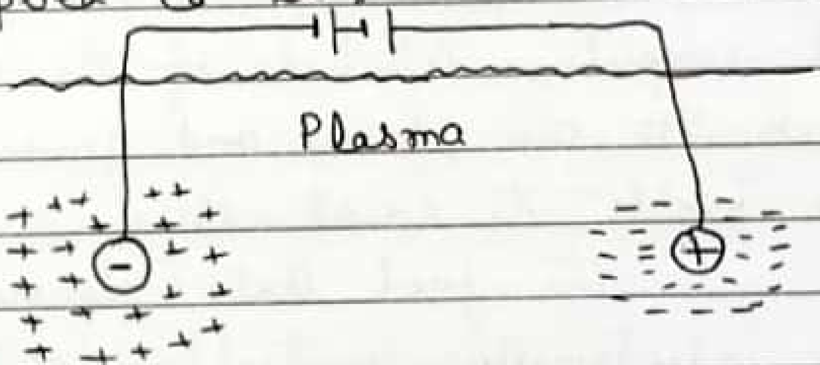
MPHYEC-1 I

Unit I →

"A plasma is a quasi neutral gas of charged and neutral particles, which exhibits collective behaviour."

A plasma system contains electrons, ions and neutral particles. Due to these components, the plasma has ability to shield out electric potentials ~~that~~ applied to it.



Let us consider two balls placed inside a plasma medium, be connected to battery for charging. The charges on balls be inserted to put an electric field inside plasma. The charged balls start attracting oppositely charged particles and a cloud of ions would be formed across negatively charged ball and a cloud of electrons would surround positively charged ball. It is assumed that a layer of dielectric ~~keeps~~ prevents the plasma from actually recombing on the surface, or that ~~the battery~~ is large enough to maintain the potential inspite of proximity of opposite charges

If the plasma is cold (neglect thermal motions) the charged balls are surrounded such that the shielding is perfect and no electric field is present outside the clouds. If the temperature is finite, the charges on the edge of cloud (where E- is weak), have enough thermal energy to escape from electrostatic potential well. The edge of the cloud appears at the radius where the P.E. is approximately equal to thermal energy KT of the particles and hence the shielding is not complete. Potentials of the order $\frac{KT}{e}$ can leak into the plasma and generate finite electric fields to exist there.

The fact that plasma is nearly electrically neutral is stated as "plasma - quasineutrality". There is no additional positive and negative charges, locally, at any place in plasma. i.e. $n_i \approx n_e$ and

$$\boxed{|n_i - n_e| << n_i \text{ or } n_e}$$

The phenomena of sheilding is called Debye sheilding and is measured in terms of Debye length (λ_D).

$$\lambda_D = \left(\frac{\epsilon_0 K T_e}{n e^2}\right)^{1/2}$$, T_e = Temperature of Plasma for electron or electronic ... of Plasma

Let us consider the situation in which neutrality ($n_i \simeq n_e$) is violated by small amount. Let $n_e \simeq 10^{17}\ m^{-3}$ in a spherical shaped plasma with radius 10^{-2} m or 1 cm. If now n_e is increased by 1%, the E-field produced at a distance r from the centre

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{\frac{4\pi}{3} r^3 (n_i - n_e) \cdot e}{r^2}$$

$$= \frac{(n_i - n_e) \cdot e \cdot r}{3\epsilon_0}$$

Since $(n_i - n_e) = \frac{n_e}{100}$

$$|\vec{E}| = \frac{10^{17} \times 1.6 \times 10^{-19} \times 0.01}{3 \times 10^{2} \times 8.85 \times 10^{-12}} \simeq 10^5\ V/m.$$

Under the effect of such huge E-field, the charged particles, which are free to move, start moving with large velocities. As a result, the charge neutrality is again restored in the region where it was disturbed. So, the disturbed charge neutrality is reestablished.

The condition $n_i \simeq n_e$

and $|n_i - n_e| << n_i$ or n_e

quasi - neutrality.

If the dimension L of a system are $>> \lambda_D$, then whenever local conc. of charge arise or external potentials are introduced into the system, these are shielded in a small region compared to L, leaving the bulk of the plasma free of electric potentials or fields.

For

In case of cluster of neutral particles, the collision between particles control particles motion. In Plasma the movement of charged particles generate local concentration of electrons and ions, which give rise to E-fields. The motion of charges also generate currents and hence magnetic fields. These fields affect the motion of other charged particles far away.

The Columbian force between charged r

For a given solid angle ($\frac{\Delta r}{r} = \text{Cnst}$), the volume of plasma that can affect a distance charge increase as r^3. The long-range columbian force affects the motion of individual particle and hence enriches the field of study of Plasma Physics. By "collective behaviour", we mean motions that depend not only on local conditions but on the state of Plasma in remote regions as well.

Av. K.E.$_{e^-}$ > Av. K.E.$_{ions}$ > Av K.E.$_{neutral}$

Since $\frac{3}{2}KT = \langle \frac{1}{2}mv^2 \rangle$, it means all the three Components of Plasma are at different temperatures.

At normal temp., gas is non-conductor but high temp. gaseous plasma are good conductors.

In Plasma Physics, temp. is measured in eV.

$1eV = 1.6 \times 10^{-19} J = 1.38 \times 10^{-23} K$ / molecule.

In Plasma, $eV = KT \Rightarrow T = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23}} K = 11600 K$.

Collective behaviour requires $\lambda_D << L$

Three conditions, a plasma must satisfy -

(i) $\lambda_D << L$ (ii) $N_D >> 1$ (iii) $\omega\tau > 1$.

ω = Frequency of typical plasma oscillation

τ = Meantime b/w Collisions with neutral atoms.

Plasma can be characterized by two parameters n and KT_e. n varies from 10^6 to $10^{34} m^{-3}$ [over an order of 28] KT_e can vary over seven orders [from 0.1eV to 10^6 eV].

Various Plasma systems :-

I. Gas Discharge (Gaseous Electronics)

The earliest work with Plasmas was done by Langmuir, Tonks and others in 1920s, The research was aimed towards development of vacuum tubes working with large currents. In the gas discharge having $KT_e \simeq 2eV$ and $10^{14} < n < 10^{18}$ m^{-3}, the sheath around electrode as dark layer was appreciable. Gas discharges are encountered nowadays in mercury rectifiers, hydrogen thyratrons, ignitrons, Spark gaps, weilding arcs, neon and fluorescent lights, and, lighting discharge.

II. Controlled Thermonuclear Fusion

Modern Plasma Physics has its ~~begi~~ beginnings from 1952, when it was proposed that the hydrogen bomb fusion reaction be controlled to make a reactor. In order to trigger or induce Deuterium (D) - Tritenium (T) fusion reaction, the incident energies must be greater than 10 KeV

Space Physics;

An important application of Plasma Physics is in the study of the earth's environment in space. Solar winds (a continuous stream of charged particles) impinges on the earth's magnetosphere (extended beyond 50 Km ~~10 times earth's ra~~ altitude) carrying $n = 5\times10^{6}\ m^{-3}$
$KT_i = 10eV$, $KT_e = 50eV$, $B = 5\times10^{-9}\ T$
with drift speed 300 Km/s and gets distorted before reaching the earth.

Modern AstroPhysics;

Stellar interiors and atmosphere are hot enough to be in the Plasma state. For ex, the temp. of the Core of the Sun is estimated to be 2 KeV, and $n = 10^{?}\ m^{-3}$ (Hydrogen) thermonuclear reactions occuring at this temperature are responsible for the sun's radiation.

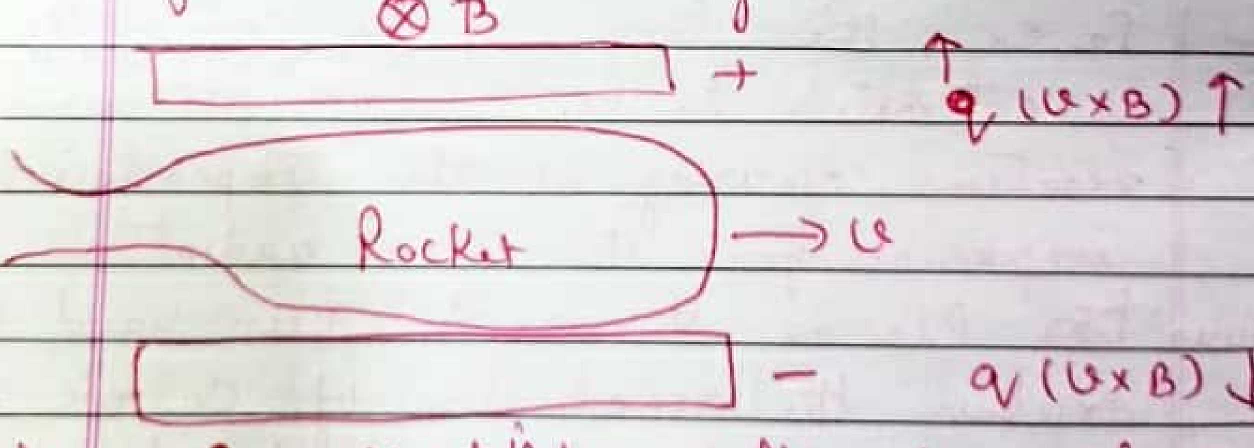
Various Plasma theories have been used to explain the acceleration of cosmic rays, to predict the development of galaxies, to explain numerious sources of radiation in Radio astronomy.

e.g. Crab nebula, and Pulsars → Rapidly rotating neutron stars

MHD Energy Conversion and Ion Propulsion

In Lab, there are two widely appreciable applications of Plasma Physics — Magnetohydrodynamic (MHD) energy conversion using dense plasma jet ~~propelltion~~ ~~a~~ stream across a magnetic field to generate electricity. The Lorentz force $q(\vec{v}_{c} \times \vec{B})$ jet velocity drift ions upward and electrons downward, which ultimately generate Potential Diff. Electric Currents can be drawn ~~from~~ across ~~through~~ electrodes. ~~without~~ ~~the~~

The same principle in reverse has been used to develop engines for interplanetary missions.



when ~~a~~ current is driven through a plasma by applying a voltage ~~to~~ b/w two electrodes, induced force $(J \times B)$ ~~&~~ shoots the plasma out of rocket, and reaction force accelerates the rocket. Here ejected Plasma is

Solid State Plasma;

In semiconductors, e^- & h^+ constitute a plasma exhibiting oscillation and instability as a gaseous Plasma. Plasma injected in to InSb exhibit very low collision frequency in comparision to that in solids. Holes in a semiconductor can have very low effective mass and hence high cyclotron frequencies even in moderate magnetic fields. In solid state Plasma $N_D < 1$ (because of low temp. & high Density) no. of Particles

but due to quantum mechanical effects N_D becomes large enough. Certain liquids such as solution of Sodium in Ammonia have been found to behave like Plasma also.

Gas Lasers ;

Pumping to create and maintain population inversion.

dc gas laser

He-Ne-lasers used for alignment and surveying

Ar and Kr lasers used in light Show.

high Pressure Avalanche Discharge

CO_2-Laser is used as a cutting tool.
Molecular-lasers are used to study far-infra red spectrum region..
Solid state Lasers like Nd-glass have wide application now a days.

All these systems depend on a plasma for their operation, since the flash tube used for pumping Contains gas discharges.

In this approach particle-particle Collisions are Considered as Ist order perturbation. This approach is not useful in which many-particles simultaneously strongly interact and when velocity distribution has prominent effect.

Different Theories of Plasma studies

The most important factor responsible for complexities in Plasma analysis is the quick local variations in ~~the~~ its densities. In case of dense fluids like water, the motion of individual molecules do not need to be considered, but in case of very less dense system single-particle behaviour needs to be taken into account for analysis.

To analyse the properties of Plasma two basic approaches are applied :—

1. Single Particle Approach, useful in low density Plasma in which motion of individual particles (electrons and positive ions) are considered.
2. Fluid behaviour Approach, specially applicable in dense plasma in which plasma may be considered as two interacting charged fluids on which equations of magnetohydrodynamics apply.

Apart from these two, we also have a macroscopic approach based on Kinetic theory of gases, which used Boltzmann transport equation. This approach is Comparatively more realistic and helps in Knowing changes in position and velocities of particles.

I. Motion of charged Particles in an electrostatic field:

$\vec{F} = q\vec{E}$ (a) Force is in the direction

$\Rightarrow \vec{a} = \frac{q\vec{E}}{m}$ of electric field

(b) If charge is initially moving along X-axis with velocity v_x and uniform E-field is along Y-axis then

$x = v_x \cdot t$ and $Y = \frac{1}{2}at^2$

$= \frac{1}{2}\frac{qE}{m}\cdot\frac{1}{v_x^2}\cdot x^2$

Path of a Parabola.

II. Motion of charge particle in a uniform magnetic field $\vec{B}$, $\vec{F} = q(\vec{v}\times\vec{B}) \rightarrow$ Lorentz force

$F = qvB\sin\theta \Rightarrow F = 0$ for $\theta = 0°$

& $F \perp v$ so work done $= 0$

Hence force has no effect on the speed of the particle and it continuously changes the direction of motion of the particle.

$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$; r = Larmor Radius

$V = r\omega$

$\omega = \frac{V}{r}$

or $r = \frac{\{2m(K.E.)\}^{1/2}}{B.q.}$

$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$\boxed{p = \sqrt{2mK}}$

$\boxed{\text{Larmor freq} = \frac{qB}{m}}$ ω

= Larmour Radius.

($qB = m\omega$)

When velocity of the particle is not perpendicular to $\vec{B}$ then component of velocity perpendicular to the velocity $V_{\perp r}$ is taken into account, and

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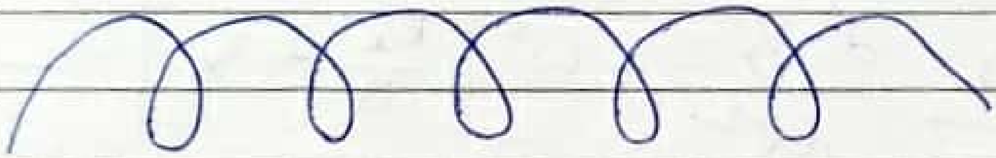
$$\boxed{r = \frac{m v_\perp}{q B}}$$ = Radius of gyration

The Component of velocity $\parallel$' to $\vec{B}$ is not affected and particle will continue to move with constant velocity along a line of force while gyrating around it with Constant frequency

$$\omega_H = \text{gyro-frequency} = \frac{qB}{m}$$



Path will be helical with its guiding centre along the line of force.

III Motion of charge particle in presence of both $\vec{E}$ and $\vec{B}$;

$$\vec{F} = m \cdot \frac{d\vec{v}}{dt} = q\left[\vec{E} + (\vec{v}\times\vec{B})\right]$$

$E_x = E$ i.e $\vec{E} = E\hat{i}$; $B_z = B$ i.e. $\vec{B} = B\hat{k}$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$\vec{v}\times\vec{B} = -\hat{j}(v_x B) + \hat{i}(v_y B)$

$$\therefore m \cdot \frac{dv_x}{dt} = q\left[E_x + (v_y \times B)\right]$$

Larmor fr. $\omega = \left(\frac{qB}{m}\right)$

$$\frac{dv_x}{dt} = \frac{q}{m}\cdot E_x + \frac{v_y}{m}\cdot \cancel{q}\cdot\omega_H = \frac{q}{m}E_x \pm \omega . V_y$$

$$\Rightarrow \frac{d^2 V_x}{dt^2} = \pm \omega \cdot \frac{dV_y}{dt} = \pm \omega \cdot \left(\frac{qB}{m}\right) \cdot V_x = \pm \omega^2 \cdot V_x \quad \text{— (1)}$$

and $$m \cdot \frac{dV_y}{dt} = q(V_x \cdot B) \Rightarrow \frac{dV_y}{dt} = -(V_x \cdot B)\frac{q}{m}$$

$$\therefore \frac{d^2 V_y}{dt^2} = -\omega \cdot \frac{dV_x}{dt} = -\omega \cdot \left[\frac{q}{m} E_x + \omega \cdot V_y\right]$$

$$= -\omega^2 \left[\frac{q}{m} \cdot \frac{E_x}{\omega} + V_y\right]$$

$$\Rightarrow \frac{d^2 V_y}{dt^2} = -\omega^2 \left[\frac{E_x}{B} + V_y\right] \quad \text{— (2)}$$

The presence of E-field ~~introduces~~ replaces V_y by $\boxed{V_y + \frac{E}{B}}$

Soln. of (1) is $\boxed{V_x = V_\perp \cdot e^{j\omega t}}$

~~and $V_y = V_\perp - e$~~

In eqn (2) Let $\frac{E_x}{B} + V_y = \psi$ then $\frac{d^2 V_y}{dt^2} = \frac{d^2 \psi}{dt^2}$

gives, $$\frac{d^2 \psi}{dt^2} = -\omega^2 \psi \Rightarrow \psi = V_\perp \cdot e^{j\omega t}$$

$$\Rightarrow \frac{E_x}{B} + V_y = V_\perp e^{j}$$

$$\Rightarrow \boxed{V_y = V_\perp e^{j\omega t} - \frac{E_x}{B}}$$

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This means there is Larmour motion with superimposed drift v_d of the guiding Centre in the negative Y-direction. So helical motion is combined with a drift $\left(= \frac{E_x}{B}\right)$

$v_\perp \cdot \omega$

IV; Motion of charged Particle in presence of inhomogeneous Magnetic field;

Space dependence of $\vec{B}$

If Space dependence of $\vec{B}$ is small (change in $\vec{B}$ with the distance of the order of Larmour radius) ~~is small~~ Compared to $|\vec{B}|$, then $\vec{B}$ may be considered as Constant and Larmor radius

$$r = \frac{mV_\perp}{eB}$$ remains Constant alongwith A.M. $mV_\perp r$

It means $mV_\perp r = mV_\perp \times \frac{mV_\perp}{eB} = \frac{m^2 V_\perp^2}{eB} =$ Constant.

The magnetic moment associated with an Orbiting charge particle is

$$\mu = \frac{1}{2}\frac{mV_\perp^2}{B} = \left(\frac{m^2 V_\perp^2}{eB}\right) \times \frac{1}{2}\left(\frac{e}{m}\right)$$
$$= \text{Constant}.$$

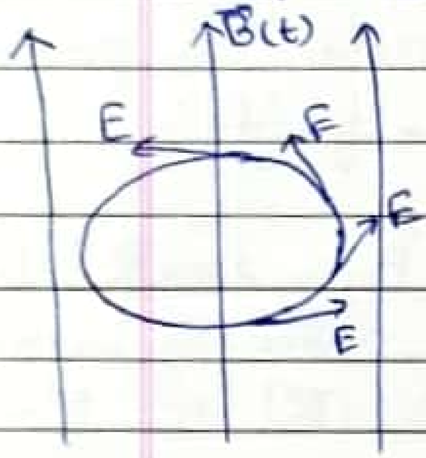
$\boxed{\mu = i \times \text{Area}}$
$= \frac{e}{t} \times \pi r^2$, $r = \frac{mr\omega}{eB}$

$t = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{eB}{m}}$

$e^{i\omega t}$

This means, if the change in the magnetic field with position is small, the μ associated with an Orbiting charge particle remains Constant

Case II When $\vec{B}$ varies with time with a rate $\frac{\partial \vec{B}}{\partial t}$ it generates electric field, which is along the tangent to circular path enclosing the magnetic field lines. This developed Electric field applies a force on the charge particle orbiting along the circular path and results in increase of K.E.



A/c to Maxwell's Eqn. ($\vec{E}$-generated is given by)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

Workdone per revolution $= \Delta W_1 = \oint_{line} e\vec{E} \cdot d\vec{l}$

$$= e \int_{surf} (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} \quad \text{, Using St... th...}$$

$$= -e \int_{surf} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}.$$

If the change in $\vec{B}$, ΔB is small in the time interval equal to Larmor periodic time $\frac{\partial B}{\partial t} = \frac{\Delta B}{\Delta t} = \frac{\Delta B}{T_c}$ can be regarded as constant, and hence it can be taken out of integration,

$$\Delta W_\perp = -e \cdot \frac{\Delta B}{T_c} \cdot \int d\vec{s}$$

$$= -e \cdot \frac{\Delta B}{\frac{2\pi r}{v_\perp}} \times (\pi r^2)\, \hat{n}$$

$\hat{n}$ → unit vector ⊥ to plane of orbit

$$= -e \cdot \frac{\Delta B\, r v_\perp}{2} = -e \cdot \Delta B \cdot \frac{m v_\perp}{eB} \cdot \frac{v_\perp}{2}$$

$$= \frac{\Delta B}{B}\left(\frac{1}{2} m v_\perp^2\right) = W_\perp \times \frac{\Delta B}{B}$$

$$\Rightarrow \frac{\Delta W_\perp}{B} - \frac{W_\perp}{B^2} \cdot \Delta B = 0 \Rightarrow \Delta\left(\frac{W_\perp}{B}\right) = 0$$

$$\Rightarrow \frac{W_\perp}{B} = \mu = \text{Constant}.$$

Stoke's theorem.

When any physical quantity changes (either with position or with time) very slowly, its variation is said to be adiabatic variation. If quantity remains constant, it is said to be adiabatic invariant.

Since ~~Thus~~ magnetic moment remains constant with slow / small variation of magnetic field, the phenomena is called adiabatic Invariance of magnetic moment.