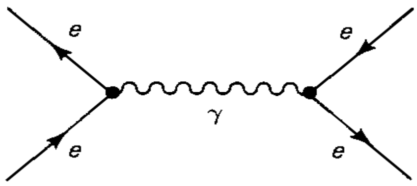
This diagram reads: Charged particle e enters, emits (or absorbs) a photon, y, and exits. Assume the charged particle is an electron, it could just as well be a quark, or any lepton except a neutrino (the latter is neutral, of course, and does not experience an electromagnetic force). To describe more complicated processes, we simply patch together two or more replicas of this primitive vertex. Consider, for example, the following:



Here, two electrons enter, a photon passes between them, and the two then exit. This diagram, then, describes the interaction between two electrons. In the classical theory we would call it the Coulomb repulsion of like charges (if the two are at rest). In QED this process **is** called Maller scattering. We say that the interaction is "mediated by the exchange of a photon," for reasons that should now be apparent. Twisting these "Feynman diagrams" around into any topological configuration, for example, we could stand the previous picture on its side:

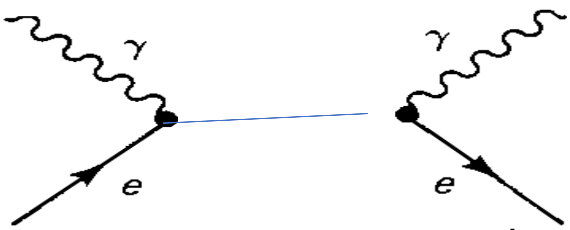


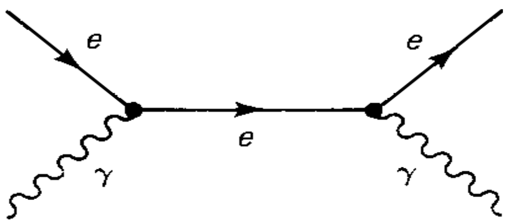
The rule of the game is that a particle line running "backward in time" (as indicated by the arrow) is to be interpreted as the corresponding antiparticle going forward. So, in this process an electron and a positron annihilate to form **a** photon, which in turn produces a new electron-positron pair. An electron and a positron went in, an electron and a positron came out. This represents the interaction of two opposite charges: their Coulomb attraction. In QED this process is called Bhabha scattering. There is a quite different diagram which also contributes:

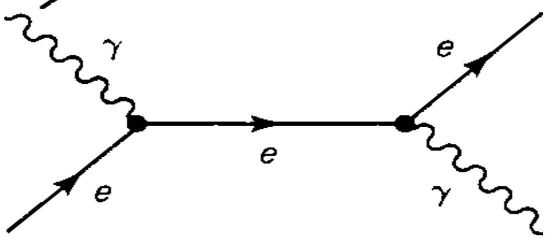


Both diagrams must be included in the analysis of Bhabha scattering.

Using just two vertices we can also construct the following diagrams, describing, respectively, pair annihilation, e- + e+ - y + y; pair production, y + y - e- + **e+;** and Compton scattering, e- + y - **e-** + y:

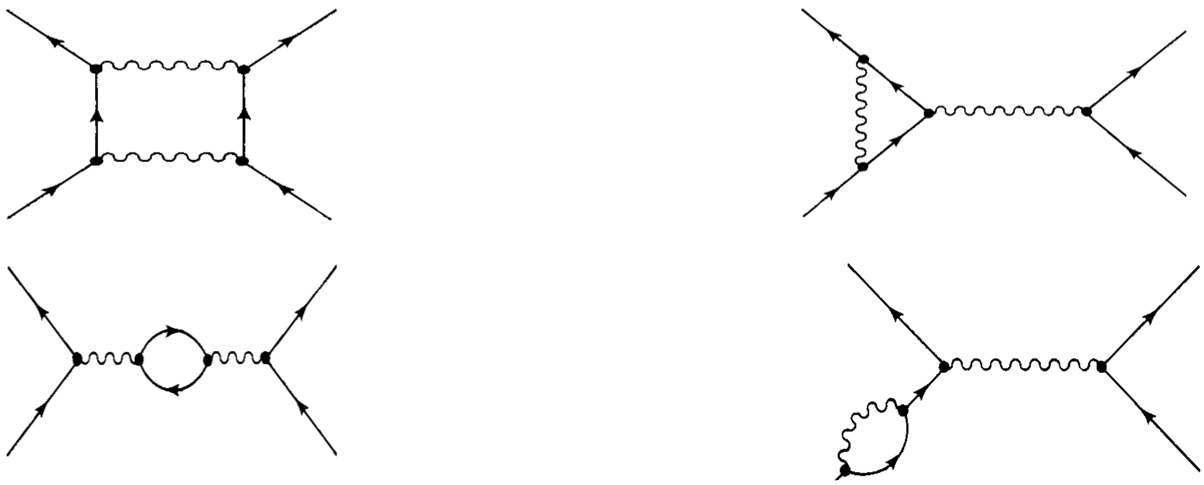






If we allow more vertices, the possibilities rapidly proliferate; for example, with four vertices

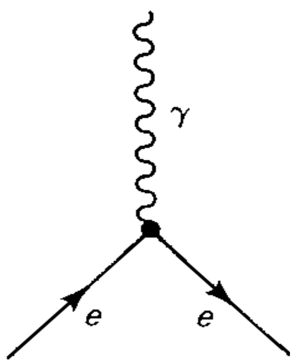
we obtain, among others, the following diagrams:



In each of these figures two electrons went in and two electrons came out. They too describe the repulsion of like charges (Maller scattering). The “innards” of the diagram are irrelevant as far as the observed process is concerned. Internal lines (those which begin and end within the diagram) represent particles that are not observed-indeed, that cannot be observed without entirely changing the process. We call them “virtual” particles. Only the external lines (those which enter or leave the diagram) represent “real” (observable) particles. The external lines, then, tell you what physical process is oncoming; the internal lines describe the mechanism involved.

The Feynman rules enforce conservation of energy and momentum at each vertex, and hence for the diagram as a whole. It follows that the primitive **QED** vertex by itself does not represent a possible physical process.

We can draw the diagram, but calculation would assign to it the number zero. The reason is purely kinematical: e- - e- + y would violate conservation of energy. Nor, for instance, is e- + e+ - y kinematically possible, although it **is** easy enough to draw the diagram:

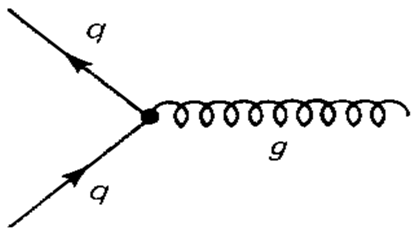


In the center-of-mass system the electron and positron enter symmetrically with equal and opposite velocities, so the total momentum before the collision is obviously zero. But their momentum cannot be zero, since photons always travel at the speed of light; an electron-positron pair can annihilate to make two

photons, but not one. Within a larger diagram, however, these figures are perfectly acceptable, because, although energy and momentum must be conserved at each vertex, a virtual particle does not carry the same mass as the corresponding free particle. In fact, a virtual particle can have any mass-whatever the conservation laws require. In the business, we say that virtual particles do not lie on their mass shell. External lines, by contrast, represent real particles, and these do carry the "correct" mass.

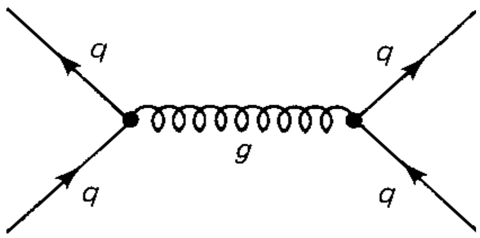
QUANTUM CHROMODYNAMICS (QCD)

In chromodynamics color plays the role of charge, and the fundamental process (analogous to e- - e- + y) is quark + quark-plus-gluon (since leptons do not carry color, they do not participate in the strong interactions):

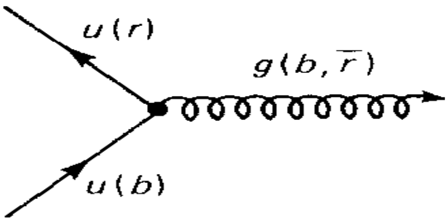


As before, we combine two or more such "primitive vertices" to represent more complicated processes. For example, the force between two quarks (which is responsible in the first instance for binding quarks together to make baryons, and indirectly for holding the neutrons and protons together to form a nucleus)

is described in lowest order by the diagram:

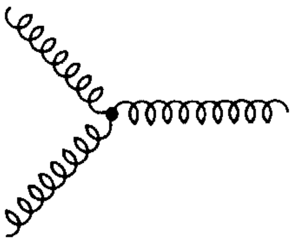


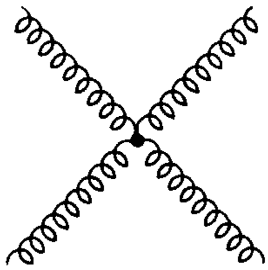
Say that the force between two quarks is "mediated" by the exchange of gluons. At this level chromodynamics is very similar to electrodynamics. However, there are also important differences, most conspicuously, the fact that whereas there is only one kind of electric charge (it can be positive or negative, to be sure, but a single number suffices to characterize the charge of a particle), there are three kinds of color (red, green, and blue). In the process **q** - **q** + g, the color of the quark (but not its flavor) may change. For example, a blue up-quark may convert into a red up-quark. Since color (like charge) is always conserved, this means that the gluon must carry away the difference-in this instance, one unit of blueness and minus one unit of redness:



Gluons, then, are "bicolored," carrying one positive unit of color and one negative unit. There are evidently **3** X **3** = 9 possibilities here, and you might expect there to be 9 kinds of gluons. For technical reasons, which we'll come to in Chapter 9, there are actually only **8.**

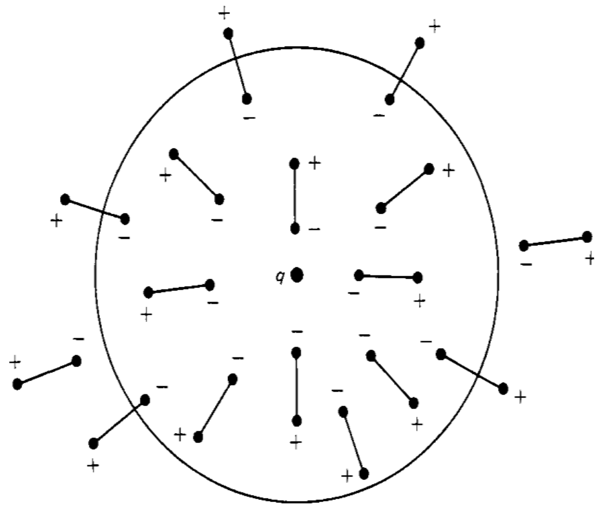
Since the gluons themselves carry color (unlike the photon, which is electrically neutral), they couple directly to other gluons, and hence in addition to the fundamental quark-gluon vertex, we also have primitive gluon-gluon vertices, in fact, two kinds: three gluon vertices and four gluon vertices:



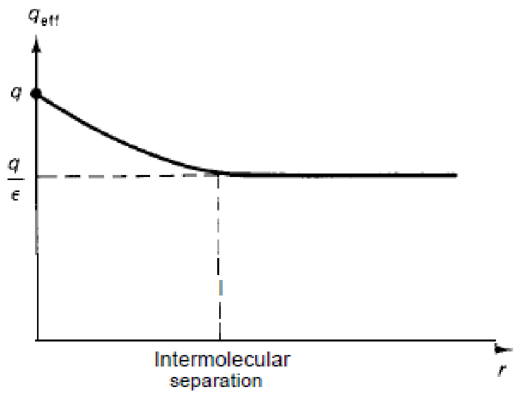


This direct gluon-gluon coupling makes chromodynamics a lot more complicated than electrodynamics, but also far richer, allowing, for instance, the possibility of glueballs (bound states of interacting gluons, with no quarks on the scene at all).

Another difference between chromodynamics and electrodynamics is the size of the coupling constant. Remember that each vertex in QED introduces a factor of $a = \frac{1}{137}$, and the smallness of this number means that we need only consider Feynman diagrams with a small number of vertices. Experimentally, the corresponding coupling constant for the strong forces, $\boldsymbol{\alpha_s}$**,** as determined, say, from the force between two protons, is greater than 1, and the bigness of this number plagued particle physics for decades. For instead of contributing less and less, the more complex diagrams contribute more and more, and Feynman's procedure, which worked so well in QED, is apparently worthless. One of the great triumphs of quantum chromodynamics (QCD) was the discovery that in this theory the number that plays the role of coupling "constant" is in fact not constant at all, but depends on the separation distance between the interacting particles (we call it a "running" coupling constant). Although at the relatively large distances characteristic of nuclear physics it is big at very short distances (less than the size of a proton) it becomes quite small. This phenomenon is known as asymptotic freedom; it means that within a proton or a pion, say, the quarks rattle around without interacting much. Just such behaviour was found experimentally in the deep inelastic scattering experiments. From a theoretical point of view, the discovery of asymptotic freedom rescued the Feynman calculus as a legitimate tool for QCD, in the high-energy regime.



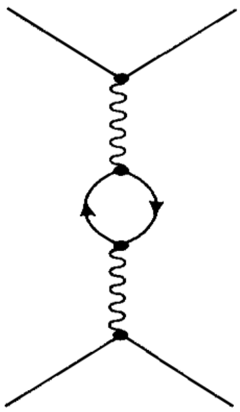
Even in electrodynamics, the effective coupling depends somewhat on how far you are from the source. This can be understood qualitatively as follows. Picture first a positive point charge **q** embedded in a dielectric medium (i.e., a substance whose molecules become polarized in the presence of an electric field). The negative end of each molecular dipole will be attracted toward **q,** and the positive end repelled away, as shown in Figure below. As a result, the particle acquires a "halo" of negative charge, which partially cancels its field.

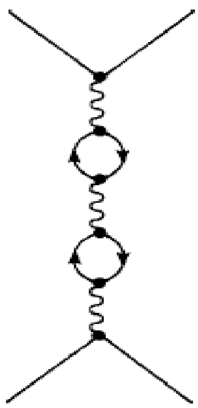


In the presence of the dielectric, then, the effective charge of any particle is somewhat reduced:

$$\mathbf{q_{eff}} = \frac{q}{\varepsilon} \qquad \textbf{(1)}$$

(The factor ε by which the field is reduced is called the dielectric constant of the material; it is a measure of the ease with which the substance can be polarized.) Of course, if you are in closer than the nearest molecule, then there is no such screening, and you "see" the full charge **q.** Thus, if you were to make a graph of the effective charge, as a function of distance. The effective charge increases at very small distances. Now, it so happens that in quantum electrodynamics the vacuum itself behaves like a dielectric; it sprouts positron-electron pairs, as shown in Feynman diagrams such as these:





The virtual electron in each "bubble" is attracted toward **q,** and the virtual positron is repelled away; the resulting vacuum polarization partially screens the charge and reduces its field. Once again, however, if you get too close to **q,** the screening disappears. What plays the role of the "intermolecular spacing" in this case is the Compton wavelength of the electron, $\boldsymbol{\lambda_c}$= h/mc = **2.43** X 10^{-10}cm. For distances smaller than this the effective charge increases, just as it did in

Figure 2.2. Notice that the unscreened ("close-up") charge, which you might regard **as** the "true" charge of the particle, is not what we measure in any ordinary experiment, since we are seldom working at such minute separation distances.

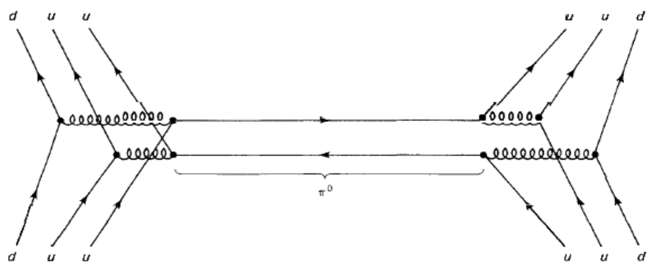
What we have always called "the charge of the electron" is actually the fully screened effective charge. **So** much for electrodynamics. The same thing happens in **QCD,** but with an important added ingredient. Not only **do** we have the quark-quark-gluon vertex (which, by itself, would again lead to an increasing coupling strength at short distances), but now there are also the direct gluon-gluon vertices. **So,** in addition to the diagrams analogous to vacuum polarization in **QED,** we must now also include gluon loops, such as these:

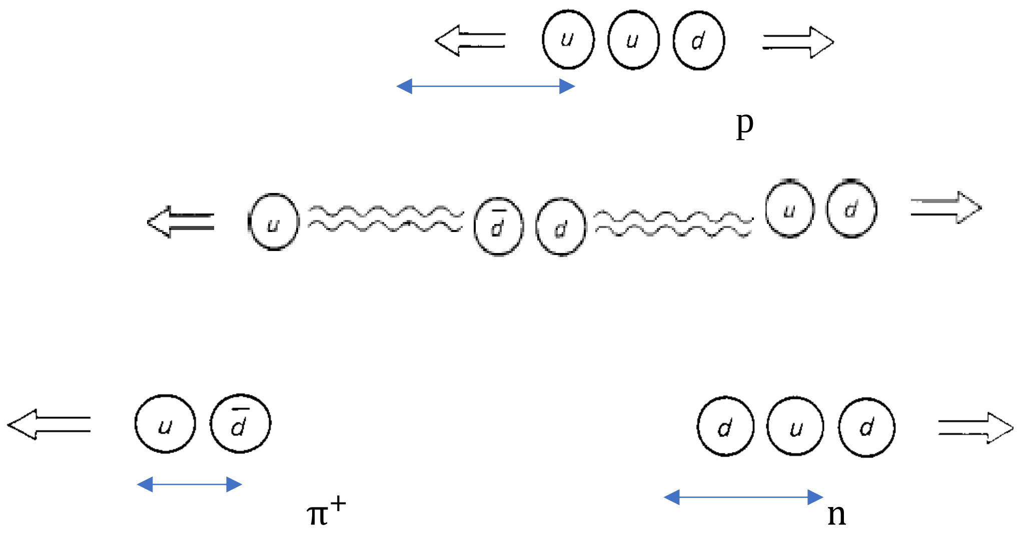
It is not clear a priori what influence these diagrams will have on the as it turns out, their effect is the opposite: There occurs a lung of competition between the quark polarization diagrams (which drive **a,** up at short distances) and gluon polarization (which drives it down). Since the former depends on the number of quarks in the theory (hence on the number flavors, f), whereas the latter depends on the number of gluons (hence on the number of colors, n), the winner in the competition depends on the relative number of flavors and colors. The critical parameter turns out to be,

$$a=2f- lln \qquad (2)$$

If this number is positive, then, as in QED, the effective coupling increases at short distances; if it is negative, the coupling decreases. In the Standard Model, f = **6** and n = **3,** so a = -2 1, and the QCD coupling decreases at short distances. Qualitatively, this is the origin of asymptotic freedom. The final distinction between QED and QCD is that whereas many particles carry electric charge, no naturally occurring particles carry color. Experimentally,

it seems that quarks are confined in colorless packages of two (mesons) and three (baryons). **As** a consequence, the processes we actually observe in the laboratory are necessarily indirect and complicated manifestations of chromodynamics. It is as though our only access to electrodynamics came from the van der Waals forces between neutral molecules. For example, the (strong) force between two protons involves (among many others) the following diagram:





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