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Date: April 16, 2020

To

The Students of Special Paper MPHYEC-1F
MSc. Physics IV Sem, Session 2018-20
Patna University, Patna.

Through Online Portal of E-content Patna University.

Subject: Regarding your Unit-1 of your special Elective Paper code MPHYEC-1F

Dear Students,

I am hopeful that you all are safe at your home during this pandemic situation. I am writing this letter to help you in maintaining the pace of your study which you are carrying out in this difficult time.

Dear Students, before this pandemic situation breakout I was taking your Unit-1 of MPHYEC-1F. I am sure that none of us has made available any study materials on this topic at our homes. In order to mitigate these hardships, I am happy to assist you all by provided the study materials on above mentioned topic through this. These are scanned copy of first three chapter of one book. You all are requested to go through it and if you face any difficulty to understand its any point please let me know through any digital communication. Your problem will be surly addressed on the digital platform (such as Telephonically, Email, WhatsApp, Online Class through various app) where this platform shall be chosen based on your problem and convenience. Feel free to contact me, as you all were doing during your normal classes.

Thanks for listening me.

Attachment:

1) 71 scanned Pages

UNIT:- 1

1 CHAPTER

Measurements and Measurement Systems

INSIDE THIS CHAPTER

1.1 INTRODUCTION

Measurement is essentially the act, or the result, of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a standard or a unit. The result of measurement is expressed by a number representing the ratio of the unknown quantity to the adopted unit of measurement. The physical embodiment of the unit of measurement, as well as that of its sub-multiple or multiple value is called a *standard*. The device used for comparing the unknown quantity with the unit of measurement or a standard quantity is called a *measuring instrument*.

One cannot stress too strongly the importance of measurements to present-day science and technology. Indeed, no physical experiment is conceivable without a sufficiently accurate technique of measurement.

As for importance of measurements to engineering, it will be suffice to recall that interchangeability of parts — the fundamental principle of modern technology — would be impossible without sophisticated and perfect measuring facilities. It will be no exaggeration to say that the quality of measuring tools and instruments is a very accurate index of technological progress in any industry.

True of any field of science and technology, this is especially true of electrical engineering and electrical physics which have now expanded to include many new applications.

The trend towards electrification has affected — in a straight forward manner and on a large scale — measuring techniques and instruments themselves. Owing to their perfection and convenience, ever wider use has of late been made of electrical methods of measurement in which unknown quantity is converted into an electrical quantity functionally related to the former, and then the electrical quantity is measured directly. Such electrical methods of measuring nonelectrical quantities have got general recognition.

What has been said would seem enough to show the importance of electrical methods of measurement. Present-day progress in science and technology, however, especially the ever greater emphasis placed on process automation, underlines this importance still more.

We apparently know that any automatic control system depends for its operation on reliable information about the state of the controlled plant or process. This information is obtained by sensing elements which are, in fact, measuring instruments. Therefore, progress in automatic control involves the perfection of measuring elements.* In most cases, these measuring elements are based on an electrical method of measurement. Thus the development and study of measuring techniques in general and of electrical methods of measurement in particular are obviously of paramount importance.

Measurement provides us with a means of describing a natural phenomena in quantitative terms. As a fundamental principle of science, Lord Kelvin stated, "*When you can measure what you are speaking about and express them as numbers, you know something about it and when you cannot measure it or where you cannot express in numbers, your knowledge is of a meagre and unsatisfactory kind. It may be the beginning of knowledge, but you have scarcely in your thought advanced to the stage of science.*" In order to make constructive use of the quantitative information obtained from the experiment conducted, there must be a means of measuring and controlling the relevant properties precisely. The reliability of control is directly related to the reliability of measurement.

* It should be noted that the measuring elements used in automatic control systems may perform independent functions in non-automated processes or plants.

1.2 INSTRUMENTS AND MEASUREMENT SYSTEMS

Instrumentation is a technology of measurement which serves not only science but all branches of engineering, medicine, and almost every human endeavour. The knowledge of any parameter largely depends on the measurement. The in-depth knowledge of any parameter can be easily followed by the use of measurement, and further modifications can also be had. Measuring instruments may be used to monitor a process or operation, or as well as the controlling process. For example, thermometers, barometers, anemometers are employed to indicate the environmental conditions. Similarly, water, gas, and electric meters are employed to keep track of the quantity of the commodity used. Special patient monitoring equipment is used in hospitals, and of course almost every moving vehicle has a number of indicator dials. Another extremely important type of application for measuring instruments is that in which instrument serves as a component of an automatic control system. In fact to control a quantity one must be able to measure it. A central heating system depends on a temperature measuring instrument, control of an industrial process needs many measurements of temperature, flow, level etc., while control of aircraft and missiles uses instruments such as accelerometers, altimeters and gyroscopes. More specialized instruments are employed in experimental scientific and engineering work.

A measuring instrument exists to provide information about the physical value of some variable being measured. In simple cases, an instrument consists of a single unit which provides an output reading or signal according to the magnitude of the unknown variable applied to it. However, in more complex measurement situations, a measuring instrument may consist of several separate elements as shown in Fig. 1.1. These elements may consist of transducing elements for conversion of measurand to an analogous form. The analogous signal is then processed by some intermediate means and then fed to the end devices which present the results of the measurement for the purpose of display and or control. These components might be contained within one or more boxes, and the boxes holding individual measurement elements might be either close together or physically separate. Because of the modular nature of the elements within it, a measuring instrument is commonly referred to as a measurement system.

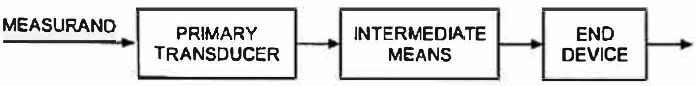


Fig. 1.1 *A Simple Measurement System*

1.3 METHODS OF MEASUREMENT

Basically, there are two methods of measurements. They are: (*i*) Direct comparison methods and (*ii*) Indirect comparison methods.

In *direct comparison methods*, the unknown quantity is determined by direct comparison with a standard of the given quantity. The result is expressed in terms of a chosen unit for the standard and a numerical multiplier. For example, a length can be measured in terms of metre and a numerical constant. Thus a 5 m length means a length of 5 times of metre. A human being can make direct length comparison with precision of about 0.25 mm. Thus due to human factors it is not possible to make very accurate measurements. The length can be measured by direct comparison method with a good degree of accuracy but in case of measurement of mass by this method, the problem becomes more intricate. It is just not possible for human being to distinguish between wide margins of mass.

Direct comparison methods of measurement are though simple, but it is not always possible, feasible and practicable to use them. The involvement of a person in these methods make them inaccurate and less sensitive. Hence direct comparison methods are not preferred and are rarely used. In engineering applications use of *measurement systems*, which are *indirect methods of measurement*, is made.

In indirect comparison method, the transducing element converts the measurand to an analogous form which is processed by some intermediate means and then supplied to the output device which presents the results of the measurement for the purpose of display and/or control.

The signal processing means either one or combination of the following:

(i) Amplification of the weak signal before being fed to the instrument.
(ii) Telemetering the data for remote reading/recording such as recording of the temperature of the surface of the moon on ground.
(iii) Extraction of the desired information from extraneous input by means of filtering.

1.4 HISTORY OF DEVELOPMENT OF INSTRUMENTS

The science and the art of measurement of physical quantities has a long history. Initially, measurements were mainly concerned with the basic and common quantities such as length, mass, force and time, and meters were developed for their measurement and direct indication under steady-state conditions. Indicating systems so developed were considered as instruments. With the passage of time, instruments came to be understood as tools in the hands of man, which were useful in accomplishing the objectives of sensing, detecting, measuring, recording, controlling, computing or communicating. Extensive modifications in the design of instruments were incorporated to achieve these objectives, while at the same time giving higher accuracy and precision where measurement was involved. Development of a variety of materials for the electrical and mechanical elements and associated solid-state electronic circuits resulted in the design of a fascinating array of instruments with multifunction capability. Considerable improvements in the characteristics of instruments, with regard to reliability, speed of response, visual display of the measured quantity etc., are noticeable in the present-day instruments. Sophisticated instruments have considerably enhanced and refined the sensory perception of the measured data and its usefulness. Although a meter is also treated as a simple instrument, a sophisticated instrument is really a complex system, in which the primary signal obtained after sensing or measurement, undergoes extensive processing before it is presented for display or recording.

The first instruments used by mankind were mechanical in nature and the principles on which they worked are even used nowadays. The earliest scientific instruments used the same three essential elements (i.e. a detector, an intermediate transfer device, and an indicator, recorder, or a storage device) as our modern instruments do.

The history of development of instruments encompasses three phases of instruments as described below.

1. ***Mechanical Instruments.*** Such instruments are very reliable for static and stable conditions but they suffer from the major drawback of inability of responding rapidly to the measurements of dynamic and transient conditions. This is because such instruments have moving parts that are rigid, heavy and bulky and consequently have a large mass. Mass results in inertia problems and, therefore, such instruments cannot faithfully follow the rapid changes which are involved in dynamic measurements. Thus, it would be almost impossible to measure a 50 Hz voltage by a mechanical instrument. However, it is comparatively easy to measure a slowly varying pressure with mechanical instruments. Another drawback of mechanical instruments is that most of them are a potential source of noise and hence cause noise pollution.

2. ***Electrical Instruments.*** Such instruments are more rapid in indicating the output of detectors as compared to mechanical instruments but unfortunately electrical instruments are also dependent upon mechanical meter movement as indicating device. Since the mechanical movements have some inertia, they have limited time and hence frequency response. For example, some electrical recorders can provide full-scale response in 0.2 second, the majority of industrial recorders have responses of 0.5–24 seconds. Some galvanometers can follow 50 Hz variations, but even these are too slow for today requirements of fast measurements.

3. ***Electronic Instruments.*** The mechanical and electrical instruments and systems cannot cope up with the very fast response requirements of the scientific and industrial measurements carried out nowadays. The necessity to step up response time and also the detection of dynamic changes in certain parameters, which needs the monitoring time of the order of milliseconds and quite often microseconds, have led to the development of electronic instruments and their associated circuitry. Such instruments make use of semiconductor devices. The response time of such instruments is extremely small as the movement involved in electronic devices is only that of electrons and electrons have very small inertia. For example, a cathode-ray oscilloscope (CRO) can follow dynamic and transient changes of the order of a few nanoseconds.

Electronic instruments are gradually becoming more reliable due to improvements in design and manufacturing processes of semiconductor devices. Another advantage of electronic devices is that very weak signals can be detected by employing preamplifiers and amplifiers. In fact hydraulic and pneumatic systems could be employed for power amplification of signals but their use is limited to slow acting control applications like servo systems, chemical processes and power systems.

Electronic instruments are light, compact and have a high degree of reliability. Their power consumption is very small. The most important use of electronic instruments is in measurement of nonelectrical quantities, where the nonelectrical quantity is converted into electrical form with the use of transducers. Electronic instruments are widely employed in detection of electromagnetically produced signals such as radio, video and microwave. The electronic instruments have higher sensitivity (due to power amplification provided by electronic amplifier) and, therefore, find wide application in the area of bioinstrumentation where bioelectric potentials are very weak (smaller than 1 mV). Electronic instruments also have the advantage of obtaining indication at a remote location that helps in monitoring inaccessible or hazardous locations. Communication is a field that is entirely dependent upon the electronic instruments and the associated circuitry. Such instruments enable us to build analog and digital computers which require very fast time response.

In brief it can be stated that, in general, electronic instruments have a higher sensitivity, a faster response, lower weight, a greater flexibility, a higher degree of reliability, and low power consumption as compared to those in case of their mechanical and purely electrical counterparts.

The new developments in electronic instruments are by virtue of digital technology, by which any precision quantity can be measured in the digital display form and can also be possibly stored in the memory for storage purpose required in future. There is a dramatic revolution in the field of medical science because of electronic instruments.

1.5 CLASSIFICATION OF INSTRUMENTS

Instruments can be subdivided into separate classes according to different criteria. Such subclassifications are useful in broadly establishing several attributes of particular instruments such as accuracy, cost and general applicability to different applications.

1.5.1. Absolute and Secondary Instruments. The various instruments, in very broad sense, are classified into two classes namely (*i*) absolute and secondary instruments.

1. ***Absolute Instruments.*** The instruments of this type provide the magnitude of the quantity to be measured in terms of instrument constant and its deflection. Such instruments do not require any comparison with any other standard instrument. The example of this type of instrument is *tangent galvanometer*, which provides the value of current to be measured in terms of tangent of the

angle of deflection produced, the horizontal component of the earth's magnetic field, the radius and the number of turns of wire used. Rayleigh's current balance and absolute electrometer are other examples of absolute instruments.

Such instruments are seldom used except in standard laboratories and in similar institutions as standardising instruments. This is because working with absolute instruments for routine work is time-consuming (every time a measurement is made it takes a lot of time in computation of the magnitude of the quantity under measurement).

2. ***Secondary Instruments.*** These instruments are so designed that the measurand can only be measured by observing the output indicated by the instrument. These instruments are required to be calibrated by comparison with an absolute instrument or another secondary instrument, which has already been calibrated against the absolute instrument. Typical examples of secondary instruments are ammeters, voltmeters, wattmeters, glass thermometers, pressure gauges etc. Secondary instruments are widely used in practice.

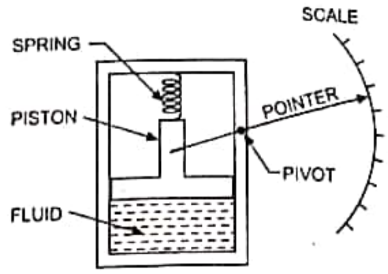
1.5.2. Direct Measuring and Comparison Instruments. Depending upon the methods used for comparing the unknown quantity with the unit of measurement, electrical measuring instruments may be classified as *direct measuring* and *comparison instru-ments.*

Direct measuring instruments convert the energy of the unknown quantity directly into energy that deflects the moving element of the instrument, the value of the unknown quantity being measured by reading the resulting deflection. Ammeters, voltmeters, wattmeters, fall in this category. *Comparison instruments* measure the unknown quantity by comparing it with a standard that is often contained in the instrument case such as resistance measuring bridges. Direct measuring instruments are most widely used in engineering practice since they are the most simple and inexpensive ones and enable the measurements to be made in the shortest possible time. Comparison instruments are used in cases when a higher accuracy of measurement is needed.

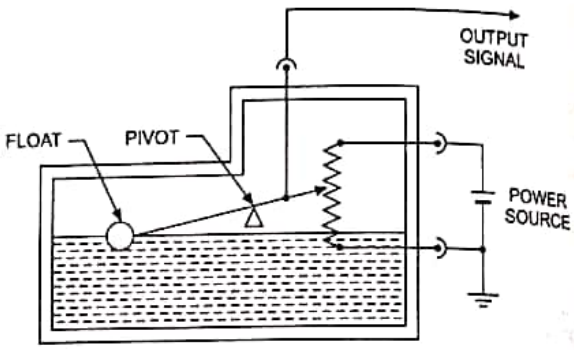
1.5.3. Active and Passive Instruments. Instruments are either active or passive according to whether the instrument output is entirely produced by the quantity under measurement or the quantity under measurement simply modulates the magnitude of some external power source. This can be illustrated by examples as below.

An example of a passive instrument is the pressure gauge shown in Fig. 1.2 (*a*). The pressure of fluid is translated into movement of a pointer against a scale. The energy expended in moving the pointer is derived entirely from the change in pressure measured; there are no other energy inputs to the system.

An example of an active instrument is a float type petrol tank level indicator, as shown in Fig. 1.2 (*b*). Here, the change in petrol level moves a potentiometer arm, and the output signal consists of a proportion of the external voltage source applied across the two ends of the potentio-



(a) Passive Pressure Gauge



(b) Petrol Tank Lever Indicator

Fig. 1.2

meter. The energy in the output signal comes from the external power source; the primary transducer float system is merely modulating the value of the voltage from this external power source. In active instruments, the external power source is usually in electrical form, but in some cases it can be in other forms of energy such as pneumatic or hydraulic.

One very important difference between active and passive instruments is the level of measurement resolution which can be obtained. With the simple pressure gauge shown, the amount of movement made by the pointer for a particular pressure change is closely defined by the nature of the instrument. Whilst it is possible to increase the measurement resolution by making the pointer longer, so that the pointer tip moves through a longer arc, the scope for such improvement is clearly restricted by the practical limit of how long the pointer can conveniently be. In an active instrument, however, adjustment of the magnitude of the external energy input allows much greater control over measurement resolution. Whilst the scope for improving this resolution is much greater incidentally, it is not infinite because of the limitations placed on the magnitude of the external energy input, in considering heating effects and for safety reasons.

In terms of cost, passive instruments are normally of a simpler construction than active ones and are, therefore, cheaper to manufacture. The choice between active and passive instruments for a particular application, therefore, involves carefully balancing the measurement resolution requirements against cost.

1.5.4. Deflection and Null Type Instruments. Another useful classification separates instruments by their operation on a null or deflection principle.

In *deflection type instruments*, the measurand (quantity under measurement) produces some physical effect which deflects or produces a mechanical displacement of the moving system of the instrument. An opposing effect is built in the instrument which tries to oppose the deflection or the mechanical displacement of the moving system. The opposing effect is closely related to the deflection or mechanical displacement that can be directly observed. The opposing effect increases until a balance is achieved, at which point the 'deflection' is measured and the value of measured quantity inferred from this. For example in a permanent magnet moving coil (PMMC) ammeter, the deflection of the movement is proportional to current I (the quantity under measurement); the torque acting on the moving coil, T_d being proportional to current. The opposing force is produced by phosphor hair springs whose torque T_c is proportional to deflection θ. Thus the value of measurand (current I), in this case depends upon the value of deflection θ, because under steady-state condition $T_d = T_c$ or $I \propto \theta$. (for details refer to Art 9.7) The instrument is made direct reading i.e. to give indication of magnitude of current under measurement in terms of deflection θ by calibration.

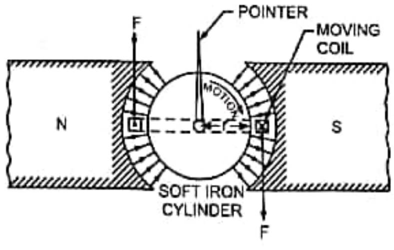


Fig. 1.3 *Deflection Type Instrument (PMMC Type Instrument)*

In contrast to the deflection type instrument, a *null type instrument* attempts to maintain deflection at zero by suitable application of an effect opposing that generated by the measurand. Necessary to such an operation are a detector of unbalance and a means (manual or automatic) of restoring balance. Since deflection is kept at zero (ideally), determination of numerical values requires accurate knowledge of the magnitude of the opposing effect. The detector should be capable of displaying unbalance i.e., a condition when the effect provided by measurand is not equal to the opposing effect.

For example a simple dc potentiometer is a null type instrument and is basically employed for measuring an unknown emf. The slide wire of the potentiometer is calibrated in terms of volts per unit length with the help of a standard emf source. The null detector is a galvanometer G whose deflection is proportional to the unbalance emf i.e. difference between the voltage drop across portion AJ of slide wire (Fig. 1.4) and unknown emf. When the two (i.e. pd across slide wire and unknown emf) are equal, then no current flows in mesh ASGJA and galvanometer will, therefore, show zero deflection.

Upon comparing the deflection and null methods of measurement examplified by the PMMC type ammeter and a simple dc potentiometer described above, we note that the accuracy attainable by the null method is of a higher level than that by deflection method. This is because the opposing effect is calibrated with the help of standards having high degree of accuracy while accuracy of the deflection type instruments depends upon their calibration which depends upon the instrument constants which are normally not known to a high degree of accuracy. Another advantage of null methods is the fact that, since the measured quantity is balanced out, the detector of unbalance can be made very sensitive because it need to cover only a small range around the balance (null) point. Also the detector need not be calibrated since it must detect only the presence and direction of unbalance, but not the magnitude of unbalance. On the other hand, a deflection instrument must be larger, more rugged, and thus less sensitive if it is to measure large magnitudes.

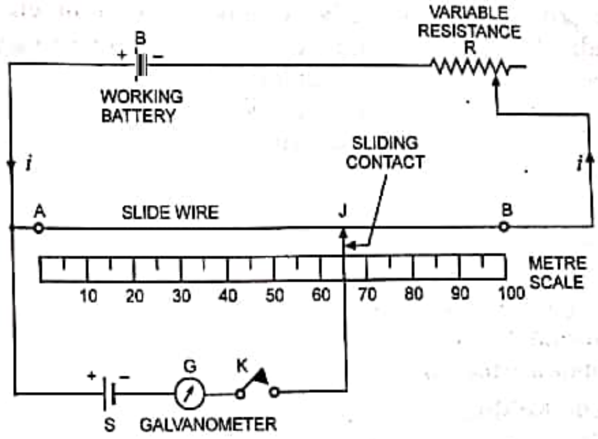


Fig. 1.4 *Simple DC Potentiometer*

The drawback of null methods is that they are not suitable for dynamic measurements wherein the measured quantity changes with time and null type instruments require many manipulations before null conditions are obtained. On the other hand, deflection type instruments can follow the variations of the measured quantity more rapidly and are, therefore, more suitable for dynamic measurements on account of their fast response. However, by use of automatic control instruments (such as self-balancing potentiometers), that maintains a continuous null under rapidly changing conditions, the requirement for manipulative operations is eliminated.

In terms of usage, the deflection type instruments are obviously more convenient. They are far simpler to read the position of a pointer against a scale. A deflection type instrument is, therefore, the one that would normally be used in the workplace. However, for calibration duties, the null type instrument is preferable because of its superior accuracy. The extra effort required to use such an instrument is perfectly acceptable in this case because of the infrequent nature of calibration operations.

In brief, it can be said that null type instruments are more accurate and highly sensitive as compared with deflection type instruments and, therefore, are more suitable for calibration purposes. The deflection type instruments are more convenient and are more suitable for use in the workplaces and for dynamic measurements.

1.5.5. Monitoring and Control Instruments. An important distinction between different instruments is whether they are suitable only for monitoring functions or whether their output is in a form that can be directly included as part of an automatic control system.

Instruments which only provide an audio or visual indication of the magnitude of the quantity under measurement, such as a liquid-in-glass thermometer, are only suitable for monitoring purposes. This class normally includes all null type instruments and most passive transducers.

For an instrument to be suitable for inclusion in an automatic control system, its output must be in a suitable form for direct input to the controller. Usually, this means that an instrument with an electrical output is required, although other forms of output such as optical or pneumatic signals are used in some systems.

1.5.6. Analog and Digital Instruments. Secondary instruments operate in two modes namely *analog mode* and *digital mode*.

Signals that vary in a continuous fashion and take on an infinite number of values in any given range are known as *analog signals*. The devices producing such signals are known as *analog devices*. On the other hand, the signals which vary in discrete steps and thus take up only finite different values in a given range are termed as *digital signals* and the devices producing such signals are called the *digital devices*.

An *analog instrument* provides an output which varies continuously as the quantity under measurement changes. The output can have an infinite number of values within the range that the instrument is designed to measure. The deflection type pressure gauge shown in Fig. 1.2 (*a*) is a good example of an analog type instrument. As the magnitude of the input changes, the pointer moves with a smooth continuous motion. Whilst the pointer can, therefore, be in an infinite number of positions within its range of movement, the number of different positions which an eye can discriminate between is strictly limited, this discrimination being dependent upon how large the scale is and how finely it is divided. A *digital instrument* has an output which varies in discrete steps and so can only have a finite number of values.

For analog signals, the precise value of the quantity (voltage, current, power, rotation angle) carrying the information is significant. However, digital signals are basically of a binary (on/off) nature, and variations in numerical value are associated with changes in the logical state (True/False) of some combinations of "switches". In a typical digital electronic system, any voltage in the range of + 2.5 to + 5 V produces the *on state*, while signals of 0 to + 1 V correspond to *off*. Thus the magnitude of voltage whether it is 2.5 V or 4 V is of no consequence. The output is same, and so the system is quite tolerant of spurious "noise" voltages which might contaminate the information signal. In a digitally represented value of, say, 4.652, the least significant digit (2) is carried by on/off signals of the same (large) size as for the most significant digit (4). Thus in an all-digital device such as a digital computer there is no limit to the number of digits which can be accurately carried; we use whatever can be justified by the particular application. In case of use of combined analog/digital systems (often the case in measurement systems), the digital portions need not limit system accuracy. These limitations generally are associated with the analog portions and/or the analog/digital conversion devices.

The majority of primary sensing elements are of the analog type. Digital revolution counter is depicted in Fig. 1.5. It is impossible for such instrument to indicate, say 0.64; it measures only in steps of 1.

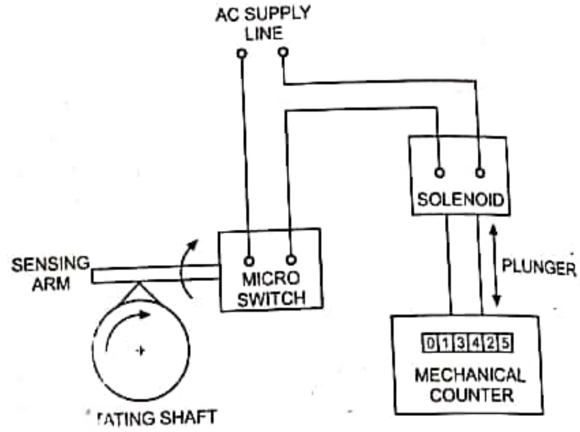


Fig. 1.5 *Digital Revolution Counter*

Though majority of the present-day instruments are analog type but the importance of digital instruments is increasing, perhaps mainly because of the widespread use of digital computers in both data processing and automatic control systems. Since digital computer works only with digital signals, any information supplied to it must be in digital form. The computer's output is also in digital form. Thus any communication with the computer at either the input or the output end must be in terms of digital signals. Since most measurements and control apparatus are of analog nature, it is necessary to have both *analog-to-digital converters* (ADC) at the input to the computer and digital-to-analog converters (DAC) at the output of the computer. Such devices* serve as "translators" that enable the computer to communicate with the outside world, which is largely of an analog nature.

Electrical measuring instruments may also be classified according to the kind of quantity being measured, kind of current for which they are designed, the principle of operation of the moving system, their accuracy class, protection against the influence of external fields, service conditions, stability against mechanical effects, field of application, method of installation and mounting, or shape and size of the instrument cases and the degree of enclosure they provide.

1.6 FUNCTIONS OF INSTRUMENTS AND MEASUREMENT SYSTEMS

An instrument or measurement system is required to sense some variable quantity, and to indicate or record its value, or to control its value. So the instruments or measurement systems, according to their functions, may be divided into

* Analog-to-digital converters and digital-to-analog converters will be discussed later on.

three categories namely, indicating instruments, recording instruments and controlling instruments.

1. ***Indicating Instruments.*** An instrument that supplies the information in the form of deflection of a pointer is known as an indicating instrument. In this way, the instrument performs a function which is commonly known as *indicating function*. For example, the deflection of pointer of an ammeter indicates the current flowing through the branch of an electric circuit in which it is connected. Pressure gauges, speedometers, thermometers, ammeters, voltmeters, wattmeters etc. fall under this category.

2. ***Recording Instruments.*** Recording instruments are those which keep a continuous record of the variations of the magnitude of an unknown quantity to be observed over a definite period of time. In such instruments the moving system carries an inked pen which touches lightly a sheet of paper wrapped over a drum moving with uniform slow motion in a direction perpendicular to that of the deflection of the pointer. Thus a curve is traced which shows the variations in the magnitude of the quantity under observation over a definite period of time. Thus the instrument performs the recording function. For example, a potentiometric type recorder, employed for monitoring temperature, records the instantaneous values of temperature on a strip chart recorder. Another example is a speed log, usually provided with commercial vehicles, that draws a graph of speed against time or distance. Temperature and pressure recorders etc. fall under this category.

3. ***Controlling Instruments.*** Controlling function is one of the most important functions, especially in the field of industrial process control. In controlling instruments, the information is used to control the original measured quantity. For example, a carburettor or fuel injection system, measures the fuel requirements of the engine for different loads, speeds, and accelerator-pedal positions, and supplies the necessary air and fuel to the engine. Thermostats, float type level control, machine tool carriage-position control etc. fall under this category.

Thus, according to functions, there are three main groups of instruments. The largest group has the indicating function. Next are the instruments which have both indicating and/or recording functions. The last group performs all the three functions namely, indicating, recording and controlling.

1.7 ELEMENTS OF A GENERALISED MEASUREMENT SYSTEM

The measurement is usually undertaken to ascertain and present the state, condition or characteristic of a system in quantitative terms. It enables the experimenter to understand the state of the system under which it exists or to distinguish the transition of the system from one state to another. To reveal the performance of a physical or chemical system, the first operation carried out on it is measurement. Measurement means determination of magnitude, extent, degree etc. of the condition of a system in terms of some standard. Measurement is the basic and primary operation, the result of which is used only to describe the system and hence treated as an independent operation with no ulterior motive other than to understand it.

Man has always extended his imaginative skills to identify physical phenomena and later developed and utilised the means for confirmation of his understanding of the phenomena. Thus the measuring techniques and systems developed are based on the basic laws of nature and the evolution of such techniques, in course of time, led to the development of indicating systems for some of the physical quantities. These can be seen to consist of mechanisms incorporated inside a housing so as to be coupled to the system under study. Identification of physical phenomena results in the development of measuring systems that interact with the quantity under measurement and develop output signals in the form of linear or angular displacements that can be transmitted to a pointer on a scale.

It is possible and desirable to describe both the operation and performance (degree of approach to perfection) of measuring instruments and associated equipment in a generalised way without recourse to specific physical hardware. The operation can be described in terms of the functional elements of instrument systems, and the performance is defined in terms of the static and dynamic performance characteristics.

Figure 1.6 represents a possible arrangement of functional elements in an instrument and includes all the basic functions considered necessary for a description of any instrument.

1. ***Primary Sensing Element.*** The primary sensing element is that which makes contact with the physical quantity under measurement, called the measurand, receives energy from the measured medium and produces an output depending in some way on the measurand. Examples are thermocouple (output emf depends upon input temperature), strain gauge (resistance depends on mechanical strain), orifice plate (pressure drop depends on flow rate). It is important to note that an instrument always extracts some energy from the measured medium. Thus

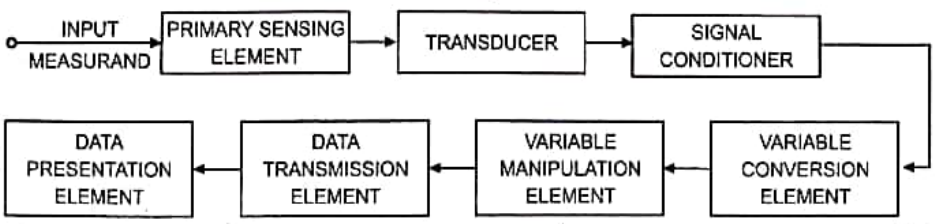


Fig. 1.6 *Functional Elements of an Instrumentation System*

the measured quantity is always disturbed by the act of measurement, which makes a perfect measurement theoretically impossible. Good instruments are designed to minimise this effect, but it is always present to some degree.

Primary sensing elements may have a nonelectrical input and output such as an orifice plate, a spring, a manometer or may have an electrical input and output such as a rectifier. In case the primary sensing element has a nonelectrical input and output, then it is converted into an electrical signal by means of a transducer. The *transducer* is defined as a device which, when actuated by one form of energy, is capable of converting it into another form of energy.

Many a times certain operations are to be performed on the signal before its further transmission so that interfering sources are removed and the signal may not get distorted. The process may be linear such as amplification, attenuation, integration, differentiation, addition and subtraction or nonlinear such as modulation, detection, sampling, filtering, chopping and clipping etc. The process is called the *signal conditioning*. So signal conditioner follows the primary sensing element or transducer, as the case may be. The examples of signal conditioning element are deflection bridge which converts an impedance change into a voltage change; amplifier which amplifies millivolts to volts; oscillator which converts an impedance change into a variable frequency voltage.

2. *Variable Conversion Element.* Now the output, which is in the form of electrical signal (voltage, current, frequency or some other electrical parameter), may or may not suit to the system. For the instrument to perform the desired function, it may be necessary to convert this output to some other more suitable form while retaining the information content of the original signal. An element that performs such a function is called a variable conversion element. For example, if output is in analog form and the next stage of the system accepts the input only in digital form, then analog-to-digital converter (ADC) will be required. Many instruments do not require any variable conversion element, while others require more than one element.

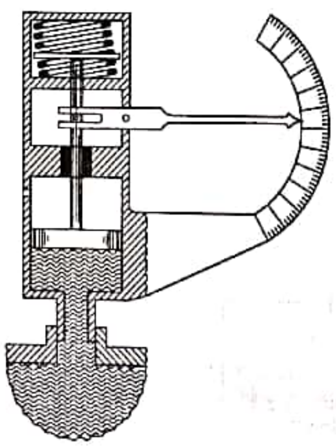
3. *Variable Manipulation Element.* The function of variable manipulation element, as the name indicates, is to manipulate the signal presented to it while preserving the original nature of the signal. For example, an electronic amplifier converts a small low voltage input signal into high voltage output signal. Thus voltage amplifier acts as a variable manipulation element. It is not necessary that this element follows the variable conversion element, as shown in Fig. 1.6. In many cases it may precede the variable conversion element.

When the functional elements of an instrument are to be physically separated out, it becomes necessary to transmit data from one to another. The element performing this function is called the *data transmission element*. It may be as simple as a shaft and bearing assembly or as complicated as a telemetry system for transmitting signals from satellites to ground equipment by radio.

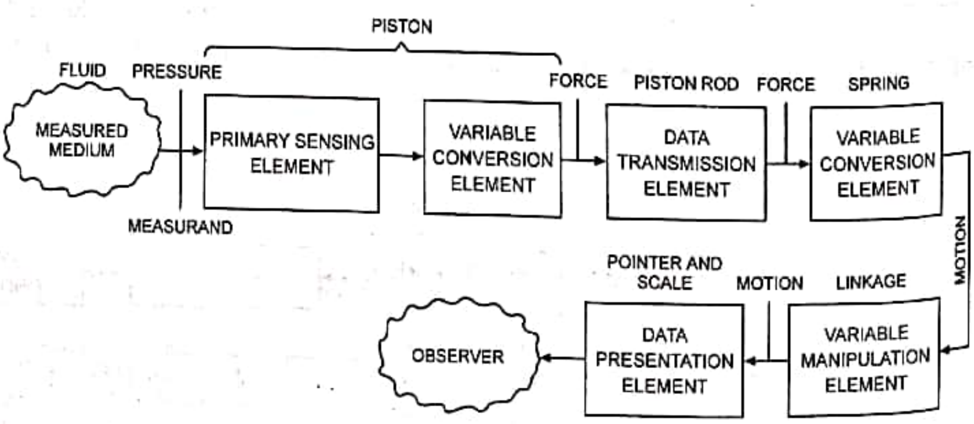
4. *Data Presentation Element.* The information regarding measurand (quantity to be measured) is to be conveyed to the personnel handling the instrument or the system for monitoring, controlling or analysis purpose. The information conveyed must be in a form intelligible to the personnel. Such devices (readout or display) may be in analog or digital format. The simplest form of a display device is the common panel meter with some kind of calibrated scale and pointer. In case, the data is to be recorded, recorders like magnetic tape may be used. For control and analysis purpose computers may be used.

The final stage in a measuring system is called the *terminating stage*. In case a control device is to be employed for final measurement stage, then it becomes necessary to apply some feedback to the input signal to accomplish the control objectives.

As an example of a measurement system, consider the rudimentary pressure gauge shown in Fig. 1.7(*a*). The primary sensing element is the piston, which also serves the function of a variable conversion element as it converts the fluid pressure (force per unit area) to a resultant force on the piston face. Force is transmitted by the piston rod to the spring, which converts force to a proportional displacement. This displacement of the piston is magnified (manipulated) by the linkage to provide a larger pointer

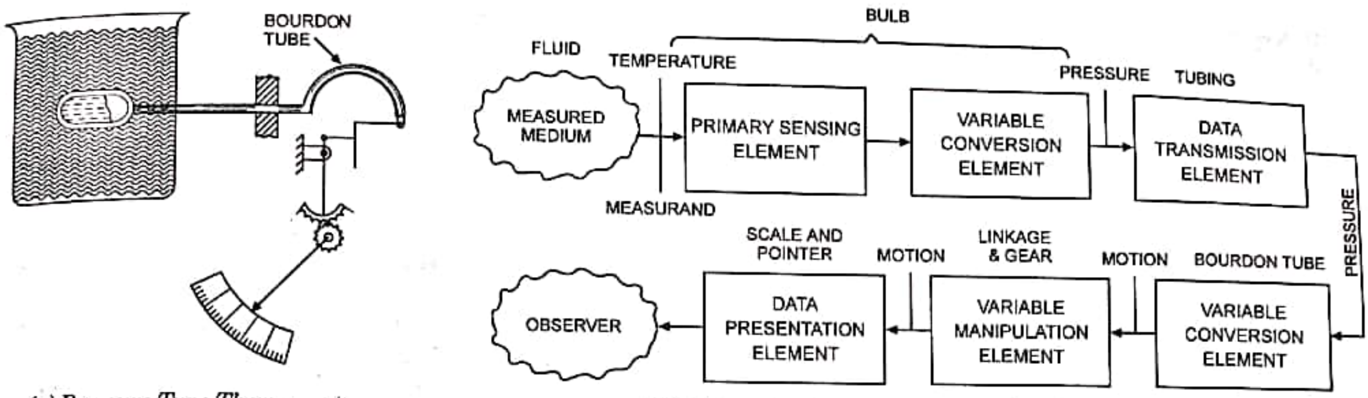


(a) Rudimentary Pressure Gauge



(b) Schematic Diagram of Pressure Measurement

Fig. 1.7



(a) Pressure Type Thermometer

(b) Schematic Diagram of Temperature Measurement

Fig. 1.8

displacement. The pointer and scale indicate the pressure, thus serving as data presentation elements. If it were necessary to locate the gauge at some distance from the source of pressure, a small tube could serve as a data-transmission element. The schematic diagram of this measurement system is depicted in Fig. 1.7(*b*).

Pressure type thermometer shown in Fig. 1.8 (*a*), is another example. The liquid-filled bulb acts as a primary sensing element and a variable conversion element since a temperature change results in a pressure buildup within the bulb, because of the constrained thermal expansion of the filling fluid. This pressure is transmitted through the tube to a Bourdon type pressure gauge, which converts pressure to displacement. This displacement is manipulated by the linkage and gearing to provide a larger pointer motion. A scale and pointer again serve for data presentation.

1.8 ART OF MEASUREMENT

We cannot think of a measurement without error inspite of the fact that very sophisticated measurement systems are available. However, the error can be reduced if a proper method of measurement is selected and some necessary precautions are observed at the time of measurement. Recording of the measured data is also very important.

A number of important steps are always involved in a measurement. The method chosen should be such that makes use of available instruments/apparatus to obtain the desired result with required accuracy. It needs consideration of the reliability of the instruments or apparatus to be employed, the effects of various conditions which will be encountered in the experiment, the time required for setting up and operating the necessary equipment, and the accuracy that may be desired or attainable in the final result. The method adopted should be as simple as possible, consistent with the requirements of the task. It is quite frequently desirable to avoid apparatus and methods whose inherent accuracy is higher than is required for the task at hand, if their use and manipulation are so time consuming or otherwise expensive that the choice is uneconomical. The appropriate choice of apparatus and experimental method becomes easier as the worker's experience becomes broader in his field.

The method and apparatus having been chosen, it is important that they be intelligently used. The function of each piece of apparatus and its method of operation should be thoroughly understood. The sketching of a preliminary wiring diagram will usually save time in assembly and may avoid blunders in connecting apparatus/instruments. Measuring instruments and other apparatus intended to be used should first be inspected to ensure that they are in good working conditions, and the setup should be carefully checked before supply is switched on. Skill in manipulation of apparatus and in recording of data is acquired by actual practice. Meticulous care and questioning attitude are essential for obtaining reliable results.

Data forming the basis of a written report should be carefully recorded. Records of data should not only be complete as regards observations, but should include a connection diagram of the circuit employed, with each instrument or other piece of apparatus identified (preferably with make, model and serial number), so that the setup can be exactly duplicated at any time in future, and so that, if any piece of apparatus proves to be defective or incorrectly calibrated, its influence on the experimental results can be determined.

While making numerical computations it is unnecessarily time consuming and fatiguing to use more digits than are actually significant in the values entering the computation. To avoid the necessity for manipulating surplus digits in arithmetic operations and the added opportunities for mistakes as a result of the extra manipulations, numerical values should always be rounded off at the point where they cease to have a real meaning in terms of measurement conditions.

Certain precautions are required to be observed for the safe and efficient use of instruments, apparatus and equipment. There are some considerations that apply in general to the electrical circuit employed, regardless of the instruments used and the type of measurement carried out. While making electrical connections ensure that contact surfaces are clean, nuts or binding posts are firmly tightened, wires or cables have sufficient x-section for the expected current, and insulation is appropriate for the voltage in use. Sliding contacts are to be cleaned occasionally with a lint-free cloth, either dry or moistened with a solvent such as benzol or varsol. Sliding contacts will remain in good working condition longer if lubricated. For high contact pressure a solid grease such as vaseline may be used while for light contacts a highly refined, neutral mineral oil is better. In case an extremely light pressure between noble metal contacts, no lubricant is to be used. When soft-soldering connections or other parts of an electrical circuit, only rosin or a solution of rosin in alcohol should be used. Almost all prepared fluxes are sufficiently corrosive to damage the components of delicate apparatus. Leads attached to the instruments should never be left hanging down over the edge of a table, or stretched between tables or across the floor where they may be accidently caught with a hand or foot and the instrument pulled to the floor. They should be twisted in pairs to each instrument to reduce the effects from magnetic fields caused by current in leads. For measurements of larger alternating currents (say above 25 A or 30 A) 5 A ammeter in conjuction with a current transformer should be used preferably to a high-range ammeter, both to avoid large currents in the neighbourhood of measuring instruments and to isolate the measuring instruments from the supply circuit. Similarly for larger alternating voltage measurement low-range voltmeter in conjuction with a potential transformer should be used in preference to a high-range voltmeter. Before putting the supply ON all components should be checked to ensure that all the connections are properly made and that the ranges of instruments and apparatus are sufficient for the quantity under measurement. Protective resistors should be inserted as per requirement. While opening a circuit connected to measuring instruments and apparatus, it is advisable where possible, to first reduce the supply voltage to a low value. The conductors or leads to the apparatus should be removed one by one, making the first break at the terminal nearest to the power supply and afterward removing the terminal connected to the equipment. While making connections, the process should be reversed making the connection to the power supply in last. Consistency in following this procedure may prevent short circuits at exposed terminals. Power circuits are usually protected by fuses or circuit breakers, but these protective devices do not guarantee that instruments and other equipment will not get damaged in case the circuit is shorted due to careless handling of leads or switches.

While using multirange instrument, the position of the range switch should be checked before putting the switch on, otherwise the pointer may bend or the coil may get burned out. When a measurement is to be made under conditions that will produce a high initial current and a much smaller steady current it may be advisable to protect the current coils of the instruments against the initial high current by a short-circuiting switch. In movement of delicate instruments such as microammeters or pivoted galvanometers, these are required to be protected against mechanical damage, by shorting the terminals to provide heavy damping. Pivoted instruments should never be placed where they may be exposed to vibration. Severe shock, such as a hammer blow on a bench or table, where an instrument is lying, can damage its pivots or jewels permanently.

Obstructions can often be detected by a "jumping" of the pointer as the operating current of the instrument is very slowly increased or reduced to cause a gradual change in deflection. If, during such a test the pointer is set to a scale mark and the instrument is lightly tapped, pivot friction will be revealed by a slight change in the position of the pointer. Where pivot friction is present its effect on instrument readings can be temporarily reduced to minimum by gently tapping the case. Hard tapping will defeat its purpose and may result in damage to the bearing. Errors due to friction and mechanical unbalance may be expected if the instrument is used in a position other than the one in which it was designed to operate. Where the moving system vibrates in operation on account of mechanical resonance at the operating frequency, errors may be expected. This condition is usually corrected by changing the balancing weights on the moving system and rebalancing it. Inelastic yield in the springs, indicated by a zero shift after the instrument has been deflected upscale for a considerable time, constitutes a source of error for which a correction cannot easily be applied. Such a spring should be replaced by the manufacturer if the amount of zero shift is significant.

As far as possible location of instruments in area of strong magnetic fields or near large masses of metal should be avoided, particularly if the instrument is not magnetically shielded. Strong fields from machinery, conductors, and even other instruments may cause significant errors in indication; and the presence of neighbouring masses of metal, for example a metal table top on which the instrument is placed, may produce appreciable eddy current errors in some types of instruments. Iron in the neighbourhood of an unshielded permanent magnet instrument may act as a magnetic shunt and change the instrument indication by a considerable amount. It must be remembered that some types of instruments require considerable power for their operation and that significant changes in circuit conditions may result due to insertion of instruments. In accurate measurements allowance must be made for such effects. Also, if the instrument or other apparatus is required to dissipate an appreciable amount of power, its resulting temperature rise may cause an error in the indicated values.

1.9 INSTRUMENTATION SYSTEMS

Measurement of nonelectrical quantities by electrical methods was given greater attention since the middle of twentieth century and this marked the beginning of one of the most fruitful and vigorous areas of activity concerning the scope of their applications. Simultaneously, the

Note : Significant figures will be explained later on.

advances made in the field of electronics substantially contributed to the birth of a new discipline known as *instrumentation*. Instrumentation deals with the science and technology of measurement of a large number of variables embracing the disciplines of physical sciences such as physics and chemistry and engineering disciplines like mechanical, electrical, electronics, communication and computer engineering. Instrumentation refers to the art and science of collection of several instruments and auxiliary equipment and their utilization for conducting successfully a test or an experiment on a system, process or plant.

An *instrumentation system* is a physical system, which is a collection of physical objects connected in such a way as to provide the desired output response. Examples of a physical system may be cited from a laboratory such as an electronic amplifier composed of many components from an industrial plant such as a steam turbine or from utility services such as a communications satellite orbiting the earth. Thus an instrumentation system may be defined as an assembly of various instruments and other components interconnected to measure, analyse and control the electrical, thermal, hydraulic and other nonelectrical physical quantities.

No physical system can be represented in its full intricacies and, therefore, idealising assumptions are always made for the purpose of analysis and synthesis of systems. An idealised physical system is known as *physical model*. A physical system can be modelled in a number of ways depending upon the specific problem to be dealt with and the required accuracy. For example, an electronic amplifier may be modelled as an interconnection of linear lumped elements, or some of these may be pictured as nonlinear elements in case the stress is on distortion analysis. Once a physical model of a physical system is obtained, the next step is to have a *mathematical model* i.e. mathematical representation of the physical model by making use of appropriate physical laws. Depending upon the choice of variables and the coordinate system, a given physical model may lead to different mathematical models. For example, a network may be modelled as a set of nodal equations employing Kirchhoff's first law (or current law) or a set of mesh equations employing Kirchhoff's second law (or voltage law). Then the mathematical model is solved for various types of inputs to have the dynamic response of the system.

Instrumentation systems can be classified into two main categories namely, *analog systems* and *digital systems*.

Analog systems deal with measurement information in analog form. An analog signal may be defined as a continuation function, such as a plot of voltage against time or displacement against force.

Digital systems handle measurement information in digital form. A digital quantity may consist of a number of discrete and discontinuous pulses whose time relationship contains information regarding magnitude or the nature of quantity.

1.10 INTELLIGENT INSTRUMENTATION AND DUMB INSTRUMENTATION

It is no exaggeration to mention that the microprocessor has made tremendous revolution in the field of instrumentation and control. Microprocessor-based systems find wide use in industries for instrumentation and control. Microprocessor-based systems may be classified as *intelligent instrumentation systems* and *dumb instrumentation systems*.

1. Intelligent Instrumentation Systems. Intelligent instrumentation system means the use of an instrumentation system for the evaluation of a physical variable by the use of a digital computer in performing all or nearly all signal and information process. In this system after the accomplishment of measurement of the physical variable, further processing (either in digital or in analog form) is carried out to refine the data, for the purpose of presentation to an observer or to other computers. Block diagram of such a system is shown in Fig. 1.9.

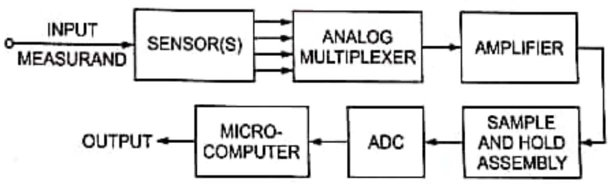


Fig. 1.9 *Generalized Block Diagram of Intelligent Instrumentation System*

From the figure, it is observed that a microcomputer has been included which gets its input through an analog-to-digital converter (ADC) in digital format.

2. Dumb Instrumentation Systems. A dumb or conventional instrumentation system is that in which the input variable is measured and displayed, but the data is required to be processed by the observer. For example, speedometer is termed as a *dumb instrument* because it can measure and display the vehicle speed, but the observer has to judge whether the indicated speed is desired one, high or low.

Intelligent instrumentation system has various advantages over the dumb instrumentation system such as higher accuracy, self-diagnosis, higher immunity to external disturbances and noise, multi-user facility, transmission over long distances etc.

1.11 INSTRUMENTATION IN AUTOMATION

The instrumentation system plays an important role in automation. An automatic control system (or automation) requires a *comparator* (or an error detector), which measures the difference between the actual and desired performance and actuates the control elements. General block diagram of an automatic control system is shown in Fig. 1.10. An error detector (or a comparator) compares a signal obtained through feedback elements, which is a function of the output response, with the reference input. Any difference between these two signals constitutes an

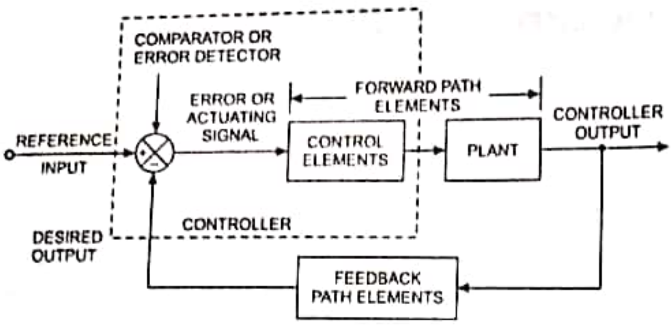


Fig. 1.10 *General Block Diagram of an Automatic Control System*

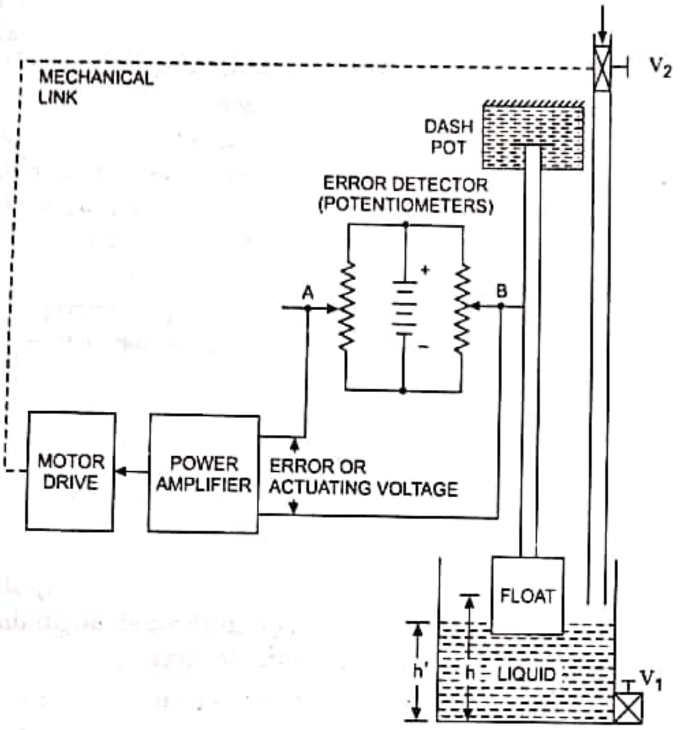


Fig. 1.11 *Automatic Tank Level Control System*

error or actuating signal, which actuates the control elements. The control elements in turn alter the conditions in the plant (controlled member) in such a manner that the original difference or error is reduced.

There are numerous examples of this type of application. A common one is the typical simple tank level control system shown in Fig. 1.11. With this control system liquid level (controlled output) in the tank can be maintained within accurate tolerance of the desired level of liquid even though the output flow rate through the valve V_1 is varied. The float (feedback path element) senses the liquid level and positions the slider arm B on a potentiometer. The slider arm A of another potentiometer is positioned corresponding to the desired liquid level h (the reference input). When the liquid level rises or falls, the potentiometers (error detector) give an error voltage (error or actuating signal) proportional to the change in liquid level. The error voltage actuates the motor through a power amplifier (control elements) which in turn conditions the plant (i.e. decreases or increases the opening of the valve V_2) in order to restore the desired liquid level. Thus, the control system automatically attempts to correct any deviation between the actual and desired liquid levels in the tank.

1.12 APPLICATIONS OF MEASUREMENT INSTRUMENTATION

The ways the instruments and measurement systems are used for different applications are given below:

1. Monitoring of processes and operations.
2. Control of processes and operations.
3. Experimental engineering analysis.

1. ***Monitoring of Processes and Operations.*** There are certain applications of measuring instruments that have essentially a *monitoring function*. For example, the thermometers, barometers and anemometers used by the weather bureau serve in such a capacity. They simply indicate the condition of the environment, and their readings do not serve any control functions in the ordinary sense. Likewise the energy meter installed in a house, keeps the track of the electrical energy consumed by the consumer for billing purposes.

2. ***Control of Processes and Operations.*** This is another extremely important type of application for measuring instruments. There has been a very strong association between measurement and control.

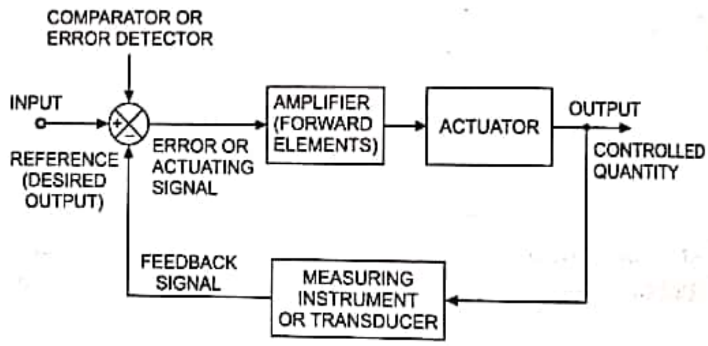


Fig. 1.12 *Block Diagram of a Simple Control System*

In order to control the process variables like temperature, pressure, humidity, liquid flow etc., these quantities are required to be measured at appropriate points in the individual plant. Figure 1.12 illustrates a simple control system. Let the output variable to be controlled be non-electrical and the control system action through electrical means. The input is reference that corresponds to the desired value of output. The input and the output are compared with the help of comparator or error detector and the error or actuating signal is amplified to operate the actuator in controlling the process. The actuator, in turn, produces power to drive the controlled circuitry. The corrective action goes on till the output under the control is at the same level as the input which corresponds to the desired output. At this stage, there is no error signal and, therefore, there is no input to the actuator and the actuator stops working. *Transducer* is connected in the feedback loop in order to convert nonelectrical output into a corresponding electrical form.

Examples of such an application are endless. A familiar application is the typical home-heating system employing some type of thermostatic control. A temperature measuring instrument (often a bimetallic element) senses the room temperature, and thus provides the information required for proper functioning of the control system. Much more sophisticated examples are found among the aircraft and missile control systems. A single control system may need information from many measuring instruments such as pitot-static tubes, angle-of-attack sensors, thermocouples, accelerometers, altimeters and gyroscopes. Many industrial machine and process controllers also make use of multi-sensor measurement systems.

3. ***Experimental Engineering Analysis.*** Many engineering problems are required to be analysed theoretically as well as experimentally. The relative amount of each depends on the nature of problem. Problems on the frontiers of knowledge often need very extensive experimental studies as adequate theories are not available yet. Thus the theory and experiment should be thought of as complementing each other. Experimental engineering analysis has many uses and some of these are listed below:

1. Testing the validity of theoretical predictions based on simplifying assumptions; improvement of theory based on measured behaviour. Frequency-response testing of mechanical linkage for resonant frequencies is an example of it.
2. Formulation of generalised empirical relationships in situations where no adequate theory exists. Determination of friction factor for turbulent flow in a pipe is an example of such an application.
3. Determination of material, component, and system parameters, variables, and performance indices. Determination of yield point of a certain alloy steel, speed-torque curves for an electric motor, thermal efficiency of a steam turbine etc. are examples.
4. Solution of mathematical equations by means of analogies. Solution of shaft torsion problems by measurements on soap bubbles is an example.
5. Study of phenomena with hopes of developing a theory. Electron microscopy of metal fatigue cracks is an example.

EXERCISES

1. What is measurement ? Explain its significance in various fields of engineering.
2. What is measurement and measuring instrument? Explain. Explain the direct measurement and indirect measurement. Which one is most commonly used method and why ?
3. Describe various methods of measurements with the help of suitable block diagram.
[G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2011-12]
4. Distinguish between direct and indirect methods of measurements giving suitable examples.
[U.P. Technical Univ. Electrical Measurements and Measuring Instruments 2013-14]
5. Briefly mention the development phases (historically) of instrument for measurement.
6. Compare the advantages and limitations of mechanical and electrical systems.
7. Enlist the advantages of electronic instruments over electrical and mechanical instruments.
[U.P. Technical Univ. Elec. Measurements and Measuring Instruments 2005-06]
8. How will you classify different instruments? Explain with examples.
[M.D. Univ. Electrical Measurements and Measuring Instruments, December-2008]
9. What is an instrument? Classify various types of electrical instruments.
[G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2012-2013]
10. What are absolute and secondary instruments? What are the advantages of electronic instruments?
[W.B. Univ. of Technology Electrical and Electronics Measurements, 2009-2010]
11. Explain primary and secondary instruments.
[M.D. Univ. Electrical Measurements and Measuring Instruments, May-2008]
12. Differentiate between primary and secondary instruments with the help of examples.
[M.D. Univ. Electrical Measurements and Measuring Instruments, May-2009]
13. Explain what is meant by active and passive instruments. Give examples of each and discuss the relative merits of these two classes of instruments.
14. Describe difference between deflection and null type of instruments giving suitable examples. Discuss about their accuracy and sensitivity.
[U.P. Technical Univ. Electrical Measurement and Measuring Instruments, 2006-07]
15. Discuss the advantages and disadvantages of null and deflection types of measuring instruments. What for are null types instruments mainly used and why?
16. Describe the difference between deflection and null type of instruments giving suitable examples. Discuss their accuracy, sensitivity and suitability for dynamic measurement.
17. Explain analog and digital modes of operation. Why the digital instruments are becoming popular now? What is meant by ADC and DAC?
18. Explain recording and integrating instruments.
[M.D. Univ. Electrical Measurement and Measuring Instruments, December-2009]
19. Draw and explain the block diagram of generalized instrumentation.
[M.D. Univ. Electrical Measurement and Measuring Instruments, December-2010]
20. Draw the block diagram of a generalized instrument and describe briefly the function of each block.
[M.D. Univ. Electrical Measurement and Measuring Instruments, May-2008]
21. What are the various elements of a measuring instrument or system? Explain with a suitable block diagram. What is variable manipulation unit?
22. What do you mean by instrumentation system? Also explain the functional block diagram of a measurement.
[G.B. Technical Univ. Electronic Instrumentation and Measurements, 2011-12]
23. Explain the term instrumentation system, its physical model and mathematical model.
24. Differentiate between analog and digital systems.
25. Explain an automatic control system giving a suitable example.
26. Distinguish between "Intelligent Instrumentation System" and "Dumb Instrumentation System".
[U.P. Technical Univ. Electronic Measurements and Instrumentation, 2007-08]

27. Cite at least two examples of measuring instrument applications in the field of (*i*) experimental engineering analysis (*ii*) control of processes and operations and (*iii*) monitoring of processes and operations.
[Madurai Univ. April 1989]

28. Write short notes on the following:
(*i*) Methods of measurement.
(*ii*) History of development of instruments.
(*iii*) Classification of instruments.
(*iv*) Analog and digital modes of operation.
(*v*) Functions of instruments and measurement systems.
(*vi*) Elements of a generalised measurement system.
(*vii*) Applications of measurement instrumentation.

SHORT ANSWER TYPE QUESTIONS WITH ANSWERS

Q. 1. What is measurement?
Ans. Measurement is essentially the act, or the result, of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a standard or a unit.

Q. 2. What is meant by the term "measurand"?
Ans. The quantity to be measured is known as the measurand.

Q. 3. What is measuring instrument?
Ans. The device used for comparing the unknown quantity with the unit of measurement or a standard quantity is called a measuring instrument.

Q. 4. Why direct comparison methods of measurement are rarely used?
Ans. It is not always possible, feasible and practicable to use direct comparison methods of measurement. These methods are inaccurate and less sensitive too. So direct comparison methods of measurement are rarely used.

Q. 5. What are the three essential elements of an instrument?
Ans. The three essential elements of an instrument are a detector, an intermediate transfer device, and an indicator or a storage device.

Q. 6. What are the drawbacks of mechanical instruments?
Ans. The drawbacks of mechanical instruments are (*i*) inability of responding rapidly to the measurements of dynamic and transient conditions and (*ii*) they are a potential source of noise.

Q. 7. What are the advantages of electronic instruments over their mechanical and purely electrical counterparts?
Ans. The electronic instruments have a higher sensitivity, a faster response, lower weight, a greater flexibility, a higher degree of reliability, and low power consumption as compared to their mechanical and purely electrical counterparts.

Q. 8. Absolute instruments are rarely used in practice. Why?
Ans. Absolute instruments are rarely used because working with absolute instruments for routine work is time-consuming.

Q. 9. Why are direct measuring instruments most widely used in engineering practice?
Ans. Direct measuring instruments are the most simple and inexpensive ones and enable measurements to be made in the shortest possible time, and, therefore, are most widely used in engineering practice.

Q. 10. What for are null type instruments mainly used and why?
Ans. The null type instruments are more accurate and highly sensitive as compared with deflection type instruments, and therefore, are more suitable for calibration purposes.

Q. 11. What is difference between analog and digital signals?
Ans. Signals that vary in a continuous fashion and take on an infinite number of values in any given range are known as *analog signals* while the digital signals vary in discrete steps and thus take up only finite different values in a given range.

Q. 12. The importance of digital instruments is increasing. Why?
Ans. The importance of digital instruments is increasing mainly because of the widespread use of digital computers in both data processing and automatic control systems.

Q. 13. Give classification of instruments on the basis of their functions.
Ans. The instruments, on the basis of their functions, are classified as indicating, recording and controlling instruments.

Q. 14. What is primary sensing element?
Ans. The primary sensing element is that which makes contact with the physical quantity under measurement, receives energy from the measured medium and produces an output depending in some way on the measurand.

Q. 15. What is meant by transducer?
Ans. The transducer is defined as a device which, when actuated by one form of energy, is capable of converting it into another form of energy.

Q. 16. What is meant by instrumentation?
Ans. Instrumentation refers to the art and science of collection of several instruments and auxiliary equipment and their utilisation for conducting successfully a test or an experiment on a system, process or plant.

Q. 17. Give classification of microprocessor-based instrumentation systems.
Ans. Microprocessor-based instrumentation systems may be classified into two categories viz. Intelligent instrumentation systems and Dumb instrumentation systems.

Q. 18. Enlist the applications of measurement instrumentation?
Ans. The applications of measurement instrumentation are (*i*) monitoring of processes and operations (*ii*) control of processes and operations and (*iii*) experimental engineering analysis.

2 CHAPTER Characteristics of Instruments and Measurement Systems

INSIDE THIS CHAPTER

2.1 INTRODUCTION

The *performance characteristics* of an instrumentation system are judged by how faithfully the system measures the desired input and how thoroughly it rejects the undesirable inputs. Quantitatively, it relates to the degree of approach to perfection. The system operation is defined in terms of static and dynamic characteristics. The former represents the nonlinear and statistical effects and the latter generally represents the dynamic behaviour of the system.

The reasons for classification of performance characteristics of instruments or measurement systems into static and dynamic characteristics are several. First, some applications involve the measurement of quantities that are constant or vary only quite slowly. Under such conditions, it is possible to define a set of performance criteria that provide a meaningful description of the quality of measurement without becoming concerned with dynamic descriptions involving differential equations. These criteria are called the *static characteristics*. Many other measurement problems involve rapidly varying quantities. Here the dynamic relations between the instrument input and output must be examined, generally by use of differential equations. Performance criteria based on these dynamic relations constitute the *dynamic characteristics*.

Actually, static characteristics also influence the quality of measurement under dynamic conditions, but the static characteristics generally show up as nonlinear or statistical effects in the otherwise linear differential equations giving the dynamic characteristics. These effects would make the differential equations unmanageable, and so the conventional approach is to treat the two aspects of problem separately. Thus the differential equations of dynamic performance generally neglect the effects of dry friction, backlash, hysteresis, statistical scatter etc., even though such effects affect the dynamic behaviour. These phenomena are more conveniently studied as static characteristics, and the overall performance of an instrument/measurement system is then judged by a semi-quantitative superposition of the static and dynamic characteristics. This approach is, of course, approximate but a necessary expedient.

2.2 STATIC CHARACTERISTICS

The static characteristics of an instrument/transducer/ measuring system are established by the process of *static calibration*. By static calibration, the relationship between the output signal and the quantity under study is experimentally determined.

In general, *static calibration* refers to a situation in which all inputs (desired, interfering, modifying) except one are kept at some constant values. Then the one input under study is varied over some range of constant values, which causes the output(s) to vary over some range of constant values. The input-output relations developed in this way comprise a static calibration *valid under the stated constant conditions of all the other inputs*. This procedure may be repeated, by varying in turn each input considered to be of interest and thus developing a family of static input-output relations. The overall instrument static behaviour may be had by some suitable form of superposition of individual effects or in some cases by variation of several inputs simultaneously. In practice there may be many modifying and/or interfering inputs, each of which might have quite small effects and which would be impractical to control. Thus the statement "all other inputs are held constant" refers to an ideal situation which can be only approached, but never reached, in practice.

The process of static calibration requires the presence of a standard with which the output signal of the measuring device is to be related. If it is an indicating or recording instrument that is under calibration, the indicated value of the pointer on the calibrated scale is checked for its correctness by comparison with a standard source of the quantity under measurement. But, in the case of measuring devices and transducers, the output signal is invariably of different forms and dimensions and by the process of calibration, the relationship between the input quantity and the output signal has to be established, and this relationship is treated as its static characteristic. It is also known as *calibration curve*, though in many cases, it is a straight line.

When a measuring system requires to be calibrated, a proper standard* whose accuracy is greater than that of the measuring system should be selected. It is better if the working standard is variable in value for calibration at several points of the scale or range. A convention often followed is that the calibration standard should be at least ten times more accurate than the system under calibration. During the calibration process, it is essential to see that the standard is not disturbed in its value due to the connection or coupling of the measuring system or device with the working standard or due to the technique adopted for comparison of the values of the standard and the measuring device.

2.2.1. Calibration Process. Calibration is an essential process to be undertaken for each instrument and measuring system as frequently as is considered necessary. A reference standard at least ten times more accurate than the instrument being tested is normally used. The calibration process is simple. It consists of reading the standard and test instruments simultaneously when the input quantity is held constant at several values over the range of the test instrument. The calibration is better carried out under the stipulated environmental conditions. All industrial grade instruments can be checked for accuracy in the laboratory by using the working standards. All modern electronic instruments that are likely to drift in sensitivity are provided with built-in calibration facility by using suitable circuits along with zener diodes.

While calibrating, it is customary to take readings both in the ascending and descending order. Calibration thus reveals some of the inherent flaws in the electro-mechanical instruments and other mechanical transducers involving elastic elements. It is essential to check the linearity between the input and output quantities of many transducers and declare the extent of non-linearity likely to exist for the range for which it is to be used.

The test instrument is calibrated under several environmental conditions in order to ensure that the grade of performance conforms to its stated specifications. While calibration in laboratory may be a simple process, statistical approach is necessary sometimes.

The process of static calibration enables the definition of several characteristics of the measuring system. The following terms specify the static characteristics of the measuring systems and devices and a better understanding of the terms enable proper selection of the measuring systems for actual application.

1. ***Accuracy***. This is a qualitative term used to relate the instrument output to the true value of measurand with declared probability limits. In measurement, it is influenced by static error, dynamic error, drift, reproducibility, non-linearity, hysteresis, temperature and vibration.

Accuracy refers to the degree of closeness or conformity to the accepted standard value or the true value of the quantity under measurement. The only time a measurement can be exactly correct is when it is a count of a number of separate items, e.g., a number of components or a number of electrical pulses. In all other cases there will be a difference between the true value and the value the instrument indicates, records or controls to i.e., there is a measurement error. The extent of this error, or the accuracy of the instrument may be specified in several different ways.

(*a*) *Point Accuracy*. Here the accuracy of an instrument is stated for only one or more points in its range i.e. the specification of such accuracy does not give any information about the general accuracy of the instrument. This is particularly applicable to temperature-measuring devices, where points are obtained at the melting and vapourising temperatures of pure solids and liquids.

(*b*) *Percentage of True Value*. When the accuracy of an instrument is expressed in this way, then the error is computed as below

$$\text{Error} = \frac{\text{Measured value} - \text{True value}}{\text{True value}} \times 100 \quad ...(2.1)$$

The percentage error stated is the maximum for any point in the range of the instrument.

(*c*) *Percentage of Full-Scale Deflection*. Here the error is calculated on the basis of maximum value of the scale, thus

$$\text{Error} = \frac{\text{Measured value} - \text{True value}}{\text{True scale value}} \times 100 \quad ...(2.2)$$

(*d*) *Complete Accuracy Statement*. In some cases, pyrometers for example, it may not be sufficient to specify accuracy at a limited number of points, and the accuracy at a larger number of points is specified in tabulated or graphical form. As a further example, the error of each individual gauge in a set of slip gauges is specified, it would not be adequate to specify a percentage error for the set based on the nominal gauge sizes.

It will be seen that an accuracy specified as a percentage of full-scale deflection implies a less accurate instrument than one having the same accuracy percentage of true value. For example, an error of ±1% of full-scale deflection on a thermometer having a range of 500 °C would mean that a true temperature of 50 °C could read from (50 – 1% of 500) to (50 + 1% of 500) i.e. 45–55 °C; as a percentage of true value it would read from 49.5–50.5 °C. Thus specification of accuracy in this manner is highly misleading.

* The standard may be absolute (or primary), secondary, and working standard. For details reference may be made to Chapter 5.

2. *Precision.* Another characteristic that is often referred to in measurement is the "precision" of the device. It is a term which describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high-precision instrument, then the spread of the readings will be very small.

Precision is a measure of the consistency or repeatability of measurements *i.e.* successive readings do not differ. (Precision is the consistency of the instrument output for a given value of input). It combines the uncertainty due to both random differences in results in a number of measurements and the smallest readable increment in scale or chart (given as the deviation of mean value).

Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about accuracy. A high-precision instrument may have a low accuracy. Low-accuracy measurements from a high-precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.

3. *Bias.* Bias describes a constant error which exists over the full range of measurement of an instrument. The error is normally removable by calibration.

Bathroom scales are a common example of instruments which are prone to bias. It is quite usual to find that there is a reading of perhaps 1 kg with no one stood on the scales. If someone of known weight 80 kg were to get on the scales, the reading would be 81 kg, and if someone of known weight of 120 kg were to get on the scales, the reading would be 121 kg. This constant bias of 1 kg can be removed by calibration: in the case of bathroom scales this normally means turning a thumbwheel with the scales unloaded until the reading is zero.

4. *Repeatability and Reproducibility.* The terms repeatability and reproducibility mean approximately the same but are applied in different contexts as given below.

Repeatability is the characteristic of precision instruments. It describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout. It is affected by internal noise and drift. It is expressed in percentage of the true value. Measuring transducers are in continuous use in process control operations and the repeatability of performance of the transducer is more important than the accuracy of the transducer, from considerations of consistency in product quality.

Reproducibility is the closeness with which the same value of the input quantity is measured at different times, and under different conditions of usage of the instrument and by different instruments. The output signals and indications are checked for consistency over prolonged periods and at different locations. Perfect reproducibility ensures interchangeability of instruments and transducers.

Both terms thus describe the spread of the output readings for the same input. The spread is referred to as repeatability if the measurement conditions are constant and as reproducibility if the measurement conditions vary.

5. *Tolerance.* Tolerance is a term which is closely related to accuracy and defines the maximum error which is to be expected in some value. Whilst it is not, strictly speaking, a static characteristic of measuring instruments, it is mentioned here because the accuracy of some instruments is sometimes quoted as a tolerance figure.

Tolerance, when used correctly, describes the maximum deviation of a manufactured component from some specified value. Electric circuit components such as resistors, for instance, have tolerances of perhaps 5%.

6. *Reliability and Maintainability.* The *reliability* of a system is defined as the probability that it will perform its assigned functions for a specific period of time under given conditions. The *maintainability* of a system is the probability that in the event of failure of the system, maintenance action under given conditions will restore the system within a specified time. Both the factors are extremely significant for complex systems. Estimates of reliability and maintainability must be included in the initial trade-off between performance cost and schedule.

The reliability of a device or system is affected not only by the choice of individual components in system but also by manufacturing methods, quality of maintenance, and the type of user.

7. *Deviation.* It is a departure from a desired or expected value or pattern and may also be described as the difference between measured value and true value for a particular input value. The deviation is given a plus or minus sign, depending on whether the measured values are above or below the true value.

8. *Scale Range and Scale Span.* Span and range are the two terms that convey the information about the lower and upper calibration points. The range of indicating instruments is normally from zero to some full-scale value and the span is simply the difference between the full-scale and lower-scale value. But some instruments operate under a bias so that they start reading, for example, voltages from 5 V to 25 V only. The zero of these instruments is suppressed from indication by means of a bias. In such a case, the scale range is said to be from 5 V to 25 V and the scale span is 25 – 5 i.e. 20 V.

Any excess value of the input signal above its upper range value or below its lower range value is called the over-range of a system or element.

On multirange instruments it is often necessary to distinguish between the limits to which the device can be adjusted versus the limits to which the device is adjusted. The following terms describe the limits in detail.

(*i*) *Low-range Limit.* The lowest value of quantity that a device can be adjusted to measure is known as *low-range limit.*

(*ii*) *Upper-range Limit.* The highest value of the measured variable that a device can be adjusted to measure is known as *upper-range limit.*

(*iii*) *Lower-range Value.* The lowest value of the measured variable a device is adjusted to measure is called the *lower-range value.*

(iv) *Upper-range Value.* The highest value of the measured variable a device is adjusted to measure is called the *upper-range value.*

9. ***Live Zero.*** Live zero is a term applied to measuring systems whose output signal is not zero for zero input quantity. Some feedback transducer systems are designed to develop output signals of dc current ranging from 4–20 mA when the input quantity changes from zero to the full-scale value. For bidirectional variation of the input quantity from zero value, the output signal variation may be affected from 12 mA in either direction over the range of 4–20 mA.

10. ***Scale Readability.*** Since the majority of the instruments that have analog (rather than digital) output are read by a human observer noting the position of a pointer on a calibrated scale, usually it is desirable for data taker to state their opinions as to how closely they believe they can read this scale. This characteristic, which depends on both the instrument and observer, is called the *scale readability.*

Scale readability varies with the design of the instrument and is partly governed by the instrument sensitivity. The term denotes the extent to which the reader is enabled to read the indications. If it is a digital instrument, the reader records the reading as obtained in all the digits, but in the case of analog instruments, his judgement of the reading decides the last digit of the indicated value. The readability of the instrument conveys the degree to which the reader can be precise in recording the reading, but it does not always ensure that the reading is accurate in its value up to the last digit.

11. ***Stability.*** Stability defines the ability of a measuring system to maintain its standard of performance over prolonged periods of time. Transducers and instruments of high stability need not be calibrated frequently.

12. ***Zero Stability.*** It defines the ability of an instrument to restore to zero reading after the input quantity has been brought to zero, while other conditions remain the same.

13. ***Resolution (or Discrimination) and Threshold.*** If the input to an instrument is increased slowly from some arbitrary nonzero value, it will be observed that the output of the instrument does not change at all until there is a certain minimum increment in the input. This minimum increment in input is called resolution of the instrument. Thus, the resolution is defined as the smallest increment of the input quantity to which the measuring system responds. Resolving power or discrimination power is defined as the ability of the system to respond to small changes of the input quantity.

One of the major factors influencing the resolution of an instrument is how finely its output scale is subdivided. Using a car speedometer as an example, this has subdivisions of typically 10 km/h. This means that when the needle is between the scale markings, we cannot estimate speed more accurately than to the nearest 5 km/h. The figure of 5 km/h thus represents the resolution of the instrument.

If the input to an instrument is increased very gradually from zero value, there will be some minimum value of input below which no output change can be observed or detected. This minimum value of input defines the *threshold* of the instrument. The phenomenon is specified by the first detectable output change which is noticeable or measurable.

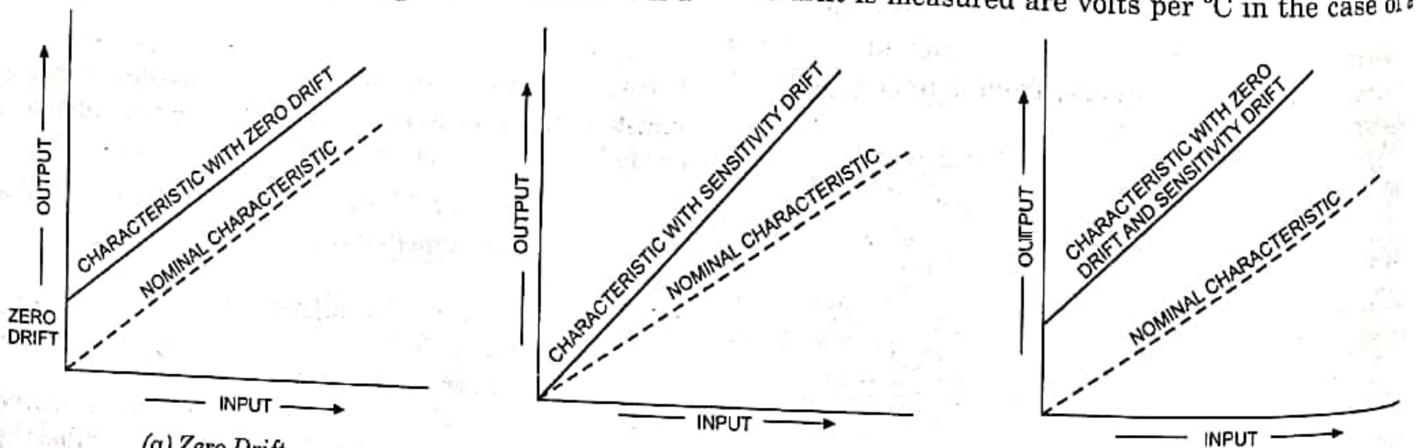
As an illustration, a car speedometer typically has a threshold of about 15 km/h. It means that if the vehicle starts from rest and accelerates, no output reading is observed on the speedometer until the speed reaches 15 km/h.

Both resolution and threshold may be expressed either in absolute value or as a percentage of full-scale reading.

14. ***Responsiveness.*** It is defined as the smallest change in the quantity under measurement, which results in an actuating effort required to cause motion of the indicating part of the instrument.

15. ***Drift.*** Drift is a slow variation in the output signal of a transducer or measuring system which is not due to any change in the input quantity. It is primarily due to changes in operating conditions of the components inside the measuring system. The drift is noticeable as zero drift and sensitivity drift.

Zero drift is the deviation observed in the instrument output with time from the initial value, when all the other measurement conditions are constant. This may be caused by a change in component values due to variation in ambient conditions or due to ageing. Typical units by which zero drift is measured are volts per °C in the case of a



(a) Zero Drift *(b) Sensitivity or Scale Factor Drift* *(c) Zero Drift Plus Sensitivity Drift*

Fig. 2.1 *Input-Output Characteristics With Drift* voltmeter affected by changes in ambient temperature. This is often called the zero drift coefficient related to temperature changes. If the characteristic of an instrument is sensitive to several environmental parameters, then it will have several zero drift coefficients, one for each environmental parameter. The effect of zero drift is to impose a bias in the instrument output readings; this is normally removable by recalibration in the usual way. The input-output characteristics with zero drift are shown in Fig. 2.1(*a*).

Sensitivity drift (also called the *scale factor drift)* defines the amount by which an instrument's sensitivity of measurement varies as ambient conditions change. It is quantified by sensitivity drift coefficients which define how much drift there is for a unit change in each environmental parameter that the instrument characteristics are sensitive to. Many components within an instrument are affected by environmental fluctuations, such as temperature changes; for instance the modulus of elasticity of a spring is temperature dependent. Fig. 2.1 (*b*) shows the characteristics with sensitivity or scale factor drift. If an instrument suffers both zero drift and sensitivity drift at the same time, then the typical modification of the output characteristic is as illustrated in Fig. 2.1 (*c*).

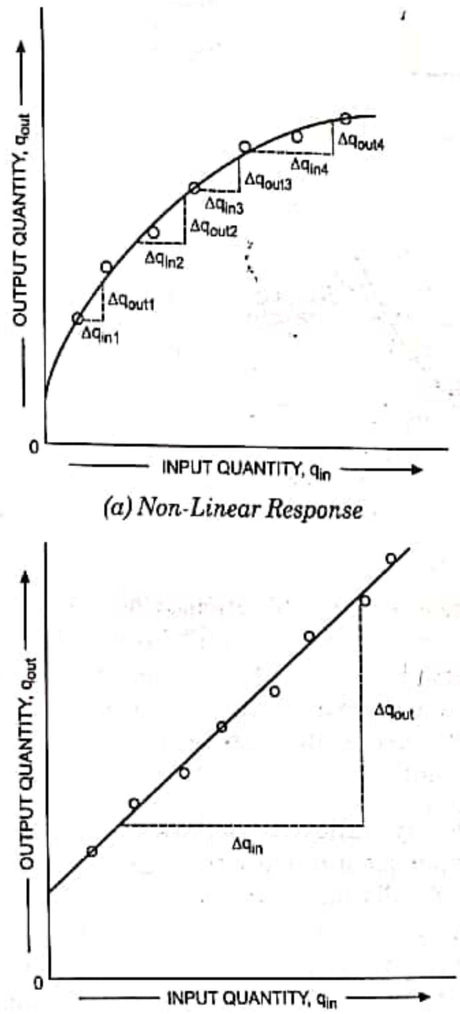
In case the drift occurs only over a portion of span of an instrument, it is called the *zonal drift.*

There are many environmental factors such as stray electric and magnetic fields, thermal emfs, mechanical vibrations, wear and tear, changes in temperature and high mechanical stresses developed in some parts of the instruments and measurement systems.

Drift is an undesirable quality because it is rarely apparent and cannot be easily compensated for. Thus it must be carefully guarded against by continuous prevention, inspection and maintenance. For instance, the effects of stray electrostatic and magnetic fields on measurement, can be avoided by proper shielding. Mechanical vibration effect can be reduced to minimum by having proper mountings. Temperature changes during measurement process should be preferably avoided or otherwise be properly compensated for.

16. *Uncertainty.* Uncertainty is expressive of the range of variation of the indicated value from the true value. It indicates the probable limits of error which the indicated value may have due to the influence of disturbing inputs. It is bipolar whereas error may be positive or negative depending on whether the indicated value is higher or lower than the true value. Statement of uncertainty signifies the quality of the measuring instrument and hence its accuracy, it is incumbent on the part of every instrumentation engineer to express the uncertainty attendant on each measured value.

17. *Static Sensitivity.* Sensitivity is defined as the ratio of the change in output signal to the change in the input quantity. It is often referred to as *incremental sensitivity* or *gain* as it relates to increments in the signals. From Fig. 2.2 (*a*) it is seen that the incremental sensitivity is different for different values of input quantity. But if the



(a) Non-Linear Response

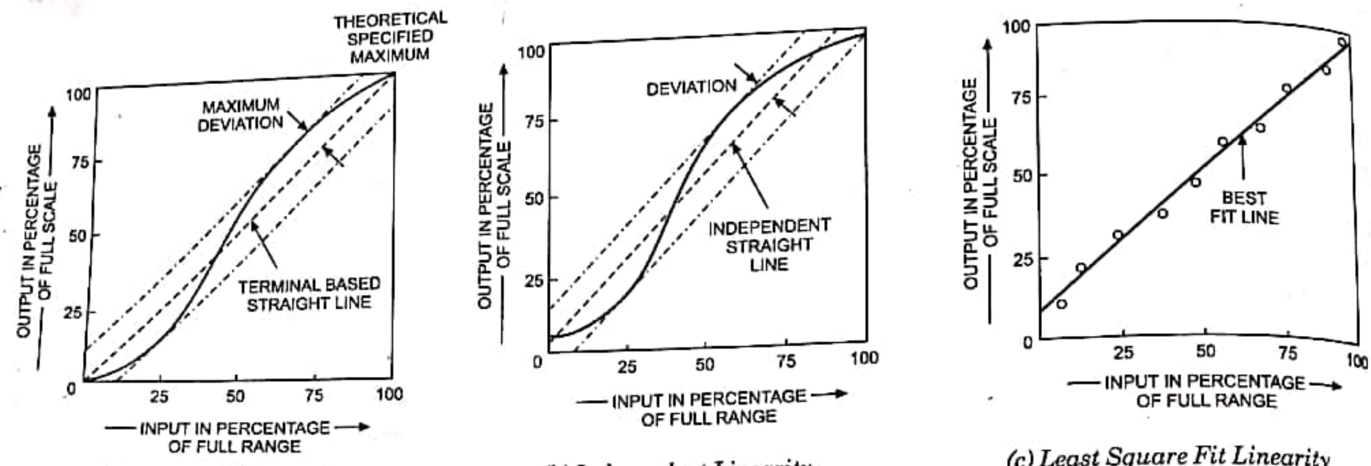
(b) Linear Response

Fig. 2.2

static calibration curve is a straight line over the entire range as depicted in Fig. 2.2 (*b*), the incremental sensitivity is constant over the entire range. In such a case, the overall sensitivity or the *static sensitivity* becomes the ratio of total change in output signal to the total change in the input quantity. For a meaningful definition of sensitivity the output quantity must be taken as the actual physical output observed, not the meaning attached to the scale numbers. For instance, the actual physical output of a voltmeter is the angular deflection of the pointer and the unit of sensitivity, therefore, will be radians per volts. Sensitivity of a measuring system should be high enough to develop a readable or detectable change in the output signal for the smallest change in the input quantity. The definition of sensitivity clearly points out that it can be increased by increasing the slope but unfortunately, this might adversely affect precision.

The reciprocal of sensitivity is called the *deflection factor* or *inverse sensitivity.*

18. *Instrument Efficiency.* It is defined as the ratio of the measured quantity and the power absorbed by the instrument at full scale. Instrument efficiency is rarely provided by the manufacturer. However, it can be determined if the instrument impedance and the full-scale voltage or current are known.



(a) *Terminal Linearity* (b) *Independent Linearity* (c) *Least Square Fit Linearity*

Fig. 2.3

In measurement applications, where the current and power are small, high efficiency instrument is a must, since otherwise the magnitude of quantity being measured will be reduced by extraction of sufficient large power from the input source by the instrument causing significant error in result.

19. *Linearity.* Linearity defines the proportionality between input quantity and output signal. If the sensitivity is constant for all values from zero to full-scale value of the measuring system, then the calibration characteristic is linear and is a straight line passing through origin. If it is an indicating or recording instrument the scale may be made linear. In case there is a zero error the characteristic assumes the form of equation given by $y = mx + c$ where y is output, x the input, m the slope, and c is the intercept.

Linearity is the closeness of the calibration curve of a measuring system to a straight line. If an instrument's calibration curve for desired input is not a straight line, the instrument may still be highly accurate. In many applications, however, linear response is most desirable. The conversion from a scale reading to the corresponding measured value of input quantity is most convenient if one merely has to multiply by a fixed constant rather than consult a nonlinear calibration curve or compute from a nonlinear calibration equation. Also, when the instrument is part of a larger data or control system, linear response of the parts often simplifies design and analysis of the whole system. Thus specifications relating to the degree of conformity to straight-line response are common.

The linearity is expressed as a percentage of the departure from the linear value, i.e. maximum deviation of the output curve from the best-fit (idealized) straight line during any calibration cycle. Absolute linearity relates to the maximum error in calibration at any point on the scale to the absolute measurement or theoretical straight line. The value is given as percentage of full scale.

The term linearity by itself means very little, and any value given is sometimes misleading. As such, the linearity is further categorised under the following classes.

Theoretical slope linearity is referred to a straight line between the theoretical end points. The line is drawn without referring to any measured values. *Terminal linearity* [Fig. 2.3 (*a*)] is a special case of theoretical slope linearity for which the theoretical end points are exactly 0% and 100% of the full-scale output.

End point linearity is referred to a straight line between the experimental end points. Such end points can be specified as those obtained during any one calibration cycle or as an average of readings during two or more consecutive calibration cycles.

Independent linearity is referred to the best straight line, a line midway between the closest possible two parallel straight lines enclosing all the output values obtained during one calibration cycle. This can be drawn only when the curve is drawn with all the output readings including the end point readings

Least square fit linearity is referred to the straight line [Fig. 2.3 (c)] for which the sum of the squares of the residuals are minimized. The residuals refer to the deviation of output readings from their corresponding points on the best-fit straight line. A relative term is the *scatter* which can be defined as the deviation of the mean value of repeated measurements from the best fit line.

20. *Instrument Hysteresis.* Hysteresis is phenomenon which depicts different output effects when loading and unloading whether it is a mechanical system or any electrical system or for that matter any system. Instrument hysteresis is the difference in the readings of an instrument, with fixed value of the input signal, which depends on whether that input value is approached from increasing or decreasing values of input. That is up-scale and down-scale deflections do not coincide when the measurement is made of the same value by method of symmetry.

The non-coincidence between the loading and unloading curves is known as *hysteresis*.

The two quantities of maximum input hysteresis and maximum output hysteresis are defined as shown in Fig. 2.4. They are normally expressed as a percentage of the full-scale input or output reading respectively i.e.

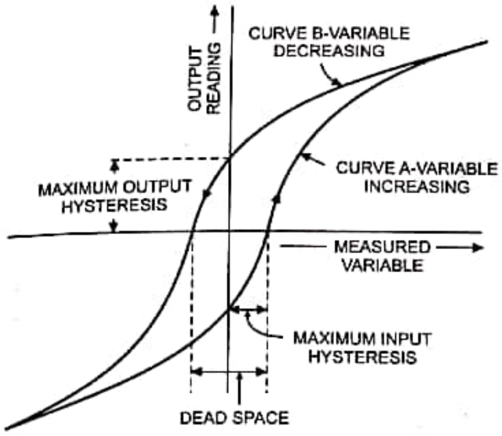


Fig. 2.4 *Instrument Characteristic With Hysteresis*

$$\text{Input hysteresis} = \frac{\text{Maximum input hysteresis}}{\text{Full-scale input}} \times 100 \quad ...(2.3)$$

$$\text{and Output hysteresis} = \frac{\text{Maximum output hysteresis}}{\text{Full-scale output}} \times 100 \quad ...(2.4)$$

Hysteresis error may be caused by backlash, friction or the characteristics of magnetic materials. Hysteresis error is reduced by proper design and selection of the mechanical components, introducing greater flexibility and providing suitable heat treatment to the materials.

21. *Dead Time.* It is defined as the time required by a measurement system to begin to respond to a change in the measurand. Thus dead time is the time before the instrument begins to respond after the measurand has been changed. In Fig. 2.5 dead time is shown by AB.

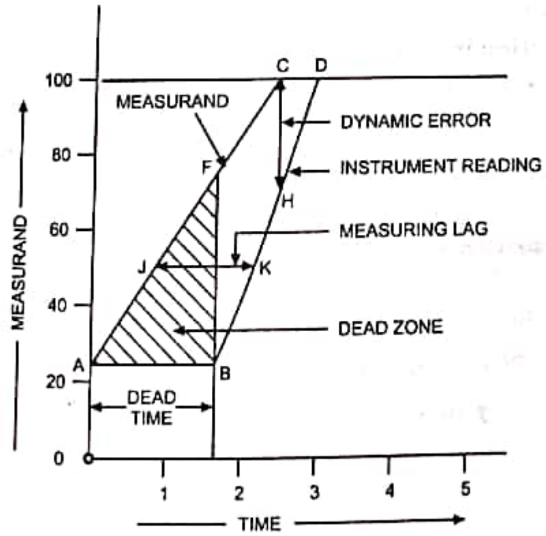


Fig. 2.5 *Dead Time and Dead Zone of The Instrument*

22. *Dead Zone.* It is the largest change of input quantity for which there is no output of the instrument. In Fig. 2.5 dead zone is shown by BFA. For instance, the input applied to the instrument may not be sufficient to overcome the friction and will, in that case not move at all.

It is due to either static friction (stiction), backlash or hysteresis. Dead zone is also known as *dead band* or *dead space*. All elastic mechanical elements used as primary transducers exhibit effects of hysteresis, creep and elastic aftereffect to some extent. Pivoted type indicating instruments and recording instruments have the drawback of small zero error due to stiction, backlash and hysteresis

23. *Friction.* *Static friction* (or *stiction*) is the force or torque that is necessary just to initiate motion from rest. *Coulomb friction* (or *dynamic friction*) is the friction force or torque which opposes motion of the output, it is independent of velocity and is normally less than the corresponding stiction. *Viscous friction* varies as a function of the velocity of a mechanism, it produces damping and affects the response of the output because it introduces lag in motion.

24. *Backlash.* The maximum distance or angle through which any part of mechanical system may be moved in one direction without applying appreciable force or motion to the next part in a mechanical sequence.

25. *Overshoot.* If measurand is applied all of a sudden to the indicating analog instrument then owing to the finite mass of pointer and the moving coil, the momentum developed by deflecting torque would cause the pointer to move beyond (or cross) the equilibrium position. This is called overshoot and is shown in Fig. 2.6. It is the maximum amount in per cent of the step magnitude by which the response exceeds the required change. In fact little overshoot is desirable to bring the pointer to rest in minimum time.

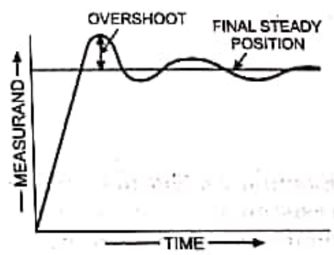


Fig. 2.6

Example 2.1. A set of independent current measurements were recorded as 10.03, 10.10, 10.11 and 10.08 A. Calculate the average current and the range of error.

[*Banaras Hindu Univ. 1989*]

Solution: Average current,

$$I_{av} = \frac{I_1 + I_2 + I_3 + I_4}{4} = \frac{10.03 + 10.10 + 10.11 + 10.08}{4}$$

$$= 10.08 \text{ A} \quad \textbf{Ans.}$$

Maximum value of current,

$$I_{max} = 10.11 \text{ A}$$

Minimum value of current,

$$I_{min} = 10.03 \text{ A}$$

$$I_{max} - I_{av} = 10.11 - 10.08 = 0.03 \text{ A}$$

$$I_{av} - I_{min} = 10.08 - 10.03 = 0.05 \text{ A}$$

Average range of error

$$= \frac{(I_{max} - I_{av}) + (I_{av} - I_{min})}{2} = \frac{0.03 + 0.05}{2}$$

$$= 0.04 \text{ A} \quad \textbf{Ans.}$$

Example 2.2. A Wheatstone bridge requires a change of 6 Ω in the unknown arm of the bridge to produce a change in deflection of 2.4 mm of the galvanometer. Calculate the static sensitivity and deflection factor.

Solution: Magnitude of output response = 2.4 mm

Magnitude of input = 6 Ω

$$\text{Static sensitivity} = \frac{\text{Magnitude of output response}}{\text{Magnitude of input}}$$

$$= \frac{2.4\text{ mm}}{6\Omega} = 0.4\text{ mm}/\Omega \text{ Ans.}$$

Deflection factor = Reciprocal of sensitivity

$$= \frac{6\Omega}{2.4\text{ mm}} = 2.5\ \Omega/\text{mm Ans.}$$

Example 2.3. A 0-5 A ammeter has a resistance of 0.01 Ω. Determine the efficiency of the instrument.

Solution: Full-scale reading of instrument, $I_f = 5.0$ A

Ammeter resistance, $R_a = 0.01\ \Omega$

Power consumption for full-scale deflection,

$$P_f = I_f^2 R_a = 5.0^2 \times 0.01 = 0.25\text{ W}$$

$$\text{Instrument efficiency, } \eta = \frac{I_f}{P_f} = \frac{5}{0.25} = 20 \text{ A per watt } \textbf{Ans.}$$

Example 2.4. A milliammeter has 100 divisions on its index scale and is provided with range multiplier switches of 1, 10 and 100. Find the range of the instrument and scale range.

Solution: Highest multiplier switch = 100 times

Range of instrument = 100 × 100 mA

= 10,000 mA = 10 A **Ans.**

Scale range = 0 – 100 **Ans.**

Example 2.5. The following resistance values of a platinum resistance thermometer were recorded at a range of temperature. Determine the measurement sensitivity of the instrument.

Temperature in °C	*Resistance in ohms*
200	305
225	310
250	315
300	325

Solution: If these values are plotted on a graph, the straight line relationship between resistance change (Δr) and temperature change (Δt) is obvious.

$$\text{Measurement sensitivity} = \frac{\Delta r}{\Delta t} = \frac{310 - 305}{225 - 200}$$

$$= 0.2\ \Omega \text{ per °C } \textbf{Ans.}$$

Example 2.6. A platinum resistance thermometer is used to measure the temperature between 0°C and 200 °C. Given that resistance at t °C as $R_t = R_0(1 + \alpha t + \beta t^2)$, $R_0 = 100.0\ \Omega$, $R_{100} = 138.50\ \Omega$ and $R_{200} = 175.83\ \Omega$, calculate the non-linearity at 100°C as a per cent of full-scale deflection. [*U.P. Technical Univ. Measurements and Instrumentation 2002-03*]

Solution:

$$\text{Length DE} = \frac{BC}{AB} \times AD = \frac{200}{175.83 - 100} \times (138.50 - 100)$$

$$= 101.543\ °C$$

i.e. point D represents a temperature of 101.543 °C corresponding to resistance of 138.50 Ω.

So deviation = 101.543 °C – 100 °C = 1.543 °C

Per cent full-scale deflection non-linearity

$$= \frac{1.543}{200} \times 100 = 0.7715\% \quad \textbf{Ans.}$$

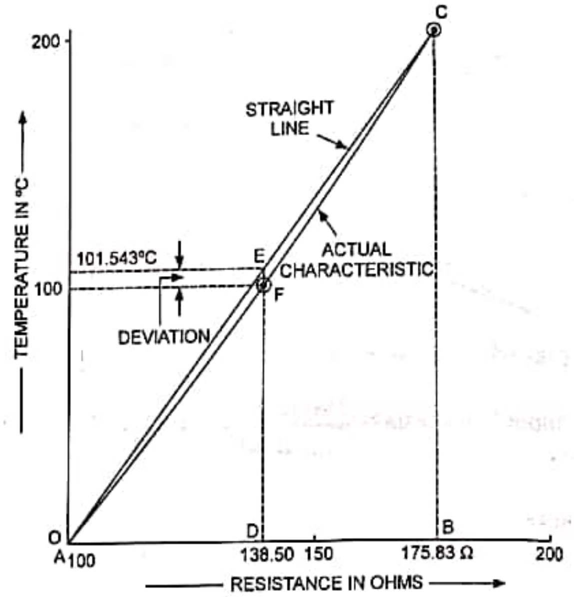


Fig. 2.7

Example. 2.7. A spring balance is calibrated in an environment at a temperature of 20°C and has the following deflection/load characteristic

Load in kg	0	1	2	3
Deflection in mm	0	10	20	30

It is then used in an environment at a temperature of 40 °C and the following deflection load characteristic is measured.

Load in kg	0	1	2	3
Deflection in mm	4	16	28	40

Determine the zero drift and sensitivity drift per °C change in ambient temperature.

Solution: At 20 °C, deflection/load characteristic is a straight line.

Sensitivity = 10 mm per kg

At 40 °C, deflection/load characteristic is a straight line.

Sensitivity = 12 mm per kg

Bias or zero drift = No-load deflection = 4 mm

Sensitivity drift = 12 – 10 = 2 mm per kg

$$\text{Zero drift/°C} = \frac{4\text{ mm}}{20°\text{C}} = 0.2\text{ mm/°C } \textbf{Ans.}$$

$$\text{Sensitivity drift/°C} = \frac{2\text{ mm}}{20°\text{C}} = 0.1\text{ mm per kg per °C } \textbf{Ans.}$$

Example 2.8. A 20 kΩ variable resistance has a linearity of 0.2% and the movement of contact arm is 300°. Find the maximum position deviation in degrees and the resistance deviation in ohms. For using this device as a potentiometer with a linear scale of 0 to 2.0 V, determine the maximum voltage error.

Solution: Per cent linearity = 0.2

Full-scale reading = 300°

Since per cent linearity

$$= \frac{\text{Maximum deviation of output from the idealised straight line}}{\text{Full-scale deflection}} \times 100$$

Maximum displacement deviation

$$= \frac{\text{Per cent linearity} \times \text{full-scale reading}}{100}$$

$$= \frac{0.2 \times 300}{100} = 0.6° \textbf{ Ans.}$$

Similarly, maximum resistance displacement

$$= \frac{0.2 \times 20 \text{ k}\Omega}{100} = 0.04 \text{ k}\Omega \text{ or } 40\ \Omega \textbf{ Ans.}$$

A displacement of 300° corresponds to 2 V and, therefore, 0.6° corresponds to a voltage of $\frac{0.6}{300} \times 2 = 4 \times 10^{-3}$ V

$\therefore$ Maximum voltage error $= 4 \times 10^{-3}$ V or 4 mV **Ans.**

Example 2.9. The dead space in a certain pyrometer is 0.12 per cent of span. The calibration is 500 °C to 1,250 °C. Determine the temperature change that might occur before it is detected.

Solution: Span = The algebraic difference between the upper and lower range values

= 1,250 – 500 = 750 °C

Dead space = 0.12% of span $= \frac{0.12}{100} \times 750 = 0.9$ °C

Thus a change of 0.9 °C must occur before it is detected.

Example 2.10. An ammeter reads 6.7 A and the true value of the current is 6.5 A. Determine the error and the correction for this instrument.

[*U.P. Technical Univ. Electronics Instrumentation and Measurements, 2010-11*]

Solution: True value of current, A = 6.5 A

Measured value of current, A_m = 6.7 A

Error of the instrument = A_m – A

= 6.7 – 6.5 = 0.2 A **Ans.**

Correction for the instrument = – Error of measurement

= – 0.2 A **Ans.**

2.3 NOISE

Noise is any kind of unwanted signal that is not related to the input. It may be defined as any signal that does not convey any useful information but is superimposed on the desired input. It may also be defined as all types of disturbances which generate unwanted input signals and may originate at any of the three stages of the instrumentation system. Thus noise may originate at the primary sensing device in a communication channel or other intermediate links. The noise may also be caused by indicating elements of the measurement system.

The common sources of noise are as follows:

1. Noise are picked up by atmospheric interferences during data transmission, by earth currents from electric appliances and machines.

2. Stray electrical and magnetic fields present in the neighbourhood of the instruments generate extraneous signals which tend to distort the original signal. The effect of these stray fields can be minimized by adequate shielding or relocation of the components of the instruments.

3. Mechanical shocks and vibrations are other sources. Their effect can be minimized by proper mounting.

4. ***Thermal* or *Johnson Noise.*** The motion of free electrons drifting around within the material constitutes a flow of many tiny random electric currents. Such currents cause minute voltage drops, which appear across the terminals of the material. The amplitude of the generated voltage increases linearly with the rise in temperature. This is because the number of free electrons available and the random motion of the electrons are increased with the increase in temperature. This undesired, randomly varying voltage is termed *thermal noise*. Thermal noise is generated within resistors.

Thermal noise is an alternating quantity and the rms value of the thermal resistance noise voltage v_n is given by expression

$$v_n = 2\sqrt{k'TBR} \quad ...(2.5)$$

where k′ is Boltzmann's constant (1.374×10^{-23} joules per Kelvin), T is absolute temperature of resistor in Kelvin, R is the resistance of the resistor in ohms and B is the circuit bandwidth in Hz over which measurements are carried out.

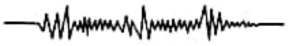


Fig. 2.8 *Thermal Noise Appearance*

The noise voltage is shown in Fig. 2.8. The varying size and shape of the noise voltage indicates that it has components of many different frequencies. The noise is uniformly distributed throughout the entire bandwidth. The noise depends on bandwidth, temperature and the resistance.

5. ***Noise From Vacuum Tubes and Transistors.*** Noise also originates from vacuum tubes and transistors often referred as *tube noise* and *transistor noise*. There are several types of noise as explained below:

Shot Noise. Among the various possible sources of noise in a tube, one of the most important is the *shot effect*. It appears that the current in a tube under dc conditions (with no input signal) is constant at every instant. Actually, however, the current from the cathode to the anode consists of a stream of individual electrons, and it is only the time average flow which is constant. Such fluctuations in the number of electrons emitted causes the shot noise. Shot noise increases with the increase in operating current and the rms value of noise current I_n in a diode is given by the expression

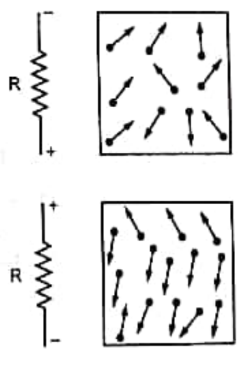


Fig. 2.9 *Thermal Agitation of Electrons Causing Varying Potential*

$$I_n = \sqrt{2eI_e B} \quad ...(2.6)$$

where *e* is electronic charge in coulomb, I_e is the emission current in amperes and B is the bandwidth in Hz.

In addition to thermal and shot noise, the other sources that cause noise in a tube are:

(i) *flicker noise* caused by the spontaneous emission of particles from an oxide-coated cathode, an effect particularly noticeable at low frequencies and decreasing with increasing value of frequencies.

(ii) *Partition noise* caused due to fluctuations in the division of charge carriers between various electrodes.

(iii) *Induced grid noise* caused due to random nature of the electron stream near the grid.

(iv) *Secondary emission noise* caused due to variations in secondary emission from plate and grid.

(v) *Gas noise* caused by the random ionization of the few molecules remaining in the tube.

In addition to thermal noise in a transistor, noise is also caused due to random motion of the charge carriers (majority as well as minority carriers) crossing the emitter and collector junctions and due to random recombination of electrons and holes in the base. There is also a partition effect developed due to random fluctuation in the division of current between collector and the base. The amount of noise produced depends upon the quiescent conditions and the source resistance R_s. Low operating currents and use of higher than normal input resistance reduce the transistor noise.

The main sources of noise in FETs are the thermal noise of the conducting channel, the shot noise caused by gate leakage current. One advantage of a FET over a BJT is that the FET usually has much lower thermal noise. This is because, unlike the bipolar transistor, there are very few charge carriers crossing a junction in the FET. This is the reason that FETs are employed near the front end of electronic equipment as the subsequent stages amplify the noise at front end along with the signal and if a FET amplifier is used at the front end, less amplified noise is obtained at the output. Thus FETs are useful for input circuits operating at low signal levels.

2.3.1. Noise Figure. A *noise figure* (NF) has been introduced in order to enable to specify quantitatively how noisy a circuit is. It is defined as the ratio of the noise power output of the circuit under consideration to the noise power output that would be obtained in the same frequency range if the only source of noise were the thermal noise in the internal resistance R_s of the signal source. Thus the noise figure is a quantity which compares the noise in an actual amplifier with that in an ideal (noiseless) amplifier. To arrive at this figure, the transistor noise output is measured under specified biased conditions and with a specified source resistance, temperature, and noise bandwidth.

If S_{pi} is the signal power input to an amplifier, N_{pi} is the noise power input due to source resistance R_s, S_{po} is the signal power output and N_{po} is the noise power output to source resistance R_s and due to other sources of noise in the device, noise figure NF is given as

$$NF = 10 \log \frac{\text{Total noise power output}}{\text{Noise power output due to } R_s}$$

$$= 10 \log \frac{N_{po}}{A_p N_{pi}}$$

where $A_p = \frac{S_{po}}{S_{pi}}$ = Power gain of amplifier

$$\text{Thus NF} = 10 \log \frac{N_{po} S_{pi}}{S_{po} N_{pi}} = 10 \log \frac{S_{pi}/N_{pi}}{S_{po}/N_{po}} \quad ...(2.7)$$

The quotient $\frac{S_p}{N_p}$ is called the signal-to-noise power ratio.

The noise figure NF is proportional to the input signal to noise-power ratio divided by the output signal to noise-power ratio.

It is advisable to keep the signal-to-noise power ratio as high as possible so as to accurately measure the wanted signal. In an amplifying system, the signal-to-noise power ratio sets an upper limit to amplification.

Example 2.11. One active and other dummy strain gauges form the opposite arms of a Wheatstone bridge. The other two arms are of equal resistances of 120 Ω each at 300 K. When a pressure of 7,000 kN/m² is applied the output voltage is 0.12 mV. Find the S/N ratio generated by the resistors. Assume bandwidth 1,000 kHz and Boltzmann constant as 1.38 × 10⁻²³ J/k.

Solution: Noise voltage,

$$v_n = 2\sqrt{k'TRB} \qquad \text{Refer to Eq. ...(2.5)}$$

$$= 2\sqrt{k'TR\Delta f}$$

$$= 2\sqrt{1.38 \times 10^{-23} \times 300 \times 120 \times 1{,}000 \times 10^3}$$

$$= 0.446\ \mu V$$

$$\text{S/N ratio} = \frac{\text{Output voltage}}{v_n} = \frac{0.12\,\text{mV}}{0.446\,\mu V} = 269 \text{ Ans.}$$

2.4 LOADING EFFECT

The ideal situation in a measurement system is that when on introducing an element, used for any purpose (may be for signal sensing, conditioning, transmission or detection), into the system, the original signal remains undisturbed i.e., introduction of any element in a measurement system should not distort the original signal in any form. However, in practical conditions it has been found that when an element is introduced in a measurement system, it extracts some energy from the system and, therefore, original signal is distorted. Such distortion may take the form of attenuation (reduction in magnitude), waveform distortion, phase shift and many a time all these undesirable features put together. Thus ideal measurement is not practicable. The incapability of the system to faithfully measure, record or control the input signal (measurand) in undistorted form is known as the *loading effect*.

A measurement system consists of three distinct stages *viz.* (*i*) detector-transducer stage, (*ii*) signal conditioning stage including original transmission stage, and (*iii*) signal presentation stage. The loading effect not only occurs in the first stage but also may occur in any of the two subsequent stages. While the first stage detector-transducer loads the input signal, the second stage loads the first stage and finally the third stage loads the second one. In fact, the loading problem may be carried right-down to the basic elements themselves.

In measurement systems, we deal both electrical and mechanical quantities and elements and so the loading effect may occur on account of both electrical and mechanical elements. The loading effects are due to impedances of various elements connected in a system.

2.4.1. Loading Effects Due To Shunt Connected Instruments. In measurement systems, voltage measuring, displaying, and recording instruments like voltmeters, oscilloscopes and strip-chart recorders are connected across the circuit i.e., in parallel (or shunt) with the circuit.

Any network can be considered equivalent to a Thevenin's voltage source E_0 in series with an output impedance Z_0, as illustrated in Fig. 2.10 (*a*). Let the load in the present case be a voltmeter of input impedance Z_L.

When the load (or any other measuring or recording device), which is a voltmeter in this case, is not connected to its terminals, the voltage across terminals AB will be equal to E_0.

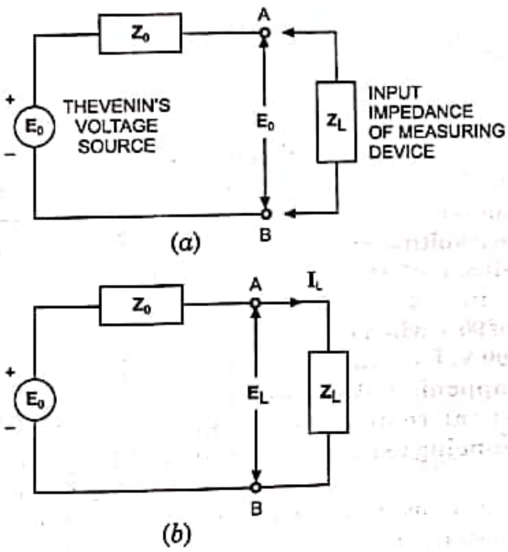


Fig. 2.10 *Voltage Source and Shunt Connected Instrument as a Load*

Ideally when the load (voltmeter in this case) is connected across the terminals A and B, the output voltage should remain the same. However, the load impedance is not infinite and, therefore, when a voltmeter with an input impedance Z_L is connected across terminals A and B, as illustrated in Fig. 2.10 (*b*), it draws some current, say I_L amperes. This causes a voltage drop of $I_L Z_0$ across the output impedance of the source. Thus output voltage under loaded condition (i.e. voltage across terminals AB when load is connected across them),

$$E_L = E_0 - I_L Z_0 = I_L Z_L$$

or $$E_0 = I_L (Z_0 + Z_L)$$

Ratio of actual voltage appearing across the load (when the instrument is connected) to the voltage under open-circuited conditions (ideal in this case) is given as

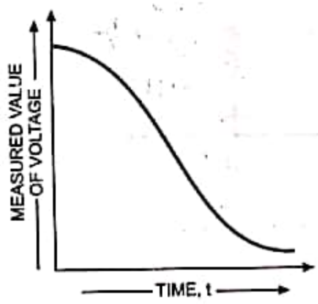
$$\frac{E_L}{E_0} = \frac{I_L Z_L}{I_L (Z_0 + Z_L)} = \frac{1}{1 + \frac{Z_0}{Z_L}} \quad ...(2.8)$$

or actual voltage measured,

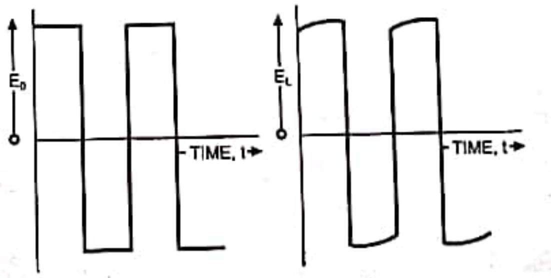
$$E_L = \frac{E_0}{1 + \frac{Z_0}{Z_L}} \quad ...(2.9)$$

Thus the voltage, which is measured, is modified both in phase and magnitude. This means that the original voltage signal is distorted due to connection of measuring device across it.

It is obvious from above Eq. (2.9) that in order to keep original signal E_0 undistorted, the value of the instrument input impedance Z_L should be infinite or the value of output impedance of the source, Z_0 should be equal to zero which cannot be attained in practice. So to have distortion, as small as possible, the input impedance of the instrument (Z_L) should be very large in comparison with source output impedance (Z_0).



(a) Effect of Frequency on Magnitude



(b) Waveform Distortion

Fig. 2.11

Since Z_0 and Z_L depend upon the frequency, the indicated voltage value will depend upon the frequency of operation. Due to input capacitance effects of the instrument, the value of input impedance Z_L becomes low at high frequencies with the result that the input signal is substantially distorted at high frequencies. Thus not only the magnitude of input signal is affected but also its phase at high frequencies. The nonsinusoidal signals are distorted in waveforms also. The magnitude of the measured signal becomes substantially small with the increase in frequency, as depicted in Fig. 2.11 (*a*).

The sharply changing nonsinusoidal waveforms are rounded off because of the finite time it takes to charge a capacitor.

2.4.2. Loading Effect Due To Series Connected Instruments. When the signal is of the form of current then series input devices are used. It is helpful to use the concept of input admittance in such cases.

Any network can be considered equivalent to Norton's constant current source I_0 in parallel with admittance Y_0, as shown in Fig. 2.12(*a*). Let the load in the present case be an ammeter of input admittance Y_{in}.

The value of current flowing between terminals A and B under ideal conditions is I_0, which is the current that flows when terminals A and B are short circuited, as illustrated in Fig. 2.12(*a*).

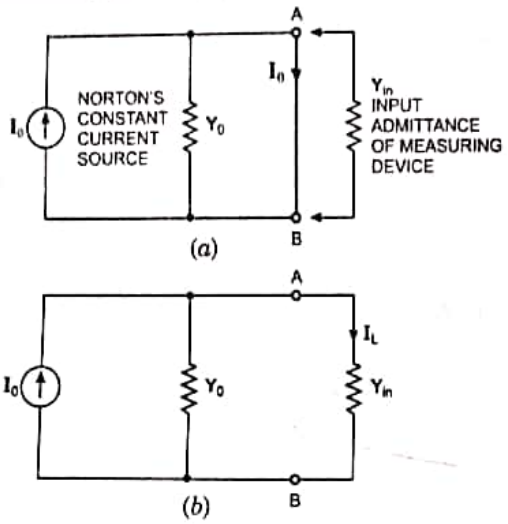


Fig. 2.12 *Current Source and Series Connected Instrument as a Load*

When we actually measure the current, an ammeter is to be connected between terminals A and B. The current flowing through the instrument is given as

$$I_L = \frac{I_0 \times Y_{in}}{Y_0 + Y_{in}} = \frac{I_0}{1 + \dfrac{Y_0}{Y_{in}}}$$

i.e. actual current measured,

$$I_L = \frac{I_0}{1 + \dfrac{Y_0}{Y_{in}}} \qquad \text{...(2.10)}$$

The above equation shows that the input admittance of the series element should be very large as compared with the output admittance of the current source so as to reduce the loading effect.

Example 2.12. A multimeter having an input resistance of 25 kΩ is used to measure the voltage across a circuit having an output resistance of 1.0 kΩ and an open-circuit voltage of 12 V. Find the error in measurement.

Solution: Measured value of voltage,

$$E_L = \frac{E_0}{1 + \dfrac{R_0}{R_L}} = \frac{12}{1 + \dfrac{1}{25}} = 11.538 \text{ V}$$

Error in measurement

= Measured value – true value

= 11.538 – 12 = – 0.462 V **Ans.**

$$\text{Percentage error} = \frac{11.538 - 12}{12} \times 100$$

= – 3.846 % or 3.846% low **Ans.**

Example 2.13. A dc circuit can be represented by an internal voltage source of 50 V with an output resistance of 100 kΩ. In order to achieve accuracy better than 99% for voltage measurement across its terminals, calculate the resistance of voltage measuring device.

[*U.P. Technical Univ. Elec. Measurements and Measuring Instruments 2005-06*]

Solution: Measured value of voltage,

$$E_m = \frac{E_o}{1 + \dfrac{R_o}{R_m}} = \frac{50}{1 + \dfrac{100 \text{ k}\Omega}{R_m}} \qquad \text{...(i)}$$

where R_m is the resistance of the voltage measuring device.

Measured value with 99% accuracy,

$$E_m = E_o \times \frac{99}{100} = 50 \times \frac{99}{100} = 49.5 \text{ V} \qquad \text{...(ii)}$$

Equating expressions (*i*) and (*ii*) we have

$$\frac{50}{1 + \dfrac{100 \text{ k}\Omega}{R_m}} = 49.5$$

$$\text{or } 1 + \frac{100 \text{ k}\Omega}{R_m} = \frac{50}{49.5}$$

$$\text{or } \frac{100 \text{ k}\Omega}{R_m} = \frac{50}{49.5} - 1 = \frac{0.5}{49.5} = \frac{1}{99}$$

or R_m = 9,900 kΩ **Ans.**

Example 2.14. What is difference between accuracy and precision of a measuring instrument? Define sensitivity of a voltmeter.

When a voltmeter is connected across either of the two 100 kΩ resistors in Fig. 2.13, it shows a reading of 90 V when it should have shown 100 V. Explain clearly why this is happening. Also calculate the internal resistance of the voltmeter being used.

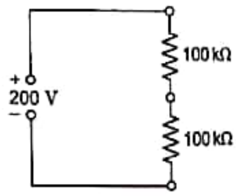


Fig. 2.13

[*U.P.S.C. I.E.S. Electronics and Communication Engineering-I, 2007*]

Solution: A voltmeter when connected across either of the two 100 kΩ resistors (say, upper one), acts as a shunt for that portion of the circuit. The voltmeter will then indicate a lower voltage drop than actually existed before the voltmeter was connected. This happens because of loading effect and mainly occurs with low sensitivity instruments.

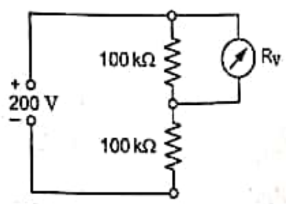


Fig. 2.14

Let the internal resistance of the voltmeter be R_V kΩ.

Equivalent resistance of the upper portion of the circuit,

$$R_{eq} = 100 \,\|\, R_V = \frac{100 \times R_V}{100 + R_V} \qquad \text{...(i)}$$

Equivalent resistance of whole circuit,

$$R_T = (R_{eq} + 100) \text{ k}\Omega$$

$$\text{Now } 90 = \frac{R_{eq}}{R_T} \times V \qquad \text{by voltage divide rule}$$

$$= \frac{R_{eq}}{100 + R_{eq}} \times 200$$

$$\text{or } R_{eq} = \frac{90 \times 100}{200 - 90} = \frac{900}{11}\ \text{k}\Omega \qquad \text{...(ii)}$$

Comparing expressions (*i*) and (*ii*) we have

$$\frac{100 \times R_V}{100 + R_V} = \frac{900}{11}$$

$$\text{or } 11\,R_V - 9\,R_V = 900$$

$$\text{or } R_V = \frac{900}{2} = 450\ \text{k}\Omega \quad \textbf{Ans.}$$

Example 2.15. A voltmeter having a sensitivity of 15 kΩ/V reads 80 V on a 100 V scale, when connected across an unknown resistor. The current through the resistor is 2 mA. Calculate the % of error due to loading effect.

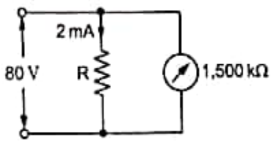


Fig. 2.15

[*J.N. Technological Univ. Hyderabad, May-2011*]

Solution: Resistance of voltmeter,

R_V = Sensitivity of voltmeter in kΩ/V × range of voltmeter

$= 15 \times 100 = 1{,}500\ \text{k}\Omega$

Voltage across unknown resistor,

V = Reading of voltmeter connected across the circuit

$= 80$ V

True value of unknown resistance,

$$R_T = \frac{\text{Voltage across resistor}}{\text{Current through resistor}}$$

$$\frac{80}{2 \times 10^{-3}} = 40{,}000\ \Omega \text{ or } 40\ \text{k}\Omega$$

Apparent value of unknown resistance,

R_m = Equivalent resistance of the circuit

$= 40\ \text{k}\Omega \,||\, 1{,}500\ \text{k}\Omega$

$$= \frac{40 \times 1{,}500}{40 + 1{,}500} = 38.961\ \text{k}\Omega$$

% error due to loading effect,

$$\varepsilon_r = \frac{R_m - R_T}{R_T} \times 100$$

$$= \frac{38.961 - 40}{40} \times 100 = -2.56\% \quad \textbf{Ans.}$$

Example 2.16. Explain briefly about sensitivity and loading effect of a voltmeter. The voltage across the 50 kΩ resistor in the circuit shown below in Fig. 2.16 are measured with two voltmeters separately. Voltmeter 1 has a sensitivity of 1,000 Ω/V. Voltmeter 2 has a sensitivity of 20,000 Ω/V. Both the meters are used on their 50 V Range. Calculate (i) the reading of each meter and (ii) the error in each reading, expressed as a percentage of true value.

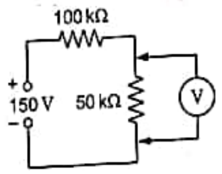


Fig. 2.16

[*U.P.S.C. I.E.S. Electrical Engineering-I, 2010*]

Solution: Resistance of voltmeter 1,

$$R_{V_1} = s_1 V = 1{,}000 \times 50\ \Omega = 50\ \text{k}\Omega$$

Resistance of voltmeter 2,

$$R_{V_2} = s_2 V = 20{,}000 \times 50\ \Omega = 1{,}000\ \text{k}\Omega$$

True value of voltage across 50 kΩ resistor,

$$V_T = \frac{50}{100 + 50} \times 150 = 50\ \text{V} \quad \text{by voltage division rule}$$

Equivalent resistance of parallel combination of 50 kΩ resistance and voltmeter 1,

$$R_{eq_1} = \frac{50 \times 50}{50 + 50} = 25\ \text{k}\Omega$$

Equivalent resistance of parallel combination of 50 kΩ resistance and voltmeter 2,

$$R_{eq_2} = \frac{50 \times 1{,}000}{50 + 1{,}000} = 47.62\ \text{k}\Omega$$

(*i*) Reading of voltmeter 1,

$$V_1 = \frac{R_{eq_1}}{R_{eq_1} + 100} \times 150 \qquad \text{by voltage division rule}$$

$$= \frac{25}{25 + 100} \times 150 = 30\ \text{V} \quad \textbf{Ans.}$$

Similarly reading of voltmeter 2,

$$V_2 = \frac{47.62}{47.62 + 100} \times 150 = 48.39\ \text{V} \quad \textbf{Ans.}$$

(*ii*) Percentage error in reading of voltmeter 1

$$= \frac{V_1 - V_T}{V_T} \times 100 = \frac{30 - 50}{50} \times 100 = -40\% \quad \textbf{Ans.}$$

(*ii*) Percentage error in reading of voltmeter 2,

$$= \frac{V_2 - V_T}{V_T} \times 100 = \frac{48.39 - 50}{50} \times 100 = -3.22\% \ \textbf{Ans.}$$

Thus, we note that loading effect is reduced with the use of high sensitivity instrument.

Example 2.17. It is required to measure the voltage across 10 kΩ resistor of the circuit shown in figure below (Fig. 2.17). Two voltmeters are available. Voltmeter A is a 1 mA movement and voltmeter B is 50 μA movement. Both use their 50 V scales. Calculate (i) the reading of each voltmeter (ii) the error from true reading.

[*U.P.S.C. I.E.S. Electrical Engineering-I, 2013*]

Solution: Resistance of voltmeter 1,

$$R_{V_1} = \frac{50}{1 \times 10^{-3}}\ \Omega = 50\ \text{k}\Omega$$

Resistance of voltmeter 2,

$$R_{V_2} = \frac{50}{50 \times 10^{-6}}\ \Omega = 1{,}000\ \text{k}\Omega$$

Equivalent resistance of parallel combination of 10 kΩ resistance and voltmeter V_1,

$$R_{eq_1} = \frac{10 \times 50}{10 + 50} = \frac{50}{6}\ \text{k}\Omega$$

Equivalent resistance of parallel combination of 10 kΩ resistance and voltmeter V_2,

$$R_{eq_2} = \frac{10 \times 1{,}000}{10 + 1{,}000}$$

$$= \frac{1{,}000}{101} \text{ k}\Omega$$

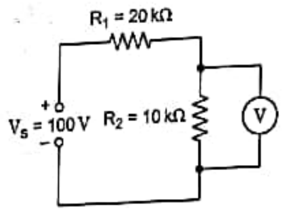


Fig. 2.17

Reading of voltmeter 1,

$$V_1 = \frac{R_{eq_1}}{R_{eq_1} + 20} \times V_s \quad \text{by voltage division rule}$$

$$= \frac{\frac{50}{6}}{\frac{50}{6} + 20} \times 100 = 29.412 \text{ V} \quad \textbf{Ans.}$$

Reading of voltmeter 2,

$$V_2 = \frac{\frac{1{,}000}{101}}{\frac{1{,}000}{101} + 20} \times 100 = 33.113 \text{ V} \quad \textbf{Ans.}$$

True value of voltage across 10 kΩ resistor,

$$V_T = \frac{V_s \times R_2}{R_1 + R_2} = \frac{100 \times 10}{20 + 10} = 33.333 \text{ V}$$

Error in reading of voltmeter 1

$$= \frac{V_1 - V_T}{V_T} \times 100 = \frac{29.412 - 33.333}{33.333} \times 100$$

$$= -11.763\% \quad \textbf{Ans.}$$

Error in reading of voltmeter 2

$$= \frac{V_2 - V_T}{V_T} \times 100 = \frac{33.113 - 33.333}{33.333} \times 100$$

$$= -0.66\% \quad \textbf{Ans.}$$

The above results indicate that the voltmeter of higher sensitivity provides more reliable results.

Example 2.18. It is desired to measure the value of current in 500 Ω resistor as shown in Fig. 2.18 by connecting a 100 Ω ammeter. Find (i) actual value of current (ii) measured value of current (iii) the percentage error in measurement and accuracy. *[R.G.P.V., June-2007]*

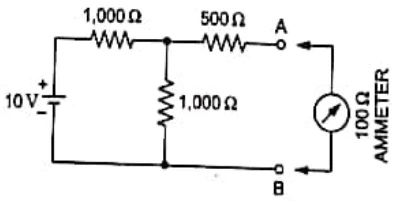


Fig. 2.18

Solution: Let us reduce the actual circuit to an equivalent Thevenin's source.

$$\text{Open-circuit voltage, } E_o = 10 - \frac{10 \times 1{,}000}{1{,}000 + 1{,}000} = 5 \text{ V}$$

Output impedance of source as looking back into terminals A and B,

$$R_{out} = \frac{1{,}000 \times 1{,}000}{1{,}000 + 1{,}000} + 500 = 1{,}000 \ \Omega$$

The Thevenin's equivalent circuit is shown in Fig. 2.19 (*a*)

(*i*) So actual value of current flowing through 500 Ω resistor

$$= I_o = \frac{5}{1{,}000} \text{ A} = 5 \text{ mA} \quad \textbf{Ans.}$$

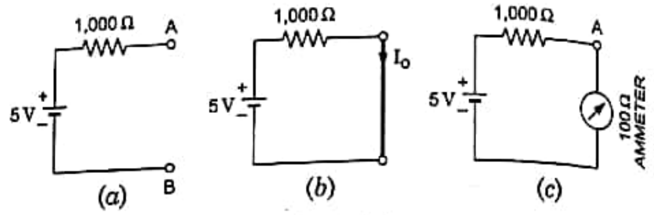


Fig. 2.19

(*ii*) When the ammeter is connected to terminals A and B, as shown in Fig. 2.19 (*c*), current through ammeter i.e. measured value of current,

$$I_L = \frac{E_o}{R_{out} + R_{Ammeter}} = \frac{5}{1{,}000 + 100} \text{ A} = 4.5455 \text{ mA}$$

(*iii*) Percentage error in measurement

$$= \frac{I_L - I_o}{I_o} \times 100 = \frac{4.5455 - 5}{5} \times 100$$

$$= -9.09\% \quad \textbf{Ans.}$$

Accuracy of measurement

$$= 100 - |\text{percentage error}|$$

$$= 100 - 9.09 = 90.91\% \quad \textbf{Ans.}$$

2.5 MAXIMUM POWER TRANSFER AND IMPEDANCE MATCHING

In measurement system many a times we are concerned with the problem of maximum power transfer from the source to the input device. Maximum power is transferred from one network to another one connected to it at two terminals, when the impedance of one is the complex conjugate of the impedance of the other. This means that for maximum power transfer from source to load, the load resistance should be equal to the source resistance and the load reactance should be equal to the source reactance in magnitude but opposite in sign, i.e. if the source is inductive the load should be capacitive and vice versa.

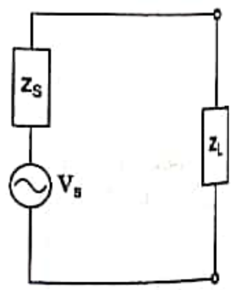


Fig. 2.20

Consider an ac source with an internal impedance $Z_s = (R_s + jX_s)$ and voltage V_s supplying to a load with impedance $Z_L = (R_L + jX_L)$

Total impedance,

$$Z = Z_s + Z_L = (R_s + R_L) + j(X_s + X_L)$$

Current delivered to load,

$$I_L = \frac{V_s}{Z} = \frac{V_s}{(R_s + R_L) + j(X_s + X_L)}$$

The magnitude of load current,

$$I_L = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

Power delivered to load,

$$P_L = I_L^2 R_L = \frac{V_s^2}{(R_s + R_L)^2 + (X_s + X_L)^2} \times R_L$$

An inspection of above equation shows that as far as variation in the value of X_L is concerned, the power delivered to load P_L will be maximum when $X_s + X_L = 0$ or $X_L = -X_s$ and power delivered will be in this condition,

$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2} \quad ...(2.11)$$

As far as variation of R_L is concerned, the power transfer will be maximum when $\frac{dP_L}{dR_L}$ is zero.

Differentiating the Eq. (2.11) with respect to R_L and equating it to zero, we have

$$\frac{dP_L}{dR_L} = \frac{V_s^2 (R_s + R_L)^2 - 2R_L (R_s + R_L) V_s^2}{(R_s + R_L)^4} = 0$$

or $(R_s + R_L)^2 - 2 R_L (R_s + R_L) = 0$

or $R_s^2 + R_L^2 + 2R_sR_L - 2R_L^2 - 2 R_sR_L = 0$

or $R_s^2 - R_L^2 = 0$

or $R_s = R_L \quad ...(2.12)$

It means that maximum power can be transferred from a source to a load if the internal resistance of the source or the output resistance of the preceding stage of an instrumentation system is equal to the load resistance or the input resistance of the succeeding source.

Substituting $R_s = R_L$ in Eq. (2.11) we have maximum power transferred and is given as

$$P_{Lmax} = \frac{V_s^2 R_L}{(R_L + R_L)^2} = \frac{V_s^2}{4R_L} \quad ...(2.13)$$

In practice, the network or generator (source) may not be capable of supplying the maximum possible power to the conjugate impedance as specified above without overheating of the source. Thus it is not always physically possible to make use of the ideal conjugate load, a higher impedance load is required to avoid burnout or damage.

In many applications it is desirable to match the impedance of input device to the output impedance of the signal source instead of making the impedance of the input device either too low or too high.

Typical cases of impedance matching are those involving applications of waveform generators like pulse generators and radio-frequency (RF) generators which utilise a transmission line to audio amplifiers feeding loud speakers and other electromechanical transducers.

The condition for impedance matching is not critical. Figure 2.21 depicts the relative amount of power transferred from one system to another for different ratios of R_L/R_s. For a 10% deviation from the correct value of impedance matching (i.e. $\frac{R_L}{R_s} = 0.9$ or 1.1), the power transfer is practically 100%. For a 20 per cent change, the power transfer reduces to 99%. Even for a 100% change (ratio $\frac{R_L}{R_s} = 0.5$ or 2), the power transferred is 89% of the maximum allowable power.

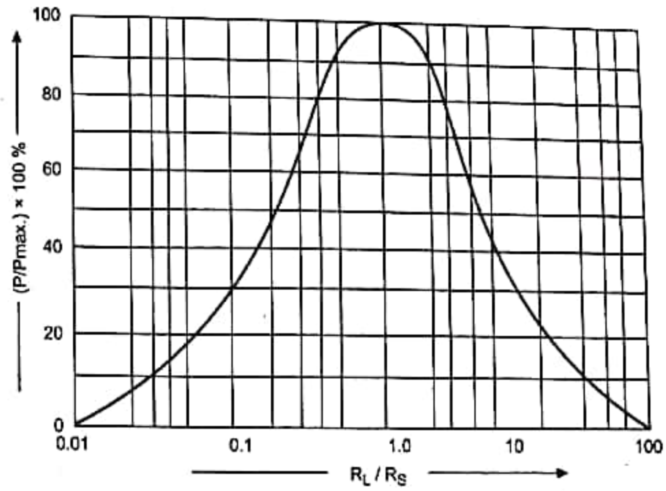


Fig. 2.21 *Maximum Power Transfer*

Example 2.19. Measurement on a human nerve cell indicates an open-circuit voltage of 80 mV, and a current of 5 nA through a 6 MΩ load. What is the maximum power available from the cell?

Solution: Load current, $I_L = 5 \text{ nA} = 5 \times 10^{-9}$ A

Open-circuit voltage, $E_0 = 80 \text{ mV} = 0.08$ V

Load resistance, $R_L = 6 \text{ M}\Omega = 6 \times 10^6\ \Omega$

It R_0 is the output resistance of the cell, then

$$R_0 = \frac{E_0}{I_L} - R_L = \frac{0.08}{5 \times 10^{-9}} - 6 \times 10^6$$

$$= 10 \times 10^6\ \Omega = 10 \text{ M}\Omega$$

Maximum power available from cell,

$$P_{max} = \frac{E_0^2}{4R_0} = \frac{(0.08)^2}{4 \times 10 \times 10^6}$$

$$= 160 \times 10^{-12} \text{ W} = 0.16 \text{ nW } \textbf{Ans.}$$

Example 2.20. The output voltage of an audio amplifier is 12 V and 4 V when delivering a power of 28.8 W and 16 W respectively. Determine load resistances in the two cases, amplifier's output voltage and output resistance. Determine also the magnitude of maximum power available from the amplifier.

Solution: Voltage across load,

$V_{L_1} = 12$ V and $V_{L_2} = 4$ V

Power delivered to load,

$P_1 = 28.8$ W and $P_2 = 16$ W

Load resistance in the first case,

$$R_{L_1} = \frac{V_{L_1}^2}{P_1} = \frac{12^2}{28.8} = 5\ \Omega \textbf{ Ans.}$$

Load resistance in the second case,

$$R_{L_2} = \frac{V_{L_2}^2}{P_2} = \frac{4^2}{16} = 1\ \Omega \textbf{ Ans.}$$

Voltage across load is given as

$$V_L = \frac{E_0 \times R_L}{R_0 + R_L}$$

So, $12 = \frac{E_0 \times 5}{R_0 + 5}$ ...(*i*) in the first case

and $4 = \frac{E_0 \times 1}{R_0 + 1}$ ...(*ii*) in the second case

Solving Eqs. (*i*) and (*ii*), we have

Open-circuit voltage of amplifier, E_0 = 24 V **Ans.**

Output resistance of amplifier, R_0 = 5 Ω **Ans.**

Maximum power available from the amplifier,

$$P_{max} = \frac{E_0^2}{4R_0} = \frac{24^2}{4 \times 5} = 28.8 \text{ W Ans.}$$

2.6 DYNAMIC CHARACTERISTICS OF MEASUREMENT SYSTEMS

On application of an input to an instrument or measuring system, it cannot attain its final steady-state position instantaneously. The fact is that the measurement system passes through a *transient state* before it reaches its final steady-state position. Some measurements are carried out in such conditions that allow sufficient time for the instrument or measurement system to settle to its final steady-state position. Under such circumstances the study of behaviour of the system under transient state, known as *transient response*, is not of much importance; only *steady-state response* is to be considered. On the other hand, in many measurement systems it becomes imperative to study the system response under both transient as well as steady-state conditions. In many cases the transient response of the system is more important than its steady-state response. As we know that the instruments and measurement systems do not respond to the input immediately due to the presence of energy storage elements (such as electrical inductance and capacitance, mass fluid and thermal capacitance etc.) in the system. The system exhibits a characteristic sluggishness due to the presence of these elements.

In measurement systems having inputs dynamic in nature, the input varies from instant to instant, so does the output. The behaviour of the system under such conditions is dealt by the *dynamic response* of the system, and its characteristics are given below in brief.

1. *Dynamic Error*. It is the difference of true value of the quantity changing with time and the value indicated by the instrument provided static error is zero. In Fig. 2.5, it is indicated by CH. Total dynamic error is the phase difference between input and output of the measurement system.

2. *Fidelity*. It is the ability of the system to reproduce the output in the same form as the input. In the definition of fidelity any time lag or phase difference is not included. Ideally a system should have 100% fidelity and the output should appear in the same form as the input and there is no distortion produced by the system.

Fidelity needs are different for different applications. Moving iron type instruments providing rms readings are required to have the same sensitivity for dc signals and ac signals of frequencies over a small range around 50 Hz. Waveform recorders and cathode-ray oscillographs are required to have excellent fidelity with no amplitude or phase distortion for signals of frequencies over a wide range, but not necessarily extending to zero. In all such cases where accurate measurement is the objective, the amplitude-frequency characteristic should remain flat within ± 2% with no appreciable phase error.

While specifying the performance of electronic amplifiers and such other electronic equipment meant for entertainment, fidelity specification is relaxed. A greater degree of distortion is tolerated in performance because the human sensory systems cannot detect the distortion at the output end. Hence fidelity is claimed for a wider range of frequencies even though they are quoted as 'hi-fi' (high fidelity) systems. The dynamic sensitivity at zero frequency (or a mid frequency) is allowed to fall down to 70.7% over the frequency range of its performance.

3. *Bandwidth*. Bandwidth of a system is the range of frequencies for which its dynamic sensitivity is satisfactory. For measuring systems, the dynamic sensitivity is required to be within 2% of its static sensitivity. Thus the amplitude-frequency characteristic is almost flat as depicted in Fig. 2.22 right up to a certain frequency from dc, and this range of frequencies is specified as the bandwidth of the measuring system.

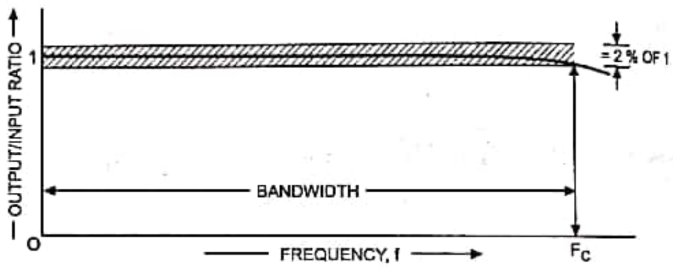


Fig. 2.22 *Amplitude-Frequency Characteristic of a Typical System*

For other physical systems, electrical filters and electronic amplifiers, the above criterion is relaxed with the result that their bandwidth specification extend to frequencies at which the dynamic sensitivity is 70.7% of that at zero or the mid frequency.

4. *Speed of response*. Speed of response of a physical system refers to its ability to respond to sudden changes of amplitude of input signal.

It is usually specified as the time taken by the system to come close to steady-state conditions, for a step input function. Hence the speed of response is evaluated from the knowledge of the system performance under transient conditions and terms such as time constant, measurement lag, settling time and dead time dynamic range are used to convey the response of the variety of systems, encountered in practice.

5. *Time Constant*. It is associated with the behaviour of a first-order system and is defined as the time taken by the system to reach 0.632 times its final output signal amplitude. A system having smaller time constant attains its final output amplitude earlier than the one with larger time constant and therefore, has higher speed of response. It is related to the system parameters.

6. ***Measuring Lag.*** It is defined as the delay in the response of an instrument to a change in the measurand. This lag is usually quite small but it becomes quite significant where high speed measurements are required. In Fig. 2.5, JK is the measuring lag.

Measurement lag is of two types. In *retardation type*, the response of the instrument begins immediately after a change in the measurand has occurred. In *time delay type*, the response of the system begins after a *delay time* after the application of the input, as illustrated in Fig. 2.23.

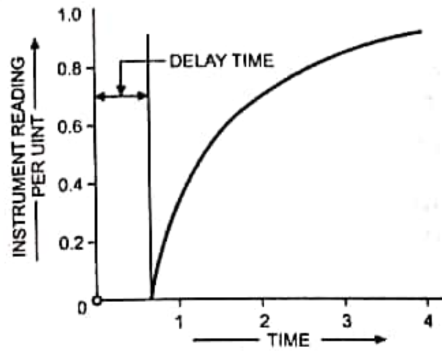


Fig. 2.23 *Delay Time in Instruments*

7. ***Settling or Response Time.*** It is the time required by the instrument or measurement system to settle down to its final steady-state position after the application of the input. For portable instruments, it is the time taken by the pointer to come to rest within ± 0.3% of its final scale length while for panel type instruments, it is the time taken by the pointer to come to rest within ± 1% of its final scale length.

A smaller settling time indicates higher speed of response. Settling time is also dependent on the system parameters and varies with the condition under which the system operates. The settling time of second-order instruments is affected by the degree of damping provided for the instrument. The effect of damping on the settling time is shown in Fig. 2.24.

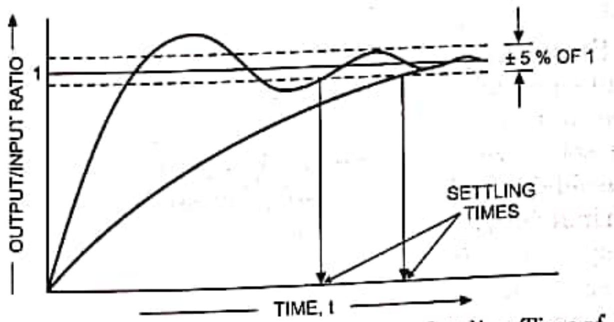


Fig. 2.24 *Effect of Damping on the Settling Time of Indicating Instruments*

Dynamic Range. Dynamic range is the range of signals which the measuring system is likely to respond faithfully under dynamic conditions. This is generally expressed as the ratio of the amplitudes of the largest (maximum) signal to the smallest (minimum) signal to which the system is subjected and the system can handle satisfactorily. The ratio is usually expressed in dB, A dynamic range of 40 dB indicates that the measuring system can handle a range of input signals of amplitudes of 100 to 1.

2.7 STANDARD SIGNALS

Dynamic response of a measuring system, when subjected to dynamic inputs which are functions of time, depends very much on its own parameters, apart from the nature and complexity of the function. Thus the dynamic response of the measuring system may be considered to consist of two components: one due to its own characteristic parameters and the other due to the nature of the input function.

Though the likelihood of measuring systems being subjected to a variety of measurands of varying description exists, it is customary to specify the performance of measuring systems for some standard inputs. These inputs, individually or together, are believed to be representative of the commonly known simple process conditions. Besides, mathematical representations of these inputs or signals are adequately known. The most common input signals are shown in Fig. 2.25.

(a) Step Input *(b) Ramp Input* *(c) Sinusoidal Input*

Fig. 2.25 *Standard Signals*

Step input function with the amplitude, given a step change,

$$f(t) = A \text{ for } t \geq 0 \qquad \text{...(2.14)}$$
$$= 0 \text{ for } t < 0$$

Ramp input function with the amplitude changing linearly with time.

$$f(t) = kt \text{ for } t > 0 \qquad \text{...(2.15)}$$

Sinusoidal input function where the amplitude varies sinusoidally with time,

$$f(t) = A \sin 2\pi ft, \text{ for } t > 0 \qquad \text{...(2.16)}$$

where f is the frequency in Hz.

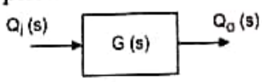
Even the above specific functions are very difficult to stimulate for all types of instruments. However, mathematical modelling of the instruments can be made and the measurement system with the prescribed input can be analytically studied for deriving the *dynamic* (or the transient) characteristics.

2.8 DYNAMICS OF INSTRUMENT SYSTEMS

An instrument system is invariably a combination of some physical elements connected together to accomplish a desired objective. A variety of mechanical and electrical components are used in making up the transducer and instrumentation systems for indication and recording of physical variables. It is important to realize the performance of these systems under static and dynamic conditions from the knowledge of the behaviour of each component of the system.

For purpose of analysis certain assumptions are usually made concerning the behaviour of each element, and a simpler mathematical model is developed. The physical laws governing the behaviour of each element are utilized in developing model of the physical system. The development of such models usually results in the formation of standard differential equations with constant coefficients and hence these equations are considered as linear models. The composite physical system is then characterized as linear time — invariant and is represented by means of a relationship between its input (excitation) and output (response) variables. For the common simple transducer and instrument systems, it is customary to relate its output variable with a single input variable and study its performance when excited by input functions, treated as standard input signals.

2.8.1. Transfer Function Representation. The transfer function approach is usually adopted for the study of physical systems and the transfer function of a linear time-invariant system is defined as the ratio of the Laplace transform of the output quantity (variable) to the Laplace transform of the input quantity (variable), under the assumption that the initial values of both variables are zero and the system wakes up to respond to the excitation function from the instant of its application. The transfer function (TF) is denoted by G(s) and is written in the form



$$G(s) = \frac{Q_o(s)}{Q_i(s)} \qquad ...(2.17)$$

Fig. 2.26 *Transfer Function Representation*

where $Q_o(s)$ and $Q_i(s)$ represent the output (response) and input (excitation) variables respectively. G(s) is also defined as the system function. The highest power of s (= the complex frequency) in the denominator of the transfer function determines the order of the physical system. Most of the transducer and instrument systems belong to either first or second order and the evaluation of their response in time domain for standard input signals is very much desired for classifying their response as satisfactory or unsatisfactory.

The transfer function is used to estimate the transient and steady-state response of the system when subjected to input signals of varying descriptions. However, the response of the instrument systems for step input and sinusoidal input functions is what is mostly desired. Performance criteria are usually specified for the system response for both the static and dynamic inputs and any lack of conformity with the specifications is revealed by actual testing of the systems. Those systems that may be found to make up the resonance or found to belong to third order and above, have to be dynamically compensated for satisfactory performance as a stable system, or are limited for use on such input functions having a frequency content far removed from the region, leading to undesirable operation.

2.9 GENERALIZED PERFORMANCE OF SYSTEMS

The most widely used mathematical model for the dynamic response studies is the ordinary linear differential equation with constant coefficients. The relationship can be represented in general as

$$a_n \frac{d^n q_o(t)}{dt^n} + a_{n-1} \frac{d^{n-1} q_o(t)}{dt^{n-1}} + ... + a_1 \frac{dq_o(t)}{dt} + a_0 q_o(t)$$

$$= b_m \frac{d^m q_i(t)}{dt^n} + b_{m-1} \frac{d^{m-1} q_i(t)}{dt^{n-1}} + ... + b_1 \frac{dq_i(t)}{dt} + b_0 q_i(t) \quad ...(2.18)$$

where $q_o(t)$ is the output quantity (variable) and $q_i(t)$ is the input quantity and a's and b's are constants representing the physical parameters of the system. Both $q_o(t)$ and $q_i(t)$ are functions of time. The value of n defines the order of the system. The above representation relates $q_o(t)$ and $q_i(t)$ in time domain. The solution of equations of this type can be obtained by either the method of **D** *operators* or the *Laplace transform method*. The latter method will be followed here.

The transfer function is defined as the ratio of the Laplace transform of the output quantity to the Laplace transform of the input quantity, when all initial conditions are zero.

The operational transfer function of Eq. (2.18) can be written as

$$G(s) = \frac{Q_o(s)}{Q_i(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + ... + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + ... + a_1 s + a_0} \quad ...(2.19)$$

where $Q_o(s)$ is the Laplace transform of the output, $Q_i(s)$ is the Laplace transform of the input, and s is the Laplace operator.

The time response $q_o(t)$ can be obtained by taking inverse Laplace transform of the right-hand side of equation

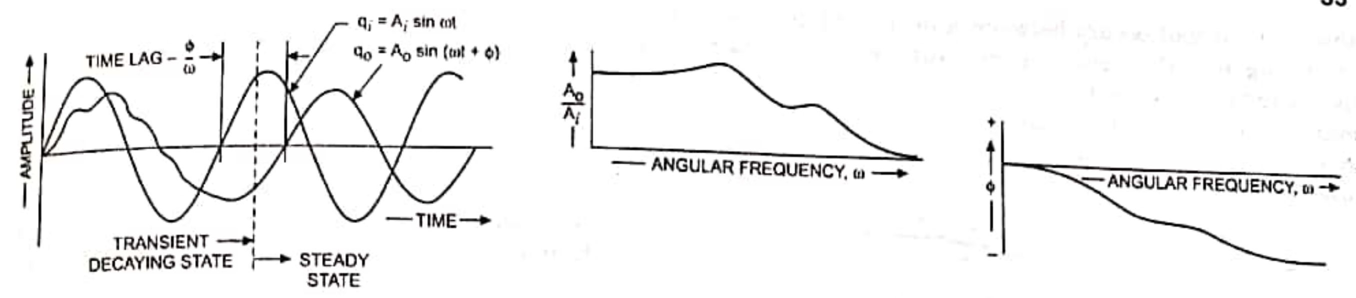
$$Q_o(s) = G(s)\, Q_i(s) \quad ...(2.20)$$

The time response $q_o(t)$ has two components. One component decays with time to zero and is independent of the type of input. It only depends on the dynamics of the system. This component is called the *transient response* of the system. Second component is the response which exists after the transient response is died out. This component is called the *steady-static response* of the system.

Sinusoidal Transfer Function. Of the various inputs prescribed, the sinusoidal input is specially important as any periodic input can be written in the form of a series representing fundamental and harmonic sinusoidal waveforms. The solution can be performed for each individual frequency input and superposed as long as the system is linear. Thus, the *frequency response* of a system, defined as that consisting in finding amplitudes and phase shifts as functions of frequency is quite important. This, however, is easily found if in transfer function, the complex variable of Laplace transform is replaced by $j\omega$ (by writing $j\omega$ for s), as shown below:

$$\frac{Q_o(j\omega)}{Q_i(j\omega)} = \frac{b_m (j\omega)^m + b_{m-1}(j\omega)^{m-1} + ... b_1(j\omega) + b_0}{a_n (j\omega)^n + a_{n-1}(j\omega)^{n-1} + ... a_1(j\omega) + a_0} \quad ...(2.21)$$

For any given frequency ω, the term $\frac{Q_o(j\omega)}{Q_i(j\omega)}$ is a complex number. This can also be represented in the polar form as



(a) Input Output Waveforms *(b) Amplitude Ratio* *(c) Phase Shift*

Fig. 2.27 *Sinusoidal Response Characteristics*

$$\frac{Q_o(j\omega)}{Q_i(j\omega)} = M \angle \phi \quad ...(2.22)$$

where M represents the amplitude ratio of the output to the input, and the angle ϕ represents the phase angle by which the output Q_o leads the input Q_i.

The frequency response of the system will consist of curves sharing amplitude ratio and phase shift as a function of frequency, often represented as in Fig. 2.27.

2.10 ZERO-ORDER SYSTEM

The zero-order system has no dynamic error and no time lag of measurement. It is represented by

$$a_o q_o = b_o q_i$$

$$\text{or } q_o = \frac{b_o}{a_o} q_i = K q_i \quad ...(2.23)$$

where K is a proportionality constant and may be identified with the static sensitivity defined earlier.

From Eq. (2.23) it is clear that, no matter how q_i might vary with time, the system output q_o follows it perfectly with no distortion or time lag of any sort. *Thus, the zero-order system represents ideal or perfect dynamic performance.*

A practical example of a zero-order system is the displacement measuring potentiometer, as illustrated in Fig. 2.28.

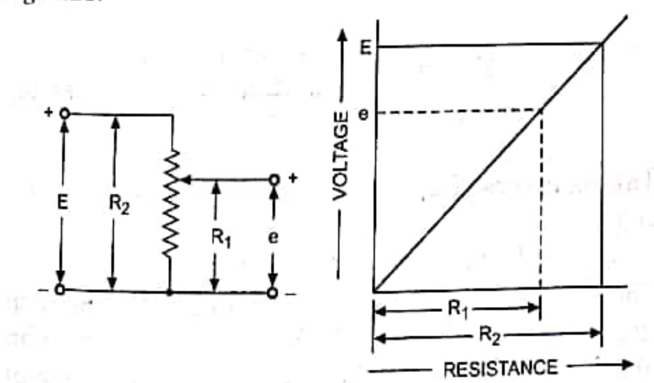


Fig. 2.28 *Potentiometric Device Representing Zero-Order System*

When the potentiometer of resistance R_2 is excited with the voltage E, the output voltage e is given as

$$e = \frac{R_1}{R_2} E = KE \quad ...(2.24)$$

where R_1 is the part of potentiometer, as depicted in Fig. 2.28.

Here it is assumed that the resistance value R_2 is linearly distributed along the length. If the measuring device is examined more critically, it may be found that the potentiometer is not exactly a zero-order system, because the potentiometer resistance element will consist of some inductance and capacitance, depending upon the winding features. Besides this, the impedance of the measuring device offers a loading effect on the circuit. Further, if R_1 is varied fast, the dynamic errors become dominant owing to parasitic inductance and capacitance effects. The reasons why a potentiometer is normally called a zero-order instrument are as follows:

1. The parasitic inductance and capacitance can be made very small by design.
2. The speeds (frequencies) of motion to be measured are not high enough to make the inductive or capacitive effects noticeable.

2.11 FIRST-ORDER SYSTEM

If in Eq. (2.18) all coefficients a's, b's other than a_1, a_o and b_o are taken to be zero, we have

$$a_1 \frac{dq_o}{dt} + a_o q_o = b_o q_i \quad ...(2.25)$$

Any system following above equation is termed as a first-order system. Eq. (2.25) may be rewritten as

$$\frac{a_1}{a_o}\frac{dq_o}{dt} + q_o = \frac{b_o}{a_o} q_i$$

By substituting $\frac{b_o}{a_o}$ = K (static sensitivity) and $\frac{a_1}{a_o}$ = τ (time constant) and by taking the Laplace transform with zero initial conditions, the transfer function of the first-order system becomes

$$\frac{Q_o(s)}{Q_i(s)} = \frac{K}{1 + \tau s} \quad ...(2.26)$$

The thermocouple is a good example of a first-order instrument. It is well known that, if a thermocouple at room temperature is plunged into boiling water, the output emf does not rise instantaneously to a level indicating 100 °C, but instead approaches a reading indicating 100 °C in a manner similar to that shown in Fig. 2.29.

A large number of other instruments also belong to this first-order class. This is of particular importance in control systems where it is necessary to take account of the time lag that occurs between a measured quantity changing in value and the measuring instrument indicating the change. Fortunately, the time constant of many first-order instruments is small relative to the dynamics of the process being measured, and so no serious problems are caused.

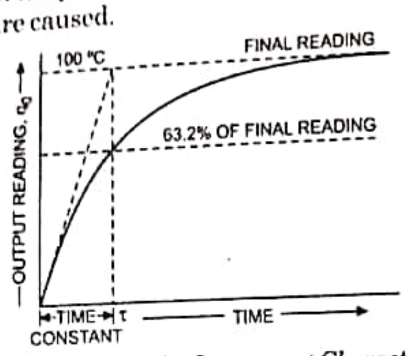


Fig. 2.29 *First-Order Instrument Characteristic*

2.11.1. Step Response of First-Order System. Consider a first-order system which is initially in equilibrium. On the application of a step input, let the input quantity be increased instantaneously by an amount Q_s. Then Eq. (2.26) becomes

$$(1+\tau s)\,Q_o = \frac{KQ_s}{s} \qquad ...(2.27)$$

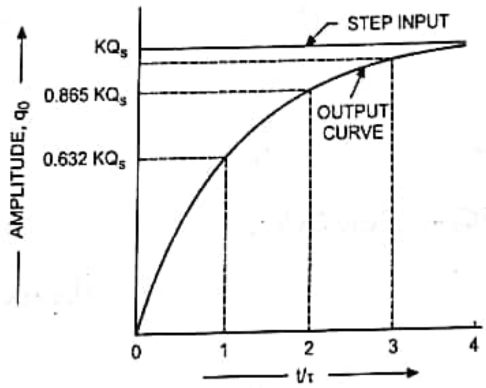


Fig. 2.30 *Step Response of a First-Order System*

A typical example of this case is a thermocouple in air suddenly immersed in boiling water, resulting in a step input change in junction temperature.

The step input can be represented mathematically by the relationship in time domain as,

$$Q_i = 0 \qquad \text{for } t < 0$$
$$\text{and } Q_i = Q_s \qquad \text{for } t \geq 0$$

In Laplace transform notation,

$$Q_i(s) = \frac{Q_s}{s}$$

Hence Eq. (2.27) becomes,

$$Q_o(s) = \frac{K}{(1+\tau s)}\cdot\frac{Q_s}{s} = KQ_s\left[\frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}\right]$$

Taking inverse Laplace transform, we have

$$q_o(t) = KQ_s\,(1 - e^{-t/\tau}) \qquad ...(2.28)$$

Above Eq. (2.28) reveals the fact that $q_o(t)$ assumes a final value $K\,Q_s$ slowly with time. The fastness with which the system responds or in other words, the speed of response, is dependent on the value of time constant τ. The smaller the value of τ, the higher the speed of response. Thus, for a first-order system, minimizing the magnitude of time constant τ helps in improving the dynamic response.

The normalized value $q_o(t)/KQ_s$ is plotted against t/τ a response of the form depicted in Fig. 2.30 is obtained. One of the important characteristics of such an exponential response is that at $t = \tau$, $q_o = 0.632\ KQ_s$. It means that in first-order system when $t = \tau$, the response reaches 63.2% of its steady-state limit.

2.11.2. Ramp Response of First-Order System. Consider the case when a ramp input is applied to a first-order system which is initially in equilibrium. Let the input change with a slope m with respect to time t, then,

$$Q_i(s) = \pounds\,(mt) = \frac{m}{s^2} \qquad ...(2.29)$$

Substituting $Q_i(s) = \frac{m}{s^2}$ from Eq. (2.29) in Eq. (2.26) we have

$$Q_o(s) = Q_i(s)\cdot\frac{K}{1+\tau s}$$

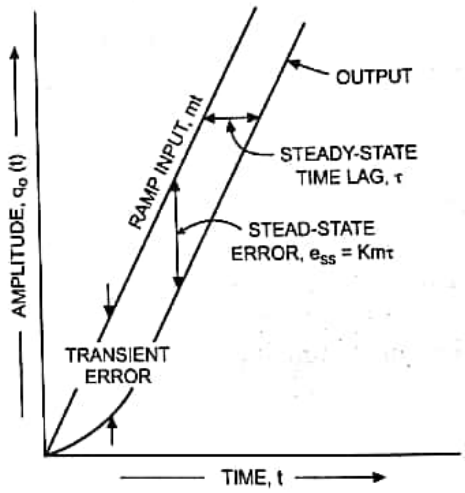


Fig. 2.31 *Response of First Order System to a Ramp Input*

$$= \frac{K}{1+\tau s}\cdot\frac{m}{s^2} = Km\left[\frac{1}{s^2} + \frac{\tau}{s+\frac{1}{\tau}} - \frac{\tau}{s}\right] \qquad ...(2.30)$$

Taking inverse Laplace transform of above equation we have

$$q_o(t) = Km\,(t - \tau + \tau e^{-t/\tau}) \text{ for } t \geq 0 \qquad ...(2.31)$$

The response of the system for ramp input is shown in Fig. 2.31. If the system is ideal, it should result in an output signal $q_o(t) = Kmt$, but there is a deviation from this value due to its time constant. Hence the dynamic error is given by

$$e_d(t) = Kmt - q_o(t) = Km\tau\,(1 - e^{-t/\tau}) \qquad ...(2.32)$$

The second term of the net dynamic error i.e. $Km\tau e^{-t/\tau}$ dies with time and hence it constitutes transient error, whereas the first term $Km\tau$ becomes the steady-state error. Under steady-state conditions, the amplitude of output attains the true value after τ second only i.e. steady-state time lag is τ seconds only.

2.11.3. Impulse Response of First-Order System. An impulse function is a rectangular pulse of infinitesimally short duration, infinitely high magnitude, and a finite area. If the area of the pulse is taken as unity, the value is termed as *unit impulse function u(t)*. Such an input plays an important role in the dynamic analysis of a system.

The Laplace transform of an impulse function is the area under the impulse. Hence for a unit-impulse input the Laplace transform of the input is unity

$$\text{i.e. } Q_i(s) = 1.0 \qquad \text{...(2.33)}$$

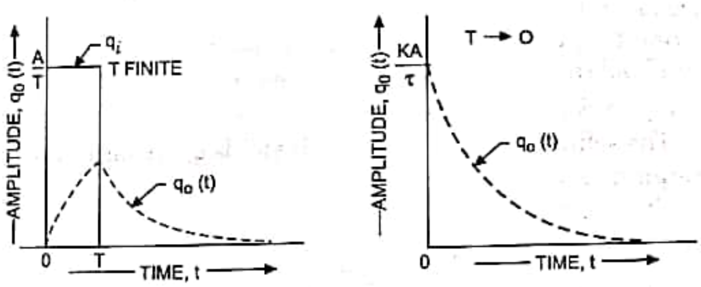
Substituting $Q_i(s) = 1.0$ from Eq. (2.33) in Eq. (2.26), we have

$$Q_o(s) = Q_i(s) \cdot \frac{K}{1+\tau s} = \frac{K}{1+\tau s} \qquad \text{...(2.34)}$$

Taking inverse Laplace transform of above equation, we have

$$q_o(t) = \frac{K}{\tau} e^{-t/\tau} \qquad \text{...(2.35)}$$

If the strength of the impulse is A units, the response becomes A times the one given by Eq. (2.35). The response depicted in Fig. 2.32 (*a*) is for an impulse of strength of A units but of duration T seconds and that in Fig. 2.32 (*b*) is for an ideal pulse.



(*a*) *For a Prolonged Impulse Input* (*b*) *For an Ideal Impulse Input*

Fig. 2.32 *Response of First-Order System*

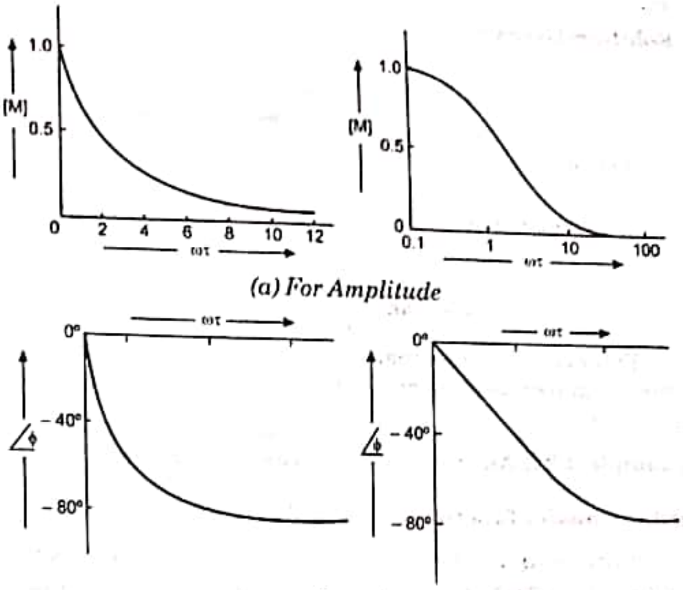
2.11.4. Frequency Response of First-Order System. For sinusoidal input functions, the frequency response is determined from the relation

$$\frac{Q_o(j\omega)}{Q_i(j\omega)} = \frac{K}{1+j\omega\tau} = \frac{K}{\sqrt{1+\omega^2\tau^2}} \angle \tan^{-1}(\omega t)$$

$$= M \angle \phi \qquad \text{...(2.36)}$$

The above equation is obtained by substituting $s = j\omega$ in Eq. (2.30) for a sinusoidal input $q_i = A_i \sin(\omega t)$.

At zero frequency i.e., under dc excitation, the value of M becomes equal to K with $\phi = 0$. Treating the natural frequency of the system, ω_n, as given by $\frac{1}{\tau}$, the frequency response curves relating M and $\angle\phi$ with $\frac{\omega}{\omega_n} = \omega\tau$ are shown in Fig. 2.33.



(*b*) *For Phase*

Fig. 2.33 *Characteristics Frequency Response of a First-Order System*

A first-order system approaches the ideal condition when time constant τ tends to zero. For any other value of τ, the measurements will be accurate below a particular value of ω only.

For a pure sinusoidal input, the amplitude and phase shift can be determined by simple calculations. If the input is complex i.e. a combination of several sinusoidal waves of different frequencies, the transfer function at each frequency is to be determined and then interpreted.

Example 2.21. In a permanent magnet moving coil spring-controlled voltmeter, the pointer moves through an angle of 60° when voltage under measurement is changed by 40 mV. Determine the instrument sensitivity.

Solution: Change in input quantity, $\Delta q_i(t) = 40$ mV

Change in output quantity, $\Delta q_o(t) = 60°$

∵ In a spring-controlled PMMC instrument the relation between the deflection and voltage is linear,

$$\text{Sensitivity, } S_i = \frac{\Delta q_o(t)}{\Delta q_i(t)} = \frac{60}{40} = 1.5° \text{ per mV } \textbf{Ans.}$$

Example 2.22. Calculate the time lag of an instrument with step input on a 95% response basis.

Solution: From Eq. (2.28)

$$\frac{q_o(t)}{KQ_s} = (1 - e^{-t/\tau}) = 0.95$$

Let the time lag be T_L seconds. Then

$$e^{-T_L/\tau} = 1 - 0.95 = 0.05$$

$$\text{or } T_L = \tau \log_e \frac{1}{0.05} = 2.996\,\tau \text{ seconds } \textbf{Ans.}$$

Example 2.23. Calculate the dynamic error and lag in a first-order instrument with a time constant τ of 10 millisecond, when it is required to measure an input $x = 10 \sin 100\, t$. Assume an unit conversion factor.

Solution: Dynamic error.

$$e_d(t) = KQ_s\left[1 - \frac{1}{\sqrt{1+\omega^2\tau^2}}\right]$$

For $\omega\tau = 1$

$$\text{Dynamic error} = 10\left[1 - \frac{1}{\sqrt{1+1}}\right] = 2.93 \text{ units Ans.}$$

$$\text{and Time lag, } T_L = \text{Tan}^{-1}\frac{\omega\tau}{\omega} = \frac{\text{Tan}^{-1}1}{100} = 7.854 \text{ ms Ans.}$$

This example shows that the error, for $\omega\tau = 1$, is 30%. For a meaningful measurement, i.e., for the error to lie within 10%, $\omega t \le 0.48$.

Example 2.24. An instrument is represented by a first-order transfer function $G(s) = \frac{1}{1+\tau s}$.

If the response to a unit step function attains 0.632 times of its final value in 2.5 seconds, determine the time constant τ of the instrument.

Solution: By definition of time constant—in a first order system the response attains 0.632 times of its final value when $t = \tau$. Hence time constant is 2.5 seconds **Ans.**

Example 2.25. Determine the steady-state error in a first-order system when excited by a unit ramp input. The dynamic error is 1.58 for t = time constant τ of the system.

Solution: The dynamic error in a first-order system excited by unit ramp input is given by equation

$$e_d = K\tau(1 - e^{-t/\tau})$$

where τ is a time constant of the system

Substituting $e_d = 1.58$ and $t = \tau$ in above equation, we have

$$1.58 = K\tau(1 - e^{-t/\tau}) = 0.632\, K\tau$$

$$\text{or } K\tau = \frac{1.58}{0.632} = 2.5 \text{ seconds}$$

Steady-state error, $e_{ss} = K\tau = 2.5$ seconds **Ans.**

2.12 SECOND-ORDER SYSTEM

If in Eq. (2.18) all coefficients a's, b's other than a_2, a_1, a_0 and b_0 are taken to be zero, we have

$$a_2\frac{d^2q_o}{dt^2} + a_1\frac{dq_o}{dt} + a_oq_o = b_oq_i \qquad ...(2.37)$$

Taking the Laplace transform of the above equation, we have

$$G(s) = \frac{Q_o(s)}{Q_i(s)} = \frac{b_o}{a_2s^2 + a_1s + a_0}$$

The above equation can be rewritten as

$$G(s) = \frac{K}{s^2\left(\frac{a_2}{a_o}\right) + s\left(\frac{a_1}{a_o}\right) + 1} \qquad ...(2.38)$$

where K is static sensitivity and is equal to b_o/a_o. The undamped natural frequency ω_n of the second-order system becomes $\sqrt{a_o/a_2}$ and the ratio a_1/a_o signifies the damping conditions of the system. Representing the ratio of the damping coefficient under any damping condition to the coefficient that would result in critical damping conditions, by a dimensionless factor δ, commonly known as *damping factor* (or damping ratio), it can be shown that

$$\delta = \frac{a_1}{2\sqrt{a_oa_2}} \qquad ...(2.39)$$

$$\text{and } \frac{2\delta}{\omega_n} = \frac{a_1}{a_o} \qquad ...(2.40)$$

Equation (2.38) can be rewritten as

$$\frac{Q_o(s)}{Q_i(s)} = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\delta s}{\omega_n} + 1} \qquad ...(2.41)$$

2.12.1. Step Response of Second-Order System. For a step input of magnitude Q_s Eq. (2.41)

$$\frac{Q_o(s)}{KQ_s} = \frac{\omega_n^2}{s(s^2 + 2\delta s\omega_n + \omega_n^2)} \qquad ...(2.42)$$

$$\left[\because \text{ the Laplace transform of step input is } \frac{1}{s}\right]$$

While solving above equation, three practical situations must be identified, depending upon the magnitude of δ. The roots of the second term in the denominator of the above equation become real and different if the damping ratio exceeds unity (i.e. overdamped system); critically damped system (i.e. when δ = unity) gives rise to real and equal roots, and the underdamped system (i.e., when $\delta < 1$) gives a complex conjugate pair of roots.

The solution of above Eq. (2.42) yields $q_o(t)$ for different damping conditions as follows:

(*i*) For overdamped system i.e. when $\delta > 1$

$$\frac{q_o(t)}{KQ_s} = -\frac{\delta + \sqrt{\delta^2-1}}{2\sqrt{\delta^2-1}}\exp\left(-\delta + \sqrt{\delta^2-1}\right)\omega_nt + \frac{\delta - \sqrt{\delta^2-1}}{2\sqrt{\delta^2-1}}\exp\left(-\delta - \sqrt{\delta^2-1}\right)\omega_nt + 1$$

$$= 1 - \frac{\exp(-\omega_n\delta t)}{\sqrt{\delta^2-1}} \times \sinh\left[\omega_n\sqrt{(\delta^2-1)}\,t + \sinh^{-1}\sqrt{\delta^2-1}\right] \qquad ...(2.43)$$

(*ii*) For critically damped systems i.e. when $\delta = 1$

$$\frac{q_o(t)}{KQ_s} = 1 - (1 + \omega_nt)e^{-\omega_nt} \qquad ...(2.44)$$

(*iii*) For underdamped system i.e. when $\delta < 1$

$$\frac{q_o(t)}{KQ_s} = 1 - \frac{e^{-\delta\omega_nt}}{1-\delta^2}\sin\left(\sqrt{1-\delta^2}\,\omega_nt + \phi\right) \qquad ...(2.45)$$

$$\text{where } \phi = \text{Sin}^{-1}\sqrt{1-\delta^2}$$

The output responses of a second-order system for different values of δ are shown in Fig. 2.34. For case A where $\delta = 0$, there is no damping and the instrument output exhibits constant amplitude oscillations when disturbed by any change in the physical quantity measured. For light damping of $\delta = 0.2$, represented by case B, the response to a step change in input is still oscillatory but the oscillations gradually die down. A further increase in value of δ reduces oscillations and overshoot still more, as shown by curves C and D, and finally the response becomes very overdamped as shown by curve E where output reading creeps up slowly towards the true reading. Obviously the extreme response curves A and E are grossly unsuitable for any measuring system/instrument.

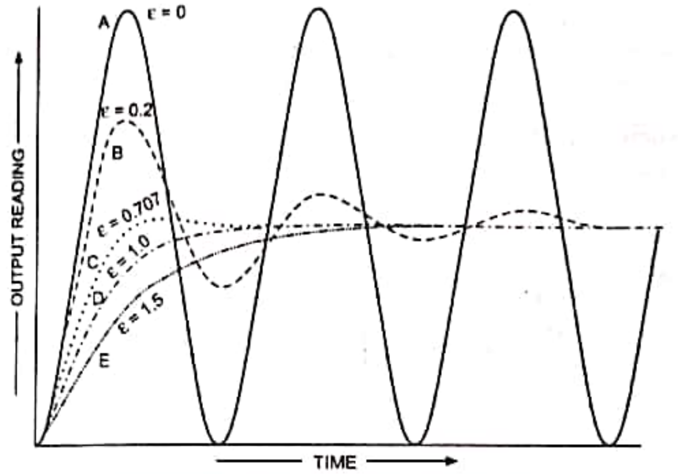


Fig. 2.34 *Responses of Second-Order System To a Step Input*

Physical systems that are likely to be subjected to step input functions are thus required to be critically damped and all the second-order type indicating instruments are designed to work under damping factors near unity. When working at δ of 0.7 or 0.8 the settling time may become larger than that of critical damping, but the oscillatory response, with $q_o(t)/KQ_s$ overshooting the final value, ensures positive response of the system without being stuck anywhere in its movement. At a damping factor of 0.68, the indicated value will be within ± 5% of the final value within the time of about $0.28/\omega_n$.

The first peak value of $q_o(t)/KQ_s$ for slightly under-damped conditions is obtained from Eq. (2.45) and is used to determine the first overshoot.

Differentiating Eq. (2.45) w.r.t. time and equating it to zero to obtain the time corresponding to maximum overshoot of the response from the final value, we have

$$\frac{dq_o(t)}{KQ_s dt} = \frac{-\delta\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin(\omega t + \phi) + \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \omega_n \sqrt{1-\delta^2} \cos(\omega t + \phi) = 0 \qquad t \geq 0 \qquad \text{...(2.46)}$$

where $\omega = \omega_n \sqrt{1-\delta^2}$

$$\phi = \sin^{-1}\sqrt{1-\delta^2} = \cos^{-1}(-\delta) \qquad \text{...(2.47)}$$

From Eqs. (2.46) and (2.47), we have

$$\frac{\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin \omega_n \sqrt{1-\delta^2}\, t = 0 \qquad t \geq 0$$

The above equation provides two values of t i.e.

$$t = \infty$$

and $\omega_n \sqrt{1-\delta^2}\, t = n\pi$ where $n = 0, 1, 2, ...$

Since the first peak of the output response occurs at $n = 1$, hence

$$t_{max} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \qquad \text{...(2.48)}$$

For this value of time the maximum value of response obtained from Eq. (2.45) is

$$\frac{q_o(t_{max})}{KQ_s} = 1 + e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}$$

Hence maximum overshoot,

$$M_p = \frac{q_o(t_{max})}{KQ_s} - 1 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \qquad \text{...(2.49)}$$

M_p is usually expressed as a percentage of final value. Experimental determination of the percentage overshoot enables the estimation of the damping ratio of the system at which it is working.

2.12.2. Ramp Response of Second-Order System. Just as in the case of a first-order system, the response of a second-order system to a ramp of slope m with respect to time can be obtained. The solution in this case will be (*a*) for overdamped case ($\delta > 1$)

$$\frac{q_o(t)}{Kq_i} = \left[\left(Kmt - \frac{2\delta}{\omega_n}\right) + \frac{2\delta}{\omega_n} e^{-\delta\omega_n t} \left(\cosh \omega_n t\sqrt{\delta^2-1} + \frac{2\delta^2-1}{2\delta\sqrt{\delta^2-1}} \sinh \omega_n t\sqrt{\delta^2-1}\right)\right] \qquad \text{...(2.50)}$$

(*b*) For critically damped case ($\delta = 1$)

$$\frac{q_o(t)}{Kq_i} = \left[\left(Kmt - \frac{2\delta}{\omega_n}\right) + \frac{2\delta}{\omega_n} \cdot e^{-\omega_n t}\left(1 + \frac{\omega_n t}{2}\right)\right] \qquad \text{...(2.51)}$$

(*c*) For an underdamped case ($\delta < 1$)

$$\frac{q_o(t)}{Kq_i} = \left[\left(Kmt - \frac{2\delta}{\omega_n}\right) - \frac{e^{-\delta\omega_n t}}{\omega_n\sqrt{(1-\delta^2)}} \sin\left(\omega_n t\sqrt{1-\delta^2} + \phi\right)\right] \qquad \text{...(2.52)}$$

$$\text{where } \phi = \text{Tan}^{-1} \frac{2\delta\sqrt{1-\delta^2}}{2\delta^2 - 1} \quad ...(2.53)$$

The response of a second-order system to a unit ramp input is shown in Fig. 2.35.

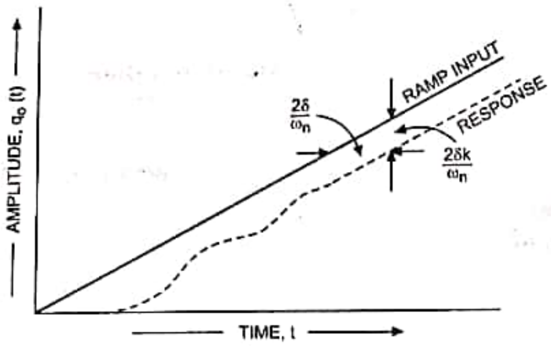


Fig. 2.35 *Response of Second-Order System To a Unit Ramp Input*

2.12.3. Frequency Response of Second-Order System. The frequency response of the second-order system is, as already seen, obtained from its transfer function and is given by

$$\frac{Q_o(j\omega)}{Q_i(j\omega)} = \frac{K}{-\left(\frac{\omega}{\omega_n}\right)^2 + \frac{2\delta j\omega}{\omega_n} + 1} \quad ...(2.54)$$

Writing $\frac{\omega}{\omega_n} = \eta$, the ratio of the frequency of the forcing function to its natural frequency, the response is given by

$$\frac{Q_o(j\omega)}{Q_i(j\omega)} = \frac{K}{\sqrt{\left[\left(1-\eta^2\right)^2 + 4\delta^2\eta^2\right]}} \angle\phi \quad ...(2.55)$$

$$\text{where } \phi = \text{Tan}^{-1} \frac{2\delta\eta}{\sqrt{1-\eta^2}} = M\angle\phi \quad ...(2.56)$$

The frequency response characteristics of a second-order system are shown in Fig. 2.36.

2.12.4. Impulse Response of Second-Order System. It is equally important to understand how second-order systems respond to impulse input functions. The impulse response in time domain reflects its natural behaviour and is used to relate the impulse response with the impulse strength, while making measurements of signals that can be treated as impulses or pulses of such duration as to activate the system for response after the pulse dies. It is essential to assume that the system is provided with certain energy within the duration of the pulse T, and its response begins only after the pulse ceases to exist.

Thus it is necessary to evaluate the initial conditions such as q_o (t) and $\dot{q}_o$ (t) at $t = 0^+$ so as to be incorporated in the solution obtained for its natural behaviour. As the impulse is no more present, the system attains its final value which is the same as that with which it starts. The basic Eq. (2.18) may be integrated to give

$$a_2 \frac{dq_o(t)}{dt} + a_1 q_o(t) + a_0 \int_0^T q_o(t)\, dt = b_o \int_0^T q_i(t)\, dt \quad ...(2.57)$$

As the pulse duration is very short, it is assumed that q_o (t) at t = T is zero and that the system has an initial rate of change given by $\frac{dq_o(t)}{dt}$. On application of these conditions, Eq. (2.57) gives the initial velocity as

$$\left.\frac{dq_o(t)}{dt}\right|_{t=T} = \frac{b_o}{a_2} \int_0^T q_i(t)\, dt = KA\omega_n^2 \quad ...(2.58)$$

where A is strength of the input pulse.

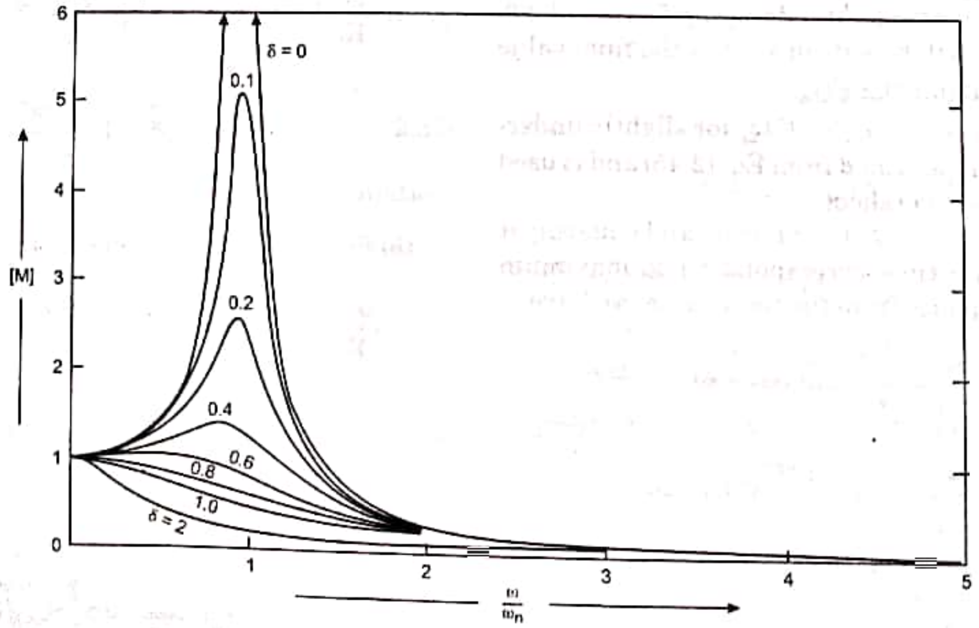
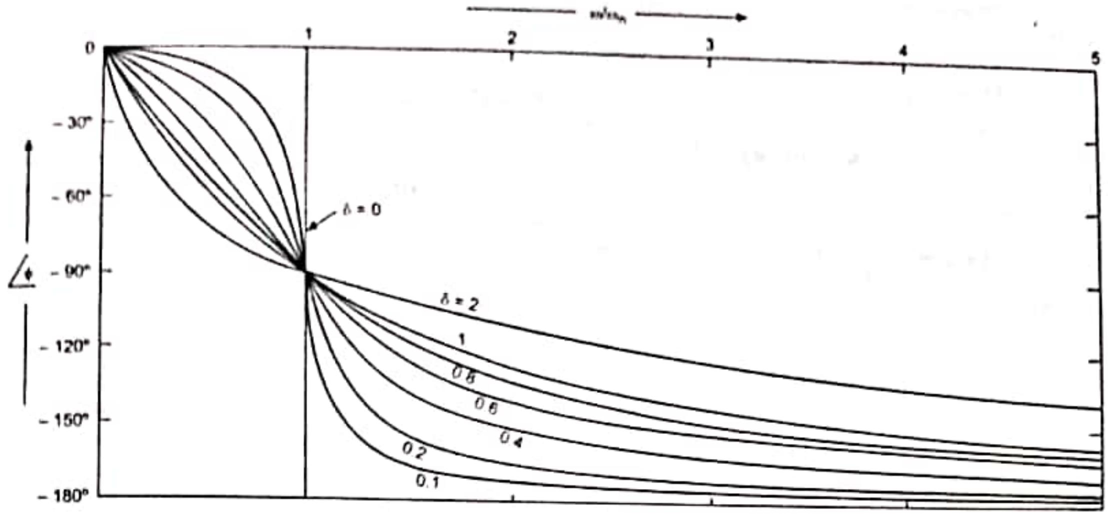


Fig. 2.36 *(a) For Amplitude*



(b) For Phase

Fig. 2.36 *Frequency Response Characteristics of a Second-Order System.*

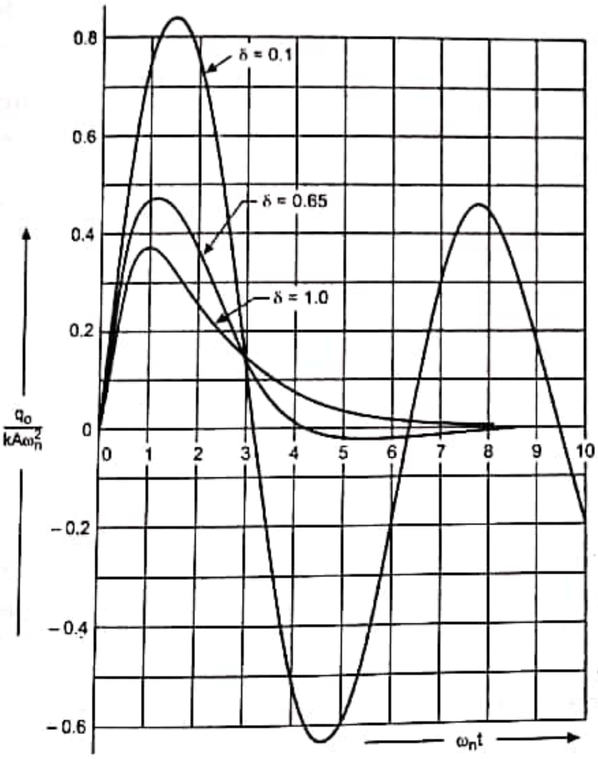


Fig. 2.37 *Non-Dimensional Impulse Response of Second-Order System*

The response of the system for unit impulse input is given by

$$Q_o(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\delta s}{\omega_n} + 1}$$

From which the value of $q_o(t)$ is obtained in a usual way as

$$\frac{q_o(t)}{KA\omega_n} = \frac{1}{\sqrt{1-\delta^2}} e^{-\delta\omega_n t} \sin\left(\omega_n \sqrt{1-\delta^2}\,t\right) \quad ...(2.59)$$

for underdamped system i.e. when $\delta < 1$

$$\frac{q_o(t)}{KA\omega_n} = \omega_n t e^{-\omega_n t} \quad ...(2.60)$$

for critically damped system i.e. when $\delta = 1$

$$\frac{q_o(t)}{KA\omega_n} = \frac{1}{\sqrt{\delta^2-1}} e^{-\delta\omega_n t} \sinh\left(\omega_n \sqrt{\delta^2-1}\,t\right) \quad ...(2.61)$$

for overdamped system i.e. when $\delta > 1$

The response curves are shown in Fig. 2.37.

2.13 HIGHER-ORDER SYSTEMS

Higher-order physical systems may be studied for their performance when they are subjected to standard input signals and the response becomes further complicated. Unless they are properly damped, they may become unstable in performance. Usually, instrument systems are designed to operate under conditions that keep the system stable and dynamic compensation is adopted in order to have the desired performance and limit its operation at frequencies far removed from the region of resonance or unstable operation.

2.14 DEAD-TIME ELEMENT

A dead-time element or transport lag is defined as a system in which the output is exactly of the same form as that of the input, but the event occurs after a time delay τ_d. Mathematically.

$$q_o(t) = Kq_i(t - \tau_d) \text{ for } t \geq \tau_d \quad ...(2.62)$$

Such cases can be observed in pneumatic signal transmission systems. A pressure input at one end of a length of pneumatic tubing will be observed at the other end after the time required for propagation through the distance between two ends. Assuming the speed as equal to the speed of sound, the dead time for a 300 m length of tubing is of the order of 1 second.

This phenomenon is important in hydraulic and pneumatic instrumentation systems.

Example 2.25. **An instrument is represented by a second-order transfer function**

$$G(s) = \frac{1}{s^2 + s + 1}$$

Determine the percentage overshoot if the instrument is excited by a unit step input. Determine, also the steady-state error in unit ramp response.

Solution: Comparing the given transfer function with Eq. (2.41) we have $\omega_n = 1$ and $\delta = \frac{1}{2\omega_n} = 0.5$

From Eq. (2.49)

$$\text{Percentage overshoot} = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \times 100$$

$$= e^{\frac{-0.5\pi}{\sqrt{1-0.5^2}}} \times 100 = 16.3\% \textbf{ Ans.}$$

Steady-state error in unity ramp response,

$$e_{ss} = \frac{-2\delta}{\omega_n} = \frac{-2\times 0.5}{1} = -1 \textbf{ Ans.}$$

2.15 CHOICE OF INSTRUMENTS

The main consideration in choosing the most suitable instrument for use in measurement of a particular quantity (variable) in a manufacturing plant or other system is the specification of the instrument characteristics required, especially parameters such as the desired measurement accuracy, resolution and sensitivity. It is also essential to know the environmental conditions that the instrument will be subjected to, as certain conditions will immediately eliminate the possibility of using some types of instruments. Provision of this type of information usually needs the expert knowledge of personnel who are intimately acquainted with the operation of the manufacturing plant or system in question. Then a skilled instrument engineer, having knowledge of all the instruments available for measuring the quantity in question, will be able to evaluate the possible list of instruments in terms of their accuracy, cost and suitability for the environmental conditions and thus to choose the most appropriate instrument. As far as possible, measurement systems and instruments should be chosen which are as insensitive as possible to the working environment, although this requirement is often difficult to meet because of cost and other performance considerations.

Published literature is of considerable help in the choice of a suitable instrument for a particular measurement situation. However, new techniques and instruments are being developed all the time, and therefore, a good instrumentation engineer must keep abreast of the latest developments by reading the appropriate technical journals regularly.

The instrument characteristics discussed earlier in this chapter are those features which form the technical basis for a comparison between the relative merits of different instruments. Generally, the better the characteristics, the higher the cost. However, in comparing the cost and relative suitability of different instruments for a particular measurement situation, consideration of durability, maintainability and constancy of performance are also very important because the instrument chosen will often have to be capable of operating for long periods without performance degradation and a requirement for costly maintenance. As a consequence, the initial cost of the instrument often has a low weightage in the evaluation exercise.

Cost is very strongly correlated with the performance of an instrument, as measured by its static characteristics. Increasing the accuracy or resolution of an instrument, for example, can only be done at a penalty of increasing its manufacturing cost. Instrument choice, therefore, proceeds by specifying the minimum characteristics, required by a measurement situation and then searching manufacturer's catalogues to find an instrument whose characteristics match those required. Selection of an instrument with characteristics superior to those required would only mean paying more than necessary for a level of performance greater than that required.

As well as purchase cost, other important factors in the assessment exercise are instrument durability and the maintenance requirements. The projected life of an instrument often depends on the conditions in which the instrument will have to operate. Maintenance requirements must also be considered, as they also have cost implications.

As a general rule, a good assessment criterion is obtained if the total purchase cost and estimated maintenance costs of an instrument over its life are divided by the period of its expected life. The figure obtained is thus an annual cost. However, this rule is modified where instruments are being installed on a process whose life is expected to be limited. Then, the total costs can only be divided by the period of time an instrument is expected to be used, unless an alternative use for the instrument is envisaged at the end of this period.

Thus, instrument choice is a compromise between performance characteristics, ruggedness and durability, maintenance requirements and purchase cost.

EXERCISES

1. What is the importance of static characteristics of measurement system?
2. Define various static characteristics of instruments.
   [M.D. Univ. Electrical Measurements and Measuring Instruments, December-2008]
3. Explain briefly the following terms as applied to characterisation of measurement systems.
   (i) Accuracy (ii) Precision (iii) Resolution (iv) Sensitivity and (v) Linearity.
   [U.P.S.C. I.E.S. Electronics & Communication Engineering-I, 2002]

4. Define the following with suitable examples:
(i) Precision (ii) Accuracy (iii) Repeatability (iv) Drift related to the instruments.
[Rajasthan Technical Univ. Electronic Measurements & Instrumentation, February-2011]

5. Define the following:
(i) Accuracy (ii) Precision (iii) Sensitivity (iv) Linearity (v) Resolution (vi) Hysteresis.
[R.G. Technical Univ. Electronic Instrumentation, February-2010]

6. Explain the following terms:
(i) Static sensitivity (ii) Range (iii) Instrument efficiency (iv) Resolution and (v) Linearity.
[U.P. Technical Univ. Elec. Measurements and Measuring Instruments 2004-05]

7. Define and explain (i) Precision (ii) Accuracy (iii) Resolution (iv) Threshold. [M.D. Univ. Electrical Measurements and Measuring Instruments, May-2011]

8. Explain the phenomenon of hysteresis in measurement system. Alongside explain the terms "Threshold", "Dead Zone" and "Backlash". Draw neat diagrams to explain your answer. [M.D. Univ. Electrical Measurements and Measuring Instruments, December-2006]

9. Explain how the non-linearity of measuring system is defined and estimated.
Define the terms 'drift', 'threshold value' and 'dead band' of a measuring system. Give an example for each.

10. Explain hysteresis and loading effect.
[R.G. Technical Univ. Electronic Instrumentation, December-2010]

11. What is loading effect and how can it be reduced?
[M.G. Univ. Kerala, November-2012]

12. What are the effects of using a voltmeter of low resistivity? Explain with an example.
[J.N. Technological Univ. Hyderabad, February/March-2012]

13. Explain about ammeter loading effect.
[J.N. Technological Univ. Hyderabad Electronic Measurements and Instrumentation, May-2011]

14. Explain the dynamic characteristics of instruments.
[U.P. Technical Univ. Electronics Measurements and Instrumentation, 2006-07]

15. What is meant by steady-state response and dynamic response of a measuring system
Explain the following terms:
(i) Dynamic error (ii) Fidelity, (iii) Speed response, (iv) Response time, and (v) Measuring lag.

16. Define the terms 'time constant' and 'settling time', of an instrument and indicate the factors responsible for them.

17. Define the term 'bandwidth'. What does this term convey when it is quoted for a measuring instrument?

18. Explain the types of test signals used for determination of dynamic characteristics.
Define 'dynamic error' and show how it differs with the type of input signal applied to the measuring system.

19. What is meant by order of a system and what does it signify ? Give the characteristics of a zero-order system.

20. Derive the transfer function of a first-order system and identify the constants by which it is characterized.
Define time constant of a first-order system and explain how its dynamic performance is affected when it is subjected to a ramp input function.

21. Explain step response of a first-order system.
[M.D. Univ. Measurements and Instrumentation, 2010]

22. Define the damping ratio of a second-order system and show how its response varies with time when subjected to a step input under different damping conditions.

23. Discuss the performance of a second-order system when subjected to a (i) step input (ii) ramp input and (iii) sinusoidal input function. Suggest suitable values for the damping factor at which the system is preferable to operate in each case.

24. What is meant by critical damping condition of a measuring system ? Explain why certain physical systems are operated under critical damping conditions.
Define the *settling time* of physical systems and show how does it vary with damping factor for second-order systems.

25. Write short notes on the following:
(i) Instrument hysteresis.
(ii) Dead Time and dead zone.
(iii) Noise and noise figure.
(iv) Loading effect.
(v) Dynamic characteristics of measuring systems.
(vi) Standard signals.
(vii) Generalized performance of systems.

SHORT ANSWER TYPE QUESTIONS WITH ANSWERS

Q. 1. How are judged the performance characteristics of an instrumentation system ?

Ans. The performance characteristics of an instrumentation system are judged by how faithfully the system measures the desired input and how thoroughly it rejects the undesirable inputs.

Q. 2. Distinguish between Reliability and Repeatability.
[B.P. Univ. of Technology, Orissa]

Ans. The *reliability* of a system is defined as the probability that it will perform its assigned functions for a specific period of time under given conditions.

Repeatability is the characteristic of precision instruments. It describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout. It is affected by internal noise and drift. It is expressed in percentage of the true value.

Q. 3. Define accuracy and precision related to measuring instruments. [P.T.U. May 2008]

Ans. Accuracy is defined as the degree of exactness (closeness) of a measurement compared to the expected (desired) value, whereas precision is a measure of the consistency or repeatability of measurements, *i.e.* successive readings do not differ. (Precision is the consistency of the instrument output for a given value of input).

In brief, accuracy can be defined as *conforming to truth* and precision can be defined as *sharply* or *closely defined*.

Q. 4. What is the difference between accuracy and precision?
[Anna Univ. Chennai (TN), May/June-2011]

Ans. *Accuracy* refers to the degree of closeness or conformity to the accepted standard value or the true value of the quantity under measurement. The only time a measurement can be exactly correct is when it is a count of a number of separate items, e.g., a number of components or a number of electrical pulses. In all other cases there will be a difference between the true value and the value the instrument indicates, records or controls to i.e., there is a measurement error.

Thus accuracy of a measurement means conformity to truth.

Precision is a term which describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high-precision instrument, then the spread of the readings will be very small.

Precision is a measure of the consistency or repeatability of measurements *i.e.* successive readings do not differ. (Precision is the consistency of the instrument output for a given value of input). It combines the uncertainty due to both random differences in results in a number of measurements and the smallest readable increment in scale or chart (given as the deviation of mean value).

Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about accuracy. A high-precision instrument may have a low accuracy. Low-accuracy measurements from a high-precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.

Q. 5. Enumerate the factors that influence the accuracy of a system.

Ans. In measurement, accuracy is influenced by static error, dynamic error, drift, reproducibility, non-linearity, hysteresis, temperatures and vibration.

Q. 6. Explain the terms: (i) Static error (ii) Static correction and (iii) Relative error. **[U.P. Technical Univ., 2009-10]**

Ans. (i) **Static error** is defined as the algebraic difference between the value indicated or conveyed by the output of the measuring system or transducer and the true value of the quantity presented to the input.

i.e. Static error = Measured valued – true value.

Error is positive quantity if measured or indicated value is higher than true value and is expressed as a percentage of the full-scale value of the measuring system.

(ii) **Static correction** is the opposite of error i.e. the algebraic difference between the true value and the indicated or measured value of the quantity.

So, Correction = True value – indicated value

Correction is a quantity which is added algebraically to the indicated value so as to have true value.

(iii) **Relative error** is defined as the ratio of static error to the true value of the quantity under measurement.

$$\text{i.e. Relative error} = \frac{\text{Static error}}{\text{True value}} = \frac{\text{Measured value} - \text{true value}}{\text{True value}}$$

The relative error may be quoted as a fraction e.g., 5 parts in 1,000 or may be expressed as a percentage.

Q. 7. Define the term resolution. **[P.T.U. December 2004]**

Ans. The resolution of any instrument is the smallest change in the input signal (quantity under measurement) which can be detected by the instrument. It may be expressed as an actual value or as a fraction or percentage of the full-scale value.

Q. 8. What is meant by drift? **[P.T.U. December-2009; December-2012]**

Ans. Drift is undesired change in the output-input relationship over a period of time.

Drift is a slow variation in the output signal of a transducer or measuring system which is not due to any change in the input quantity. It is primarily due to changes in operating conditions of the components inside the measuring system.

Q. 9. What is meant by bias?

Ans. Bias describes a constant error which exists over the full range of measurement of an instrument.

Q. 10. What is tolerance? Is it a static characteristic of measuring instrument?

Ans. Tolerance is a term which is closely related to accuracy and defines the maximum error which is to be expected in some value. Strictly speaking it is not a static characteristic of measuring instrument.

Q. 11. What is meant by overshoot?

Ans. If measurand is applied all of a sudden to the indicating analog instrument then owing to the finite mass of pointer and the moving coil, the momentum developed by deflecting torque would cause the pointer to move beyond (or cross) the equilibrium position. This is called overshoot and is shown in Fig. 2.6. It is the maximum amount in per cent of the step magnitude by which the response exceeds the required change. In fact little overshoot is desirable to bring the pointer to rest in minimum time.

Q. 12. What is meant by bandwidth of a system?

Ans. Bandwidth of a system is the range of frequencies for which its dynamic sensitivity is satisfactory.

Q. 13. What is meant by loading effect?

Ans. The incapability of the system to faithfully measure, record or control the input signal (measurand) in undistorted form is known as loading effect.

Q. 14. What is settling time? **[P.T.U. May 2006]**

Ans. It is the time required by the instrument or measurement system to settle down to its final steady-state position after the application of the input. For portable instruments, it is the time taken by the pointer to come to rest within ± 0.3% of its final scale length while for panel type instruments, it is the time taken by the pointer to come to rest within ± 1% of its final scale length.

A smaller settling time indicates higher speed of response. Settling time is also dependent on the system parameters and varies with the condition under which the system operates. The settling time of second-order instruments is affected by the degree of damping provided for the instrument.

Q. 15. Explain briefly about sensitivity and loading effect of a voltmeter. [U.P.S.C. I.E.S. E.C.E., 2010]

Ans. The sensitivity of a voltmeter is defined as

$$S_V = \frac{1}{I_{fs}} = \frac{1}{I_m} \, \Omega/V$$

where I_{fs} is the current required for full-scale deflection.

A voltmeter when connected across two points in a highly resistive circuit, acts as a shunt for that portion of the circuit. The meter will then provide a lower voltage drop than actually existed before the meter was connected. This effect is called the *loading effect.*

Loading effect of the instrument is caused principally by low sensitivity instrument.

Q. 16. Give a practical example of zero-order system.

Ans. Displacement measuring potentiometer.

Q. 17. Give an example of first-order system ?

Ans. A good example of first-order system is a thermocouple.

Q. 18. The dynamic error of first-order system is given as

$$e_d = Km\tau - Km\tau e^{-t/\tau}$$

Separate out steady-state error and transient error.

Ans. Steady-state error, $e_{ss} = Km\tau$

Transient error, $e_t = Km\tau e^{-t/\tau}$

PROBLEMS

1. A set of independent voltage measurements were recorded as 20.05, 20.10, 20.12, and 20.09 volts. Determine the average voltage and the range of error. [Ans. 20.09; 0.035 V]

2. A diaphragm type pressure measuring instrument is calibrated for absolute pressure of 6 to 760 mm of mercury. It has an accuracy of ± 1%. Calculate the scale range, scale span and maximum static error. [Ans. 6–760 mm: 754 mm: ± 7.54 mm]

3. Determine the linearity of a potentiometer to obtain an error not to exceed 5 parts in 10,000. [Ans. 0.05%]

4. A 10.0 V voltmeter has a resistance of 1,000 Ω. Determine the efficiency of the instrument. [Ans. 100 V per volt]

5. A thermocouple has an output emf as shown below when its hot (measuring) junction is at the temperatures shown. Determine the sensitivity of measurement for the thermocouple in mV/ °C.

mV	2.35	4.70	7.05	9.4
°C	200	400	600	800

[Ans. 0.01175 mV per °C]

6. (a) An instrument is calibrated in an environment at a temperature of 25 °C and the following output readings *y* are obtained for various input values *x*:

x	5	10	15	20	25
y	4.7	9.4	14.1	18.8	23.5

Determine the measurement sensitivity, expressed as the ratio $\frac{y}{x}$.

(b) When the instrument is subsequently used in an environment at a temperature of 55 °C, the output-input characteristic changes to the following:

x	5	10	15	20	25
y	5.9	11.8	17.7	23.6	29.5

Determine the new measurement sensitivity. Hence determine the sensitivity drift due to change in ambient temperature of 30 °C. [Ans. (*a*) 0.94, (*b*) 1.18; 0.24]

7. A load cell is calibrated in an environment at a temperature of 25 °C and has the following deflection/load characteristic.

Load in kg	0	50	100	150	200
Deflection in mm	0.0	1.5	3.0	4.5	6.0

When used in an environment at 40 °C, its characteristic changes to the following:

Load in kg	0	50	100	150	200
Deflection in mm	0.3	1.9	3.5	5.1	6.7

(*a*) Determine the sensitivity at 25 °C and 40 °C.

(*b*) Calculate the total zero drift and sensitivity drift at 40 °C.

(*c*) Hence determine the zero drift and sensitivity drift coefficients in units of μm/°C and μm/kg/°C.

[Ans. (a) 30 μm/kg; 32 μm/kg, (b) 0.3 mm; 2 μm/kg (c) 20.0 μm/ °C, 0.133 μm/kg/°C]

8. The dead zone of a certain pyrometer is 0.125% of the span. The calibration is 800 °C to 1,800 °C. What temperature change must occur before it is detected ? [Ans. 1.25 °C]

9. An ammeter reads 6.7 A and the true value of current is 6.54 A. Determine the error and the correction of the instrument.
[U.P. Technical Univ. Electronic Measurements & Instrumentation, 2006-07]
[Ans. 0.16 A; – 0.16 A]

10. A multimeter having a sensitivity of 3,000 Ω/V is used to measure the voltage across circuit having an output resistance of 10 kΩ. The open-circuit voltage of the circuit is 6 V. Find the reading of the multimeter when it is set to its 10-V scale. Find the percentage error. [Ans. 4.5 V, 2.5% low]

11. A voltmeter with an internal resistance of 4,750 Ω is used to measure the voltage across a resistance of 600 Ω connected in series with a resistance of 400 Ω. What is the error in measurement? [U.P.S.C I.E.S. Electronics & Telecommunication Engineering-I, 1998]
[Ans. – 4.8333%]

12. A multimeter having a sensitivity of 4,000 Ω/V is used to measure the voltage across circuit having an output resistance of 20 kΩ. The open-circuit voltage of the circuit is 7.5 V. Find the reading of the multimeter when it is set to its 10-V scale. Also find the percentage error.
[U.P. Technical Univ. Elec. Measurements and Measuring Instruments 2004-05]
[Ans. 5 V; 33.3%]

13. [illegible]

14. [illegible]

Ans. [illegible]

15. [illegible]

16. Determine the [illegible]

17. [illegible]

18. [illegible]

19. An [illegible]

Determine [illegible] factor and natural [illegible] find the [illegible]

20. [illegible] frequency and [illegible]

Ans. [illegible]

3 CHAPTER

Errors in Measurement

INSIDE THIS CHAPTER

3.1 INTRODUCTION

Measurement is the process of comparing an unknown quantity with an accepted standard quantity and a measuring instrument is a device used for comparing the unknown quantity with the standard quantity or a unit of measurement. No measurement can be made with the complete accuracy, therefore, a study of errors is necessary in the study of measuring processes. The fact that we are studying the subject of errors does not mean that all measurements can be made with an extreme degree of accuracy. Measurements cannot be called good or bad merely on the basis of the degree of accuracy, but rather on their adequacy under the given conditions. In general, closer results can be achieved by time, care and expense; whether the improvement is justified in a particular case is, of course another question. In any event, we must know what the accuracy of our results really is, whether high or low, otherwise we are in the dark, and have figures which we do not know how to evaluate. We cannot study the matter of accuracy without knowing something of the different types of errors that may enter, and what we can do about them.

3.1.1. Absolute Error. The value of the unknown quantity obtained on making measurements with standards and measuring instruments is considered to be its *true value*, though practically it is never. There is always some difference, small or large, between the measured value and the true or exact value of the unknown quantity. The difference between the measured value A_m and the true value A of the unknown quantity is known as the *absolute error of measurement*, δA*

$$\text{i.e., } \delta A = A_m - A \qquad ...(3.1)$$

3.1.2. Relative Error and Percentage Error. The absolute value of error δA does not indicate precisely the accuracy of measurements. For example, an absolute error of one cm is infinitesimal in determining the circumference of earth, negligible in setting out a cricket pitch, but intolerable in a metre stick. The quality of measurement is, therefore, indicated preferably in terms of *relative error*. The relative error is the ratio of absolute error to the true value of the quantity to be measured.

$$\text{i.e., Relative error, } \varepsilon_r = \frac{\delta A}{A} = \frac{\varepsilon_0}{A} = \frac{\text{Absolute error}}{\text{True value}} \qquad ...(3.2)$$

When the absolute error $\varepsilon_0 = \delta A$ is negligible, i.e., when the difference between measured value A_m and true value A is negligible then relative error may be expressed as

$$\varepsilon_r = \frac{\delta A}{A_m} \qquad ...(3.3)$$

The relative error may be quoted as a fraction e.g., 5 parts in 1,000 or may be expressed as a percentage.

$$\text{i.e., Percentage error } = \varepsilon_r \times 100 = \frac{\varepsilon_0}{A} \times 100 \qquad ...(3.4)$$

The amount of relative error is often an indication of the class of use to which a piece of apparatus may be put. For example, a resistor which is within 5% of its nominal value may be quite satisfactory in a shunt field regulator or in a radio set but useless as a standard or in a decade resistance box. A resistor within 1% of its nominal value can be quite useful for general measurements but not for very precise measurements. In order to assure the purchaser of the quality of the circuit components or measuring instruments, the manufacturers guarantee a certain accuracy. Circuit components such as resistors, inductors and capacitors are guaranteed to be within a certain percentage of the rated value whereas in indicating instruments the accuracy is mostly guaranteed to be within a certain percentage of full-scale reading.

3.1.3. Limiting or Guarantee Errors. Manufacturers of equipment/apparatus give guarantee about the accuracy

* **Note:** δA is also called the reading correction or simply correction, being of the same magnitude as the error, but of opposite sign.

of the equipment/apparatus with some limiting deviations from the specified value in order to enable the purchaser to make proper selection according to his requirement. As mentioned above in Art. 3.1.2, in most of the indicating instruments the accuracy is guaranteed to be within a certain percentage of full-scale reading while circuit components such as resistors, inductors and capacitors are guaranteed to be within a certain percentage of the rated value. The limits of these deviations from the specified values are known as *limiting errors.*

For example, if a certain ammeter of 0–25 A range is guaranteed to be accurate within ± 2 per cent of full-scale reading by the manufacturer, it means that the magnitude of limiting error will be 2 per cent of 25 A i.e. $\frac{2 \times 25}{100}$ = 0.5 A.

If the resistance of a resistor is given as 800 ± 5% the manufacturer guarantees that resistance falls between the limits of (800 ± 40) Ω i.e., between 760 Ω and 840 Ω. Here the maker is not specifying a standard deviation* or a probable error* but assures that the error is not more than the limits set.

The magnitude of a given quantity having a measured value A_m and a maximum or a limiting error $\pm \delta A$ must have a magnitude between the limits $A_m - \delta A$ and $A_m + \delta A$

$$\text{or } A = A_m \pm \delta A \quad ...(3.5)$$

For example, the measured value of a capacitor is 100 µF with a limiting error of ± 5 µF. The true value of the capacitor will be between the limits 100 ± 5 i.e. between 95 and 105 µF.

Example 3.1. The measured value of a resistance is 10.25 Ω, whereas its value is 10.22 Ω. Determine the absolute error of measurement.

Solution: Measured value, $A_m = 10.25\ \Omega$

True value, $A = 10.22\ \Omega$

Absolute error, $\delta A = A_m - A$

$= 10.25 - 10.22 = 0.03\ \Omega$ **Ans.**

Example 3.2. A wattmeter reads 25.34 watts. The absolute error in the measurement is – 0.11 watt. Determine the true value of power.

Solution: Measured value,

$A_m = 25.34$ watts

Absolute error, $\delta A = -0.11$ watt

True value, A = Measured value – absolute error

$= A_m - \delta A = 25.34 - (-0.11)$

$= 25.45$ watts **Ans.**

Example 3.3. The measured value of a capacitor is 205.3 µF, whereas its true value is 201.4 µF. Determine the relative error.

[B.P. Univ. of Technology Orissa Electronics Measurements and Measuring Instruments, 2007]

Solution: Measured value,

$A_m = 205.3 \times 10^{-6}$ F

True value, $A = 201.4 \times 10^{-6}$ F

Absolute error, $\varepsilon_0 = A_m - A = 205.3 \times 10^{-6} - 201.4 \times 10^{-6}$

$= 3.9 \times 10^{-6}$ F

Relative error, $\varepsilon_r = \frac{\varepsilon_0}{A} = \frac{3.9 \times 10^{-6}}{201.4 \times 10^{-6}}$

$= 0.0194$ or 1.94% **Ans.**

Example 3.4. The expected value of the voltage to be measured is 150 V and the measured value is 148 V. Calculate (i) Relative accuracy (ii) Absolute error.

[U.P. Technical Univ. Electronic Instrumentation and Measurements, 2009-10]

Solution: Measured value of voltage, $A_m = 148$ V

Expected (or true) value of voltage, $A = 150$ V

Absolute error, $\varepsilon_o = A_m - A = 148 - 150 = -2$ V **Ans.**

Relative accuracy $= 1 - \left|\frac{\varepsilon_o}{A}\right| = 1 - \frac{2}{150} = 0.98667$ **Ans.**

Example 3.5. The expected value of the voltage across a resistor is 80 V. However, the measurement gives a value of 79 V. Calculate:

(i) absolute error (ii) % error (iii) relative accuracy (iv) % of accuracy.

[M.D. Univ. Electrical Measurement and Measuring Instruments, December-2010]

Solution: Measured value of voltage, $A_m = 79$ V

Expected value of voltage, $A = 80$ V

(*i*) Absolute error, $\varepsilon_o = A_m - A = 79 - 80 = -1$ V **Ans.**

(*ii*) % error $= \frac{A_m - A}{A} \times 100 = \frac{79 - 80}{80} \times 100$

$= -1.25\%$ **Ans.**

(*iii*) Relative accuracy $= 1 - \left|\frac{\varepsilon_o}{A}\right| = 1 - \frac{1}{80} = 0.9875$ **Ans.**

(*iv*) % of accuracy = 100 × relative accuracy

$= 100 \times 0.9875 = 98.75\%$ **Ans.**

Example 3.6. A resistor of value 4.7 kΩ is read as 4.65 kΩ in a measurement. Calculate (i) absolute error, (ii) % error and (iii) accuracy. *[U.P.S.C. I.E.S. Electronics and Telecommunication Engineering-I, 2009]*

Solution: Measured value of voltage, $A_m = 4.65$ kΩ

True value of resistor, $A = 4.7$ kΩ

(*i*) Absolute error, $\varepsilon_o = A_m - A$

$= 4.65 - 4.7$

$= -0.05$ kΩ or $-50\ \Omega$ **Ans.**

(*ii*) % error $= \frac{A_m - A}{A} \times 100 = \frac{4.65 - 4.7}{4.7} \times 100$

$= -1.064\%$ **Ans.**

(*iii*) Accuracy = 100 – | % error | %

$= 100 - 1.064 = 98.936\%$ **Ans.**

Example 3.7. The inductance of an inductor is specified as 20 H ± 5 per cent by a manufacturer. Determine the limits of inductance between which it is guaranteed.

* **Note:** The terms standard deviation and probable error will be explained later on in Arts. 3.8 and 3.11 respectively.

Solution: Relative error,

$$\varepsilon_r = \frac{\text{Percentage error}}{100} = \frac{5}{100} = 0.05$$

Limiting value of inductance,

$$A = A_m \pm \delta A$$
$$= A_m \pm \varepsilon_r A_m = A_m(1 \pm \varepsilon_r)$$
$$= 20\ (1 \pm 0.05) = 20 \pm 1 \text{ H} \quad \textbf{Ans.}$$

Example 3.8. A freshman student of electrical engineering wanted to calibrate a power meter. He recorded the current flowing through a resistor (value marked as 0.0105 Ω) as 30.4 A. Later on, a second year student discovered that the ammeter reading taken by the freshman was lower by 1.2% and the value marked on the resistor was 0.3% lower. Find the true value of the power as a percentage of the calculated by the freshman.

[*U.P.S.C. I.E.S. Electronics and Telecommunication Engineering-I, 2005*]

Solution: Power calculated by freshman,

$$P = I^2R = (30.4)^2 \times 0.0105 = 9.70368 \text{ watts}$$

True value of current,

$$I' = \frac{100}{100 - 1.2} I$$

∵ the reading taken by freshman was lower by 12%.

$$= \frac{100}{98.8} \times 30.4 = 30.769 \text{ A}$$

True value of resistance,

$$R' = \frac{100}{100 - 0.3} \times 0.0105$$

∵ value marked on the resistor was 0.3% lower

$$= 0.010532\ \Omega$$

True power, $P' = (I')^2 R' = (30.769)^2 \times 0.010532 = 9.971$ watts

$$\frac{\text{True value of power}}{\text{Power calculated by freshman}} \times 100 = \frac{9.971}{9.70368} \times 100$$
$$= 102.755\% \quad \textbf{Ans.}$$

Example 3.9. Define limiting errors.

A 0–10 A ammeter has a guaranteed accuracy of 1.5 per cent of full-scale reading. The current measured by the instrument is 2.5 A. Calculate the limiting values of current and the percentage limiting error.

[*U.P. Technical Univ. Elec. Measurements and Measuring Instruments 2005-06*]

Solution: The magnitude of limiting error of the instrument,

$$\delta A = \varepsilon_r \times A = \frac{1.5}{100} \times 10 = 0.15 \text{ A}$$

The magnitude of current being measured is 2.5 A. The relative error at this current is

$$\varepsilon_r = \frac{\delta A}{A} = \frac{0.15}{2.5} = 0.06$$

Hence, the current under measurement is between the limits of

$$A = A_m(1 \pm \varepsilon_r) = 2.5\ (1 \pm 0.06) = (2.5 \pm 0.15) \text{ A}$$

So limiting values of current under measurement are 2.35 A and 2.65 A **Ans.**

$$\text{Limiting error} = \pm\frac{0.15}{2.5} \times 100 = \pm 6\% \quad \textbf{Ans.}$$

Example 3.10. A wattmeter having a range of 1,000 W has a error ± 1% of full-scale deflection. If the true power is 100 W, what would be range of the reading? Suppose the error is specified as percentage of true value, what would be the range of the readings?

[*U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2006-07*]

Solution: The magnitude of limiting error of the instrument,

$$\delta P = \varepsilon_r \times P = 0.01 \times 1{,}000 = 10 \text{ W}$$

The magnitude of power being measured is 100 W

Range of the readings

$$= P \pm \delta P$$
$$= (100 \pm 10) \text{ W i.e. } 90 \text{ to } 110 \text{ watts} \quad \textbf{Ans.}$$

When the error is specified as a percentage of true value

$$\delta P = 0.01 \times 100 = 1 \text{ W}$$

and Range of the readings

$$= (100 \pm 1) \text{ W i.e. } 99 \text{ to } 101 \text{ W} \quad \textbf{Ans.}$$

From the above example it is noteworthy that meters are guaranteed for better accuracies for full-scale reading but when the meters are used for lower readings the limiting error increases. (say 1.5% to 6% in case of Example 3.9). If the quantity (current or voltage or any other one) under measurement is further reduced, the limiting error will further increase because the magnitude of the limiting error is a fixed quantity based on the full-scale deflection of the meter. The above examples also show the importance of taking measurements as close to full scale as possible.

Measurements or computations, combining guarantee errors, are usually made. Such a computation is illustrated in the following examples.

Example 3.11. In a Wheatstone bridge three decade resistance boxes are used which are guaranteed for ± 0.2%. An unknown resistor of R ohms is measured with this bridge. Determine the limits on resistor R imposed by the decade boxes.

Solution: Under balanced condition of Wheatstone bridge, unknown resistance R can be determined in terms of the resistances of three decade boxes, i.e. $R = \frac{R_1 R_2}{R_3}$. Here R_1, R_2 and R_3 are the resistances of the decade boxes and each is guaranteed to ± 0.2%. In worst case, the two terms in numerator may both be + ve to the full limit of 0.2% and the denominator may be –ve to the full limit of 0.2% giving a resultant error of 0.6%.

So the guarantee error is obtained by taking the direct sum of all the possible errors, adopting the algebraic signs giving the worst possible combination.

Example 3.12. The current passing through a resistor of 100 ± 0.2 Ω is 2.00 ± 0.01 A. Using the relationship, $P = I^2R$, calculate the limiting error in the computed value of power dissipation. [*R.G. Technical Univ. Bhopal Electronic Instrumentation, Nov./Dec.-2007*]

Solution: Percentage limiting error to resistance

$$= \pm \frac{0.2}{100} \times 100 = \pm 0.2\%$$

Percentage limiting error to current

$$= \pm \frac{0.01}{2} \times 100 = \pm 0.5\%$$

The power dissipated in resistance R due to flow of current I is given by the expression

$$P = I^2 R$$

In worst possible combination of errors the limiting error in the power dissipation is

$$\pm(2 \times 0.5\% + 0.2\%) = \pm 1.2\%$$

So power dissipation is given by the expression as

$$P = I^2 R$$
$$= (2.0)^2 \times 100$$
$$= 400 \text{ W} \pm 1.2\% \text{ of } 400 \text{ watts or } 400 \pm 4.8 \text{ watts}$$ **Ans.**

3.2 RESOLUTION AND SENSITIVITY

If the input is slowly increased from some arbitrary value it will be noticed that the output does not change at all until the increment exceeds the certain value called the *resolution* or *discrimination* of the instrument. Thus the resolution or discrimination of any instrument is the smallest change in the input signal (quantity under measurement) which can be detected by the instrument. It may be expressed as an actual value or as a fraction or percentage of the full-scale value. In some texts this quantity is referred to as '*sensitivity*'.

The sensitivity gives the relationship between the input signal to an instrument or a part of an instrument system and the output. Thus the sensitivity is defined as the ratio of output signal or response of the instrument to a change of input signal or the quantity under measurement.

The sensitivity will, therefore, be a constant in a linear instrument or element *i.e.* where equal changes in the input signal cause equal changes in the output.

Sensitivity is usually needed to be high, and an instrument should, therefore, not have a range largely exceeding the quantity under measurement, although some margin of excess over the expected values should be allowed in order to prevent accidental overload.

Example 3.13. A moving coil voltmeter has a uniform scale with 100 divisions and gives full-scale reading of 200 V. The instrument can read up to $\frac{1}{5}$th of a scale division with a fair degree of certainty. Determine the resolution of the instrument in volt.

Solution: Full-scale reading = 200 V

Number of divisions of scale = 100

$$1 \text{ scale division} = \frac{200}{100} = 2 \text{ V}$$

$$\text{Resolution} = \frac{1}{5}\text{th of a scale division}$$

$$= \frac{2}{5} \text{ V} = 0.4 \text{ V}$$ **Ans.**

3.3 ACCURACY AND PRECISION

Accuracy is a closeness with which the instrument reading approaches the true value of the variable under measurement while precision is a measure of the reproducibility of the measurement i.e., precision is a measure of the degree to which successive measurements differ from one another. In brief, accuracy can be defined as *conforming to truth* and precision can be defined as *sharply* or *closely defined*.

For illustration of distinction between accuracy and precision, we may compare two decade resistance boxes, each with four dials, giving resistance increments of 1, 10, 100 and 1,000 Ω per step. Suppose one box is cheaper one of low grade quality guaranteed to be within 1% while another one is high grade quality guaranteed to be within 0.1%. Both boxes may be set to the same precision i.e., both boxes can be set to any value up to 10,000 Ω by one ohm step but the accuracies, however, are very different.

Difference between accuracy and precision can be illustrated by another example in which two voltmeters of the same make and model are compared. Both voltmeters have carefully calibrated scales and knife-edged pointers with mirror backed scales to avoid parallax. So both instruments may be read to same precision. If the value of the series resistance in one meter changes considerably, its reading may be in error by fairly large amount. Hence accuracy of the two meters may be quite different. For determination of the meter in error a comparison measurement with a standard meter should be made.

Here it should be clearly understood that accuracy of a reading is not necessarily guaranteed by its precision and good measurement technique demands continuous skepticism as to the accuracy of the results.

Precision is composed of two characteristics; *conformity* and the number of *significant* figures to which a measurement may be made. Consider an example in which a voltage reading of true value 1.952 V is measured by a voltmeter which consistently and repeatedly indicates 2 V. This is as close to the true value as an observer can read the scale by estimation. Although there are no deviations from the observed value, the error caused by the limitation of the scale reading is a precision error. So, conformity is a necessary, but not sufficient condition, for precision, because of lack of significant figures obtained. Similarly precision is a necessary, but not sufficient condition, for accuracy as explained below.

If a magnitude is to be determined with accuracy to a required number of digits, it is necessary that the measuring instrument have precision of this order. Thus precision is an essential condition for achievement of the accuracy. Accuracy is a matter of careful measurement in terms of an accurately known standard.

If the results of measurement of the same variable agree among themselves then that set of readings shows precision but it does not guarantee the accuracy as there may be some systematic disturbing effects which introduce error in the readings. For this reason, in important measurements, an independent set of observations by second method is also taken to avoid the effects of some systematic errors, where this is not possible to apply second method then method under use should be studied carefully to find out and eliminate any systematic disturbing error.

3.4 SIGNIFICANT FIGURES

An indication of the precision of the measurement is obtained from the number of significant figures in which result is expressed. Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity. More the significant figures, greater the precision of measurement.

For example if the value of resistance is specified as 138 Ω then its resistance is closer to 138 Ω than to 137 Ω or 139 Ω and there are three significant figures. If the resistance is specified as 138.0 Ω then its resistance is closer to 138.0 Ω than to 137.9 Ω or 138.1 Ω. In this case there are four significant figures. The latter, with more significant figures, expresses a measurement of greater precision than the former. A resistance of 138 Ω might also be written as 0.000138 M Ω. Here there are also three significant figures and one should not count zeros between the decimal point and 138 as significant figures because these zeros are present on account of the size of the unit used but give no greater precision to the result.

In some situations, the total number of digits may not represent the measurement precision. Frequently larger number of zeros before a decimal point are used for approximate population or amounts of money. For example, the population of a city is reported in six figures as 150,000. This may imply that the true value of the population lies between 149,999 and 150,001, which is six significant figures. What is meant, however, is that the population is closer to 150,000 than to 140,000 or 160,000. So this information has only two significant numbers, and a more technically correct notation uses power of ten as 15×10^4 or 1.5×10^5. This method indicates that the population figure is only accurate to two significant figures.

In measurement works, all those digits are recorded of which we are sure nearest to the true value. For example in reading an ammeter a current of 5.7 A simply indicates that the current read by the observer to best estimation, is closer to 5.7 A than to 5.6 A or 5.8 A. Another way of expressing this result indicates the range of possible error i.e., the current may be expressed as 5.7 ± 0.05 amperes, indicating that the value of the current lies between 5.65 A and 5.75 A.

The number of significant figures in a quantity is the one measure of precision, though not definite as a percentage statement. For example 100 A covers values between 99.5 and 100.5, which is ± 0.5% or a total range of doubt of 1%. The value of 999 A also has three significant figures and a range of doubt of 1 A, but as a percentage, the range is not 0.1%. So three significant figures are indefinite as a measure of precision as it covers total range of doubt from 1 to 0.1%. It is, however, a convenient approximate index, and is used to a considerable extent.

When two or more measurements with different degrees of accuracy are added, the result is only as accurate as the least accurate measurement. Suppose two quantities 15.3 and 6.245 are added, the sum of these two quantities will be 21.545. Because one of the quantity is accurate only to three significant figures, so the result will also be reduced to three significant figures i.e., 21.5. Here one point is to be added that in such cases extra digits accumulated in the answer should be discarded or rounded off. In usual practice, if the digit in the first place to be discarded is less than 5 then this digit and the following digits are discarded from the answer.

But if the first digit to be discarded is 5 or higher, the previous digit is increased by 1. For three digit precision 5.4546 should be rounded to 5.45 and 5.4556 to 5.46.

In multiplication, significant figures increase rapidly, but again only the approximate figures are retained in the answer and extra digits accumulated in the answer are discarded or rounded off.

In addition of figures with a range of doubt, doubtful are parts also added, since the ± sign means that one number may be high and the other low. The worst possible combination of range of doubt should be taken in the answer. For example if two quantities $A = 430 \pm 5$ and $B = 225 \pm 3$ are added then answer will be represented as 655 ± 8.

$$A = 430 \pm 5\ (= \pm 1.163\%)$$
$$B = 225 \pm 3\ (= \pm 1.333\%)$$
$$C = 655 \pm 8\ (= \pm 1.221\%)$$

Here one important point is to be noted that in case of addition of two quantities, the percentage doubt in the original figures A and B does not differ from the percentage doubt in the final result C.

If the same two numbers taken above are subtracted, then we find a very interesting comparison between addition and subtraction with respect to range of doubt

$$A = 430 \pm 5\ (= \pm 1.163\%)$$
$$B = 225 \pm 3\ (= \pm 1.333\%)$$
$$\text{Difference } D = 205 \pm 8\ (= \pm 3.902\%)$$

In subtraction also, the doubtful parts are added for the same reason as in addition. Now comparing the results of addition and subtraction of the same numbers A and B, note that the precision of the result, when expressed in percentage, differs greatly. The final result after subtraction shows a large increase in percentage doubt compared to the percentage doubt after addition. The percentage doubt increases even more when the difference between the numbers is relatively small.

So one should avoid measurement technique depending on subtraction of experimental results because the range of doubt in the final result may be greatly increased.

3.5 TYPES OF ERRORS

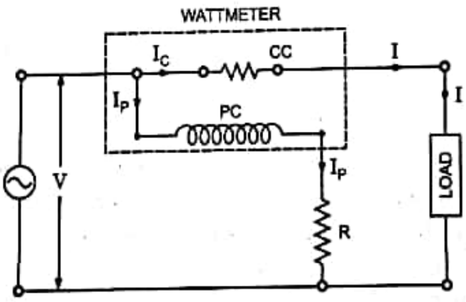
No measurement can be carried out with complete accuracy, so a study of errors is necessary in the study of measuring process. A study of errors is a first step in finding ways to reduce them and such a study also allows to determine accuracy of the final test results.

To study the matter of accuracy, it is very important to know the different types of errors that may enter during measurement of any quantity. Errors may originate in a variety of ways, but it can be grouped in three main categories.

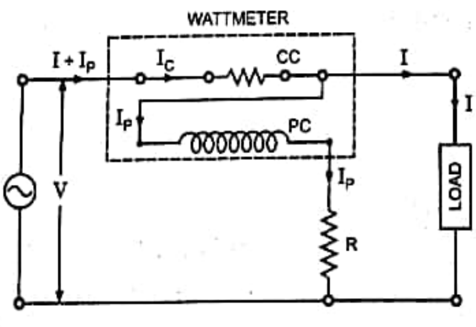
1. ***Gross Errors.*** In this category, errors occur because of mistakes in reading or using instruments and in recording and calculating measurement results. These errors are usually because of human mistakes and these may be of any magnitude and cannot be subjected to mathematical treatment.

The complete elimination of gross errors is not possible, but one can minimize them. Some errors are easily detected while others may be elusive.

One common gross error frequently committed during measurement is improper use of measuring instrument. Any indicating instrument changes conditions to some extent when connected into a complete circuit so that the reading of measured quantity is altered by the method used. For example in Figs. 3.1 (*a*) and (*b*) two possible connections of pressure coil and current coil of wattmeter are shown.



(*a*) *Wattmeter Connections*



(*b*) *Wattmeter Connections*

Fig. 3.1

The connection shown in Fig. 3.1 (*a*) is used when applied voltage is high and current flowing in the circuit is low while the connection shown in Fig. 3.1 (*b*) is used when applied voltage is low and current flowing in the circuit is high. If these connections of wattmeter are used in opposite order then error is liable to enter in wattmeter readings. Another example of this type of error is in use of a well calibrated voltmeter for the measurement of voltage across a resistance of very high value. The same voltmeter, when connected in a low resistance circuit may give a more dependable reading. This shows that the voltmeter has a loading effect on the circuit, which alters the original situation during the measurement.

Errors caused by loading effect of the meters can be avoided by using them intelligently. For example, when measuring a low resistance by voltmeter-ammeter method a high resistance voltmeter, such as a VTVM or TVM, should be used.

Gross errors are also contributed by the other factors such as improper reading of an instrument, failure to eliminate parallax or recording the result different from the actual reading taken or adjusting the instrument incorrectly.

For example a multirange instrument has different scale for each range. During measurements operator may use a scale which does not correspond to the setting of the range selector of the instrument. Gross error may also be there because of improper setting of zero before the measurement and this will affect all the readings taken during measurement.

As mentioned above that these errors cannot be treated mathematically so a great care should be taken during reading and recording the data to avoid these errors. Apart from this, at least two, three or more readings should be taken of the quantity under measurement, preferably by a different observer at different reading points. Then if the readings differ by an unreasonably large amount, the situation can be investigated and the bad reading eliminated.

2. ***Systematic Errors.*** These are errors that remain constant or change according to a definite law on repeated measurement of the given quantity. These errors can be evaluated and their influence on the results of measurement can be eliminated by the introduction of proper corrections.

These errors are sometimes referred to as *bias*, and they influence all measurements of a quantity alike. A constant uniform deviation of the operation of an instrument is known as a systematic error. There are basically three types of systematic errors: (*a*) instrumental, (*b*) environmental and (*c*) observational.

(*a*) *Instrumental Errors*. Such errors are inherent in the measuring instruments because of their mechanical structure and calibration or operation of the apparatus used. For example in the D'Arsonval movement, friction in bearings of various components may cause incorrect readings. Improper zero adjustment has a similar effect. Poor construction, irregular spring tensions, variations in the air gap may also cause instrumental errors. Calibration errors may also result in the instrument reading either being too low or too high.

These errors may be detected by checking for erratic behaviour, stability and reproducibility of results. A quick and easy way to check an instrument is to compare it with another instrument of the same characteristics or compare it with a comparatively more accurate instrument.

Such errors may be avoided by: (*a*) selecting a proper measuring device for the particular application, (*b*) applying correction factors after determining the magnitude of instrumental errors, and (*c*) calibrating the measuring device or instrument against a standard.

(b) *Environmental Errors.* Such errors are much more troublesome as these change with time in an unpredictable manner. These errors are introduced due to use of an instrument in different conditions than in which it was assembled and calibrated.

Change in temperature is the major cause of such errors as temperature affects the properties of materials in many ways, including dimensions, resistivity, spring effect and others. Other environmental changes also affect the results given by the instruments such as humidity, altitude, earth's magnetic field, gravity, stray electrostatic and magnetic fields etc. These errors can be eliminated or reduced by taking the following precautions:

(*i*) Using instrument in controlled conditions of pressure, temperature and humidity in which it was originally assembled and calibrated.

(*ii*) If above is not possible then deviations in local conditions must be determined and suitable corrections to instrument readings applied.

(*iii*) Using equipment which is immune to these effects. For example, variation in resistance with temperature can be minimized by using resistance materials of very low resistance temperature coefficient.

(*iv*) Using techniques that eliminate the effects of these disturbances. For example the effect of humidity, dust etc., can be entirely eliminated by hermetically sealing the equipment.

(*v*) The effects of external electrostatic or magnetic fields can be avoided by providing electrostatic or magnetic shields.

(*vi*) Altogether new calibrations may be made in the new conditions.

(c) *Observational Errors.* Such errors are introduced by the observer. The most common error is the *parallax error* introduced in reading a meter scale, and the error of estimation when obtaining a reading from a meter scale.

Parallax error is caused by the observer not having his line of sight on the pointer exactly at right angles to the plane of the scale. Such an error can be eliminated by providing a mirror beneath the scale and a knife-edged pointer.

Tests have been conducted in which a number of persons read fractional divisions on an ammeter scale under carefully controlled conditions and consistent individual differences were found. In measurements involving the timing of an event, one observer may tend to anticipate the signal and read too soon. Very considerable differences are likely to appear in determination of light intensities or sound levels. Important readings which may be subject to this type of error should be shared by two or more observers to minimize the possibility of a constant bias.

Modern electrical instruments have digital display of output which completely eliminates the errors due to human observational or sensing powers as the output is in the form of digits.

Systematic errors can also be subdivided into *static* or *dynamic* errors. Static errors are caused due to limitations of the measuring device or the physical laws governing its behaviour. A static error is introduced in a micrometer when excessive pressure is applied in torquing the shaft.

Static error is defined as the difference between the measured value and true value of a quantity i.e.

$$\delta A = A_m - A$$

where δA is error, A_m is measured value of quantity and A is true value of quantity.

Dynamic errors are caused by the instruments not responding fast enough to follow the variations in a measured variable.

In this case, the instrument readings lag behind the measured value in time.

3. ***Random*** **(or *Accidental*) *Errors.*** These are errors that remain even after systematic errors have been substantially reduced or at least accounted for. Random errors are generally an accumulation of a large number of small effects and may be of real concern only in measurements requiring a high degree of accuracy.

These errors are of variable magnitude and sign and do not obey any known law. The presence of random errors becomes evident when different results are obtained on repeated measurements of one and the same quantity. The effect of random errors is minimized by measuring the given quantity many times under the same conditions and calculating the arithmetical mean of the values obtained. The mean value can justly be considered as the most probable value of the measured quantity since random errors of equal magnitude but opposite sign are of approximately equal occurrence when making a great number of measurements.

The problem of random errors is treated mathematically as one of the probability and statistics, and is beyond the scope of this book. The following observations may, however, be useful.

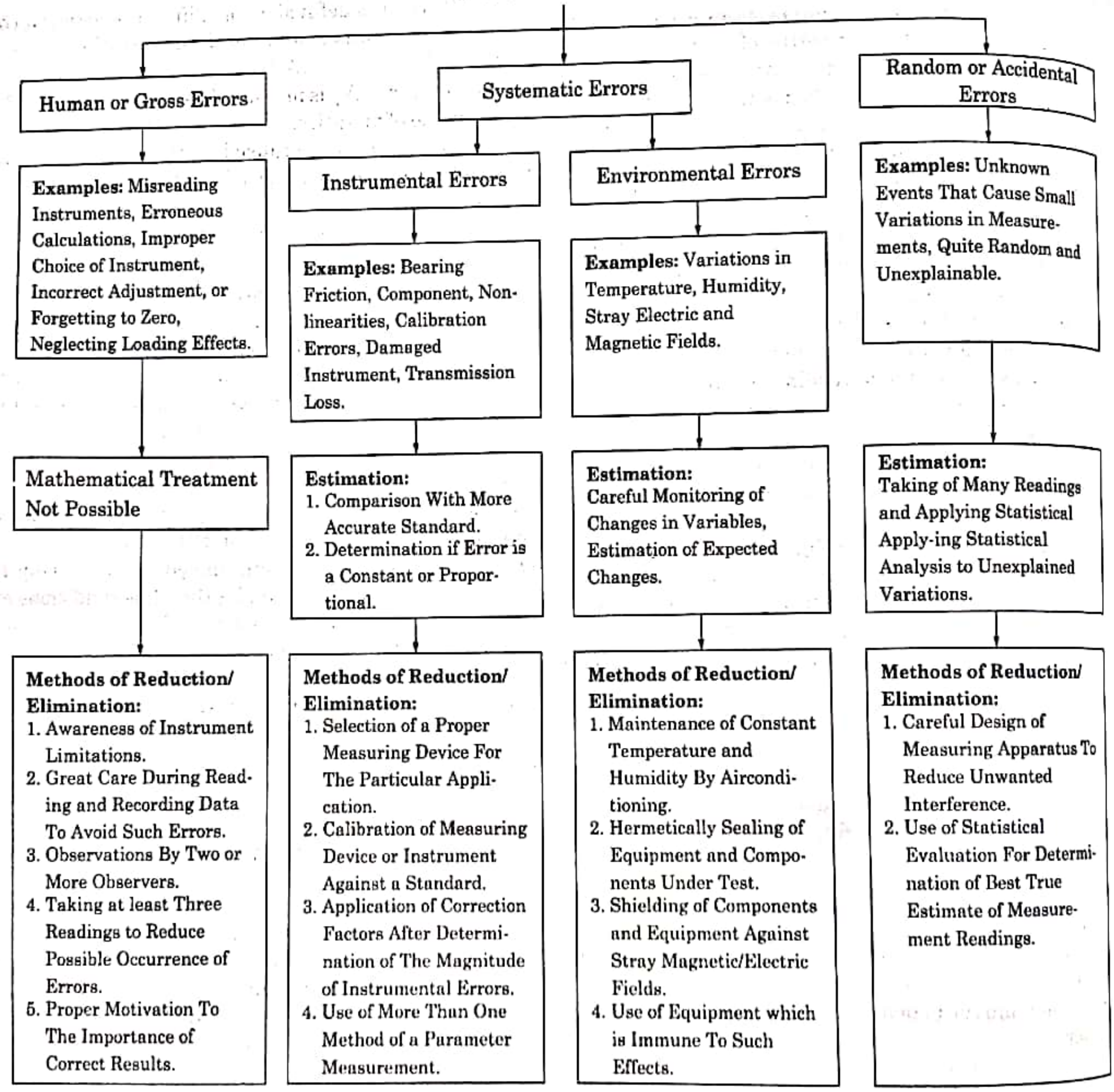
If the errors are truely random, a plot of the number of readings of each value as ordinates, against the reading values as abscisae gives a curve of a particular shape known as the *error curve*. If the plot does not give such a curve then the errors are not truely random. This test cannot, of course be applied to a set of as few, half a dozen.

The sources of errors, other than the ability of a piece of hardware to provide a true measurement, are given below:

1. Poor design.
2. Change in process parameters, irregularities, upsets etc.
3. Poor maintenance.
4. Certain design limitations.
5. Insufficient knowledge of process parameters and design conditions.
6. Errors caused by the person operating the instrument/equipment.

Table 3.1 classifies the main categories of errors and describes some causes and methods of their elimination.

TABLE 3.1
Measurement Errors
(Estimation and Reduction/Elimination)



3.6 DETERMINATION OF MAXIMUM SYSTEMATIC ERROR

When two or more quantities, each of which is subject to error, are combined, it is necessary to determine the maximum systematic error.

It should be noted particularly that when the only information about the errors of two pieces of apparatus is that they are within certain limits, they may be anywhere within these limits up to the extreme in either direction. In calculating the maximum possible systematic error it is necessary to assume that the individual errors connected may all be of such a sense as to affect the result in the same direction.

Provided that the errors are small, their effect on the final result is readily obtained from the simple rules outlined below:

(i) *Sum of Two or More Quantities.* Let the final result y be the sum of measured quantities, u, v, and z each of which is subjected to possible systematic error $\pm \delta u$, $\pm \delta v$, $\pm \delta z$.

Then the nominal result is

$$y = u + v + z$$

The relative increment of the function is given by

$$\frac{dy}{y} = \frac{d(u+v+z)}{y} = \frac{du}{y} + \frac{dv}{y} + \frac{dz}{y}$$

Expressing the result in terms of relative increment of the component quantities

$$\frac{dy}{y} = \frac{u}{y}\cdot\frac{du}{u} + \frac{v}{y}\cdot\frac{dv}{v} + \frac{z}{y}\cdot\frac{dz}{z}$$

Since the errors in the component quantities are represented by $\pm\delta u$, $\pm\delta v$ and $\pm\delta z$ then corresponding limiting error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y}\cdot\frac{\delta u}{u} + \frac{v}{y}\cdot\frac{\delta v}{v} + \frac{z}{y}\cdot\frac{\delta z}{z}\right) \quad ...(3.6)$$

The above expression shows that the resultant systematic error is equal to the sum of the products formed by multiplying the individual systematic errors by the ratio of each term to the function.

Since no approximation has been made in working out this particular result, it is true for all values of the errors and is not restricted to the case of small errors.

(ii) Difference of Two Quantities.

Let $y = u - v$

$$\frac{dy}{y} = \frac{du}{y} - \frac{dv}{y}$$

Expressing the result in terms of relative increments of component quantities

$$\frac{dy}{y} = \frac{u}{y}\cdot\frac{du}{u} - \frac{v}{y}\cdot\frac{dv}{v}$$

If the errors in u and v are $\pm\delta u$ and $\pm\delta v$ respectively, the signs may be interpreted to give the worst possible discrepancy i.e., when the error in u is $+\delta u$ and the error in v is $-\delta v$ and vice versa, then the corresponding relative limiting error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y}\cdot\frac{\delta u}{u} + \frac{v}{y}\cdot\frac{\delta v}{v}\right) \quad ...(3.7)$$

This expression is the same as obtained in first case. It may, however, be mentioned that in this case when u and v are almost equal in magnitude then the relative error in y would be very large.

(iii) Product of Two or More Quantities.

Let $y = uvz$

$$\log_e y = \log_e u + \log_e v + \log_e z$$

Differentiating with respect to y, we get

$$\frac{1}{y} = \frac{1}{u}\cdot\frac{du}{dy} + \frac{1}{v}\cdot\frac{dv}{dy} + \frac{1}{z}\cdot\frac{dz}{dy}$$

or $$\frac{dy}{y} = \frac{du}{u} + \frac{dv}{v} + \frac{dz}{z}$$

Representing the errors in u, v, and z as $\pm\delta u$, $\pm\delta v$ and $\pm\delta z$ respectively, the error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v} + \frac{\delta z}{z}\right) \quad ...(3.8)$$

From the above expression we conclude that the relative limiting error of the product of the terms is equal to the sum of the relative errors of the terms.

(iv) Quotient of Two Quantities.

Let $y = \frac{u}{v}$

$$\log_e y = \log_e u - \log_e v$$

Differentiating with respect to y, we have

$$\frac{1}{y} = \frac{1}{u}\cdot\frac{du}{dy} - \frac{1}{v}\cdot\frac{dv}{dy}$$

or $$\frac{dy}{y} = \frac{du}{u} - \frac{dv}{v}$$

Representing the errors in u and v as $\pm\delta u$ and $\pm\delta v$ respectively, the relative error in y is given by

$$\frac{\delta y}{y} = \pm\frac{\delta u}{u} \mp \frac{\delta v}{v}$$

The maximum possible error occurs when $\delta u/u$ is +ve and $\delta v/v$ is –ve or vice versa.

∴ Relative limiting error in y is given by the expression

$$\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v}\right) \quad ...(3.9)$$

(v) Power of a Factor

Let $y = u^n$

where n may be +ve or –ve, integral or fractional

$$\log_e y = n \log_e u$$

Differentiating with respect to y, we get

$$\frac{1}{y} = n\cdot\frac{1}{u}\cdot\frac{du}{dy}$$

or $$\frac{dy}{y} = n\frac{du}{u}$$

Hence the relative limiting error of y is

$$\frac{\delta y}{y} = \pm n\frac{\delta u}{u} \quad ...(3.10)$$

(vi) Composite Factors

Let $y = u^n v^m$

$$\log_e y = n \log_e u + m \log_e v$$

Differentiating with respect to y we get

$$\frac{1}{y} = \frac{n}{u}\frac{du}{dy} + \frac{m}{v}\frac{dv}{dy}$$

or $$\frac{dy}{y} = n\frac{du}{u} + m\frac{dv}{v}$$

∴ Relative limiting error of y is

$$\frac{\delta y}{y} = \pm\left(n\frac{\delta u}{u} + m\frac{\delta v}{v}\right) \quad ...(3.11)$$

Example 3.14. Explain the limiting error in terms of true value. Two capacitors $C_1 = 150 \pm 2.4$ μF, $C_2 = 120 \pm 1.5$ μF connected in parallel, what is the limiting error of the resultant capacitance C?

[*G.B. Technical Univ. Electronic Instrumentation and Measurements, 2011-12*]

Solution: We have $u = 150 \pm 2.4$ μF

$v = 120 \pm 1.5$ μF

When the two capacitors are connected in parallel, the resultant capacitance is

$$y = u + v = (150 \pm 2.4) + (120 \pm 1.5)$$
$$= (270 \pm 3.9)\ \mu F$$

Therefore, the limiting error is ± 3.9 μF **Ans.**

Relative limiting error is

$$\frac{\delta y}{y} = \pm\frac{3.9}{270} = \pm 0.0144 \text{ or } \pm 1.44\% \text{ Ans.}$$

Example 3.15. The limiting errors for a four dial resistance box are:

Units : ± 0.2%
Tens : ± 0.1%
Hundreds : ± 0.05%
Thousands : ± 0.02%

If the resistance value is set at 4,325 Ω, calculate the limiting error for this value.

[U.P.S.C. I.E.S. Electrical Engineering-I, 2007]

Solution: Error in thousands $= \pm\frac{0.02}{100}\times 4{,}000 = \pm 0.8\ \Omega$

Error in hundreds $= \pm\frac{0.05}{100}\times 300 = \pm 0.15\ \Omega$

Error in tens $= \pm\frac{0.1}{100}\times 20 = \pm 0.02\ \Omega$

Error in units $= \pm\frac{0.2}{100}\times 5 = \pm 0.01\ \Omega$

Total error $= \pm(0.8 + 0.15 + 0.02 + 0.01)$
$= \pm 0.98\ \Omega$

Limiting error $= \pm\frac{0.98}{4{,}325}\times 100$
$= \pm 0.02266\%$ **Ans.**

Example 3.16. Two resistors having the following ratings: $R_1 = 200\ \Omega \pm 10\%$ and $R_2 = 500 \pm 5\%$. Calculate:

(i) the magnitude of error in each resistor.
(ii) the limiting error in ohms when the resistors are connected in series.
(iii) the limiting error in ohms when the resistors are connected in parallel.

[J.N. Technological Univ. Hyderabad Electronic Measurements and Instrumentation, February/March 2012]

Solution: (*i*) Magnitude of error in resistor R_1,

$$\delta R_1 = \pm\frac{10}{100}\times 200 = \pm 20\ \Omega \quad \textbf{Ans.}$$

Magnitude of error in resistor R_2,

$$\delta R_2 = \pm\frac{5}{100}\times 500 = \pm 25\ \Omega \quad \textbf{Ans.}$$

(*ii*) When the two resistors are connected in series

Equivalent resistance,

$$R_{se} = R_1 + R_2 = 200 + 500 = 700\ \Omega$$

Limiting error, $\delta R = \delta R_1 + \delta R_2 = \pm 20\ \Omega \pm 25\ \Omega$
$= \pm 45\ \Omega$ **Ans.**

(*iii*) When the two resistors are connected in parallel

Equivalent resistance,

$$R_P = \frac{R_1\times R_2}{R_1+R_2} = \frac{200\times 500}{200+500} = \frac{1{,}000}{7}\ \Omega$$

Let $R_P = \frac{X}{Y}$

Then $X = R_1\times R_2 = 200\times 500 = 100{,}000$

$Y = R_1 + R_2 = 200 + 500 = 700$

Error in X $= \frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} = \pm 10 \pm 5 = \pm 15\%$

Error in Y $= \frac{\delta R_1}{Y} + \frac{\delta R_2}{Y}$

$$= \frac{R_1}{Y}\times\frac{\delta R_1}{R_1} + \frac{R_2}{Y}\times\frac{\delta R_2}{R_2}$$

$$= \pm\frac{200}{700}\times 10 \pm \frac{500}{700}\times 5 = \pm\frac{45}{7}\%$$

So percentage error (maximum possible) in equivalent parallel resistance

= Error in X + Error in Y

$$= \pm 15\% \pm \frac{45}{7}\% = \pm\frac{150}{7}\%$$

Limiting error in ohms $= \pm\frac{150}{700}\times\frac{1{,}000}{7} = \pm 30.6122\ \Omega$ **Ans.**

Example 3.17. Three resistors have the following ratings:

$R_1 = 200\ \Omega \pm 5\%$, $R_2 = 100\ \Omega \pm 5\%$, $R_3 = 50\ \Omega \pm 5\%$

Determine the magnitude of resultant resistance and limiting errors in percentage and ohms, if the above resistances are connected in (*a*) series and (*b*) parallel.

[U.P. Technical Univ. Electrical and Electronics Measurements and Instruments, 2013-14]

Solution: (*i*) When the resistances are connected in series

Equivalent resistance,

$R_{se} = R_1 + R_2 + R_3$
$= 200 + 100 + 50 = 350\ \Omega$ **Ans.**

Relative limiting error of series resistances in percentage

$$= \frac{R_1}{R_{se}}\cdot\frac{\delta R_1}{R_1} + \frac{R_2}{R_{se}}\cdot\frac{\delta R_2}{R_2} + \frac{R_3}{R_{se}}\cdot\frac{\delta R_3}{R_3}$$

$$= \pm\left(\frac{200}{350}\times 5 + \frac{100}{350}\times 5 + \frac{50}{350}\times 5\right) = \pm 5\% \quad \textbf{Ans.}$$

Relative limiting error of series equivalent resistance in ohms

$$= \pm 350 \times \frac{5}{100} = \pm 17.5\ \Omega \quad \textbf{Ans.}$$

(*ii*) When the resistances are connected in parallel

The equivalent resistance is given by the expression

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}$$

$$R_p = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$$

$$= \frac{200\times 100\times 50}{100\times 50 + 200\times 50 + 200\times 100} = 28.57\ \Omega$$

Let $R_p = \frac{X}{Y}$

then $X = R_1R_2R_3 = 200\times 100\times 50 = 10{,}00{,}000$

and $Y = R_1R_2 + R_2R_3 + R_3R_1$
$= y_1 + y_2 + y_3$
$= 200\times 100 + 100\times 50 + 50\times 200 = 35{,}000$

$$\text{Error in X} = \frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} = \pm(5+5+5) = \pm 15\%$$

$$\text{Error in } y_1 = \frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} = \pm(5+5) = 10\%$$

$$\text{Error in } y_2 = \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} = \pm(5+5) = 10\%$$

$$\text{Error in } y_3 = \frac{\delta R_3}{R_3} + \frac{\delta R_1}{R_1} = \pm(5+5) = \pm 10\%$$

Percentage error in Y

$$= \left(\frac{y_1}{Y}\frac{\delta y_1}{y_1} + \frac{y_2}{Y}\frac{\delta y_2}{y_2} + \frac{y_3}{Y}\frac{\delta y_3}{y_3}\right) \times 100$$

$$= \pm\left(\frac{20,000}{35,000} \times 10 + \frac{5,000}{35,000} \times 10 + \frac{10,000}{35,000} \times 10\right)$$

$$= \pm 10\%$$

∴ Percentage error (maximum possible) in equivalent parallel resistance

$$= 15 + 10 = \pm 25\% \quad \textbf{Ans.}$$

Error (maximum possible in equivalent parallel resistance in ohms)

$$= 28.57 \times \frac{25}{100} = 7.1425\ \Omega \quad \textbf{Ans.}$$

Example 3.18. A resistor is measured by the voltmeter-ammeter method. The voltmeter reading is 123.4 V on the 250 V scale and the ammeter reading is 283.5 mA on the 500 mA scale. Both meters are guaranteed to be accurate within ±1 per cent of full-scale reading. Calculate (a) the indicated value of the resistance (b) the limits within which the result can be guaranteed.

[R.G.P.V. Electronic Instrumentation, December-2005; G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2012-13]

Solution: (*a*) Indicated value of resistance,

$$R = \frac{V}{I} = \frac{123.4}{283.5 \times 10^{-3}} = 435.27\ \Omega \quad \textbf{Ans.}$$

The magnitude of limiting error of the voltmeter,

$$\delta V = \varepsilon_r V = \pm\frac{1}{100} \times 250 = \pm 2.5\ V$$

The magnitude of voltage under measurement,

$V = 123.4$ V

The percentage limiting error at this voltage

$$= \pm\frac{2.5}{123.4} \times 100 = \pm 2.0259\%$$

The magnitude of limiting error of the ammeter,

$$\delta I = \pm\frac{1}{100} \times 500 = \pm 5\ mA$$

The magnitude of current under measurement,

$I = 283.5$ mA

The percentage limiting error at this current

$$= \pm\frac{5\ mA}{283.5\ mA} \times 100 = \pm 1.7637\%$$

(*b*) Relative limiting error in resistance measurement,

$$\frac{\delta R}{R} \times 100 = \pm\left[\frac{\delta V}{V} + \frac{\delta I}{I}\right] \times 100$$

$$= \pm(2.0259 + 1.7637) = \pm 3.7896\% \quad \textbf{Ans.}$$

Example 3.19. The value of unknown R is calculated in terms of current drawn by it and the power it dissipates. If the limiting error involved in the measurement of power and current are 1.5% and 1% respectively. The relative limiting error in the measurement of R is?

[G.B. Technical Univ. Electronic Instrumentation and Measurements, 2012-13]

Solution: Unknown resistance R is given as

$$R = \frac{P}{I^2}$$

Relative limiting error in measurement of R

$$\frac{\delta R}{R} \times 100 = \pm\left(\frac{\delta P}{P} + \frac{2\delta I}{I}\right) \times 100$$

$$= \pm(1.5 + 2 \times 1) = \pm 3.5\% \quad \textbf{Ans.}$$

Example 3.20. An 820 Ω resistance with an accuracy of ±10% carries a current of 10 mA. The current was measured by an analog ammeter, on a 25 mA range, with an accuracy of ±2% of full-scale. Calculate the power dissipated in the resistor and determine the accuracy of the result. *[G.B. Technical Univ. Electronic Instrumentation and Measurements, 2012-13]*

Solution: The magnitude of limiting error of the ammeter,

$$\delta I = \varepsilon_r I = \pm\frac{2}{100} \times 25 = \pm 0.5\ mA$$

The magnitude of current under measurement, $I = 10$ mA

The percentage limiting error at this current

$$= \frac{\delta I}{I} \times 100 = \pm\frac{0.5}{10} \times 100 = \pm 5\% \quad \textbf{Ans.}$$

The percentage limiting in given resistor = ± 10%

Power dissipated in resistor,

$$P = I^2 R = \left(\frac{10}{1,000}\right)^2 \times 820 = 0.082\ W \quad \textbf{Ans.}$$

Percentage error

$$\frac{\delta P}{P} \times 100 = \pm\left[\frac{2\delta I}{I} + \frac{\delta R}{R}\right] \times 100$$

$$= \pm(2 \times 5 + 10) = \pm 20\% \quad \textbf{Ans.}$$

Percentage accuracy = (100 − 20)% = 80% **Ans.**

Example 3.21. Calculate the relative error in power factor if the relative error in power, current and voltage are respectively 0.5%, 1% and 1%. Quantities have SI units.

[Pb. Technical Univ. Electrical Measurements and Measuring Instruments, May-2008]

Solution: Power factor is given by

$$PF = \frac{P}{VI}$$

Relative limiting error in power factor,

$$\frac{\delta PF}{PF} \times 100 = \pm\left[\frac{\delta P}{P} + \frac{\delta V}{V} + \frac{\delta I}{I}\right] \times 100$$

$$= \pm(0.5 + 1 + 1) = \pm 2.5\% \quad \textbf{Ans.}$$

Example 3.22. The unknown resistance is determined by expression:

$$R_X = \frac{R_2 R_3}{R_1}$$

where $R_1 = 900 \pm 0.5\%\ \Omega$; $R_2 = 900 + 0.8\%\ \Omega$;

$R_3 = 825 \pm 0.6\%\ \Omega$

Determine the magnitude of R_X and limiting error in %age. [M.D. Univ. Electrical Measurements and Measuring Instruments, May-2009]

Solution: Unknown resistance,

$$R_X = \frac{R_2 R_3}{R_1} = \frac{900 \times 825}{900} = 825\ \Omega \quad \textbf{Ans.}$$

Relative limiting error in percentage,

$$\frac{\delta R_X}{R_X} \times 100 = \pm\left[\frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3}\right] \times 100$$

$$= \pm(0.5 + 0.8 + 0.6) = \pm 1.9\% \quad \textbf{Ans.}$$

Example 3.23. **If $R_x = \frac{R_1 R_2}{R_3}$ where $R_1 = 100 \pm 1\%$, $R_2 = 200 \pm 2.5\%$ and $R_3 = 100 \pm 2\%$. Find:**

(i) The nominal value

(ii) The limiting error and

(iii) The percentage limiting error of R_x

[U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2013-14]

Solution: (i) Nominal value of R_x

$$= \frac{R_1 \times R_2}{R_3} = \frac{100 \times 200}{100} = 200\ \Omega \quad \textbf{Ans.}$$

(ii) Limiting error $= \pm\left[\frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} \pm \frac{\delta R_3}{R_3}\right] \times R_x$ ohms

$$= \pm\left(\frac{1}{100} + \frac{2.5}{100} + \frac{2}{100}\right) \times 200 = \pm 11\ \Omega$$

(iii) Percentage limiting error

$$= \pm\frac{11}{200} \times 100 = \pm 5.5 \quad \textbf{Ans.}$$

Example 3.24. **$\mu = \frac{\pi r^4 (P_1 - P_2)}{8Ql}$ determines the dimensions of μ (r and l are radius and length, P_1 and P_2 are pressures and Q is flow. If $r = (0.5 \pm 0.01)$ mm; $P_1 = (200 \pm 3)$ kPa; $P_2 = (150 \pm 2)$ kPa; $Q = 4 \times 10^{-7}\ m^3/s$ and $l = 1$ m. Calculate the absolute error in μ.**

[U.P.S.C. I.E.S. Electrical Engineering-I, 2003]

Solution:

$$\mu = \frac{\pi r^4 (p_1 - p_2)}{8Ql}$$

$$= \frac{\pi (0.5 \times 10^{-3})^4 \times (200 - 150) \times 10^3}{8 \times 4 \times 10^{-7} \times 1}$$

$$= 30.68 \times 10^{-4}\ \text{kg/m-s}$$

$$\frac{\delta r}{r} = \pm\frac{0.01}{0.5} \times 100 = \pm 2\%;$$

$$\frac{\delta P_1}{P_1} = \pm\frac{3}{200} \times 100 = \pm 1.5\%;$$

$$\frac{\delta P_2}{P_2} = \pm\frac{2}{150} \times 100 = \pm 1.333\%$$

$$\frac{\delta Q}{Q} = 0 \quad \text{and} \quad \frac{\delta l}{l} = 0$$

Let $P = P_1 - P_2 = 200 - 150 = 50$ kPa

Percentage error in P

$$= \pm\left[\frac{P_1}{P} \times \frac{\delta P_1}{P_1} + \frac{P_2}{P} \times \frac{\delta P_2}{P_2}\right] \times 100$$

$$= \pm\left[\frac{200}{50} \times 1.5 + \frac{150}{50} \times 1.333\right] = \pm 10\%$$

Percentage error in μ

$$= \pm\left[4\frac{\delta r}{r} + \frac{\delta p}{p} + \frac{\delta Q}{Q} + \frac{\delta l}{l}\right]$$

$$= \pm(4 \times 2 + 10 + 0 + 0) = \pm 18\%$$

Absolute error in μ

$$= \pm\mu \times \frac{\delta\mu}{\mu} = \pm 30.68 \times 10^{-4} \times \frac{18}{100}$$

$$= \pm 5.5224 \times 10^{-4}\ \text{kg/m-s} \quad \textbf{Ans.}$$

Example 3.25. **The unknown inductance is determined by Anderson bridge and is given by the expression**

$$L_x = \frac{CP[r(Q+S) + Q \cdot S]}{S}$$

where $C = 1\ \mu F \pm 1.0\%$; $P = 1{,}000\ \Omega \pm 0.4\%$; $Q = 2{,}000\ \Omega \pm 1.0\%$; $r = 200\ \Omega \pm 0.5\%$ and $S = 2{,}000\ \Omega \pm 0.5\%$.

Determine the magnitude of unknown inductance in Henry and limiting error in per cent.

[U.P. Technical Univ. Elec. Measurements & Measuring Instruments 2001-02]

Solution: Unknown inductance,

$$L_x = \frac{CP}{S}[r(Q+S) + QS]$$

$$= \frac{1 \times 10^{-6} \times 1{,}000}{2{,}000} \times [200\,(2{,}000 + 2{,}000) + 2{,}000 \times 2{,}000]$$

$$= 0.5 \times 10^{-6} \times [0.8 \times 10^6 + 4 \times 10^6]$$

$$= 2.4\ \text{Henry} \quad \textbf{Ans.}$$

Let $u = Q + S = 2{,}000 + 2{,}000 = 4{,}000\ \Omega$

Percentage error in u

$$= \left[\frac{Q}{u} \cdot \frac{\delta Q}{Q} + \frac{S}{u} \cdot \frac{\delta S}{S}\right] = \pm\left[\frac{2{,}000}{4{,}000} \times 1.0 + \frac{2{,}000}{4{,}000} \times 0.5\right]$$

$$= \pm 0.75\%$$

Let $v = r(Q+S) = ru = 200 \times (2{,}000 + 2{,}000) = 0.8 \times 10^6$

Percentage error in v

$$= \frac{\delta r}{r} + \frac{\delta u}{u} = \pm 0.5 \pm 0.75 = \pm 1.25\%$$

Let $x = QS = 2{,}000 \times 2{,}000 = 4 \times 10^6$

Percentage error in x

$$= \frac{\delta Q}{Q} + \frac{\delta S}{S} = \pm 1.0 \pm 0.5 = \pm 1.5\%$$

Let $y = r(Q+S) + QS = v + x = 0.8 \times 10^6 + 4 \times 10^6$

$$= 4.8 \times 10^6$$

Percentage error in y

$$= \left[\frac{v}{y} \cdot \frac{\delta v}{v} + \frac{x}{y} \cdot \frac{\delta x}{x}\right]$$

$$= \pm\left[\frac{0.8 \times 10^6}{4.8 \times 10^6} \times 1.25 + \frac{4 \times 10^6}{4.8 \times 10^6} \times 1.5\right] = \pm 1.458\%$$

Percentage error in inductance

$$L_x = \frac{\delta C}{C} + \frac{\delta P}{P} + \frac{\delta S}{S} + \frac{\delta y}{y} = \pm 1.0 \pm 0.4 \pm 0.5 \pm 1.458$$

Example 3.26. The impedance of R-L circuit operating on ac is given by:

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

The resistance R is known to be 100 Ω with percentage error of 5%, L is known to be 2H with percentage error of 10% and ω is known exactly 2π × 50. Determine the percentage error in the measurement of Z.

Solution:

Let $u = R^2 = 100^2 = 10{,}000$

Percentage error in u

$$= \frac{2\,\delta R}{R} = 2 \times 5 = 10\%$$

Let $v = \omega^2 L^2 = (2\pi \times 50)^2 \times 2^2 = 394{,}784$

Percentage error in v

$$= \frac{2\,\delta L}{L} = 2 \times 10 = 20\%$$

Let $x = u + v = R^2 + \omega^2 L^2$

$$= (100)^2 + (2\pi \times 50)^2 \times 2^2 = 404{,}784$$

Percentage error in x

$$= \frac{u}{x}\cdot\frac{\delta u}{u} + \frac{v}{x}\cdot\frac{\delta v}{v}$$

$$= \left[\frac{10{,}000}{404{,}784} \times 10 + \frac{394{,}784}{404{,}784} \times 20\right]$$

$$\cong 0.24 + 19.5 = 19.74\%$$

Now $Z = x^{1/2}$

So $\frac{\delta Z}{Z} = \pm \frac{1}{2}\frac{\delta x}{x} = \pm \frac{1}{2} \times 19.74 = 9.87\%$ **Ans.**

3.7 STATISTICAL ANALYSIS

No measurement is made with 100 per cent accuracy and, therefore, there is always some error, which varies from one determination to another, and gets introduced in the value of the quantity under measurement. It is a function of statistics to separate, as far as possible, the truth from error by narrowing and defining the region of doubt. But statistical study is mainly concerned with precision of measurement and so it cannot remove systematic errors from set of data. So systematic errors should be small as compared with residual or random errors.

To make statistical methods and interpretations meaningful, a large number of measurements is usually required.

Sometimes simple approach is required for describing and summarizing the results of the measurements. Some of these methods are described below.

(i) Arithmetic Mean. The most probable value of a measured variable is the arithmetic mean of the number of readings taken. Theoretically the best approximate value will be obtained when number of observations of the quantity under measurement is infinite but in practice, only a finite number of observations can be made. The arithmetic mean is given by the following expression

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots x_n}{n} = \frac{\Sigma x}{n} \qquad \ldots(3.12)$$

where $\bar{x}$ is arithmetic mean and $x_1, x_2, x_3, \ldots x_n$ are the readings taken and n is the number of readings taken.

(ii) Deviation from the Mean. The deviation of a reading is the amount by which it differs from the mean. If we have a set of readings $x_1, x_2, x_3 \ldots$ with mean $\bar{x}$, the deviations of the individual readings are

$$\text{Deviation of } x_1 = d_1 = x_1 - \bar{x}$$
$$\text{Deviation of } x_2 = d_2 = x_2 - \bar{x} \qquad \ldots(3.13)$$

Deviation from the mean may have a + ve or – ve value but the algebraic sum of all the deviations is always zero.

(iii) Average Deviation. Average deviation is the sum of the scaler values (without sign) of the deviations divided by the number of readings. Average deviation may be expressed as

$$D = \frac{|d_1| + |d_2| + |d_3| + \ldots |d_n|}{n}$$

$$= \frac{\Sigma |d|}{n} \qquad \ldots(3.14)$$

Average deviation gives an indication of the precision of the instruments used in carrying out measurements. Low average deviation between readings shows that instruments used for measurements are highly precise.

(iv) Standard Deviation. The standard deviation of an infinite number of data is the square root of the sum of all the individual deviations squared, divided by the number of readings.

Standard deviation,

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots d_n^2}{n}} = \sqrt{\frac{\Sigma d^2}{n}} \qquad \ldots(3.15)$$

The standard deviation is also known as *root mean square deviation*, and is the most important factor in the statistical analysis of measurement data. Reduction in this quantity effectively means improvement in measurement.

In practice, the possible number of observations is finite. When the number of readings exceeds 20, the standard deviation is denoted by σ but if it is less than 20 the symbol 's' is used to denote the same. The standard deviation of a finite number of observations is given as

$$s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots d_n^2}{n-1}} = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad \ldots(3.16)$$

Another expression for essentially the same quantity is the *variance* or *mean square deviation*, which is same as the standard deviation except that the square root is not extracted.

So, variance, V = Mean square deviation = σ^2 ...(3.17)

Variance is a convenient quantity for use in many computations because variances are additive. The standard deviation, however, has the advantage of being of the same units as the variable, making easy to compare magnitudes. Nowadays most scientific results are expressed in terms of standard deviation.

(v) Standard Deviation of Mean. When we have a multiple sample data, it is evident that the mean of various sets of data can be analyzed by statistical means. This may be accomplished by taking standard deviation of the mean given as

$$\sigma_m = \frac{\sigma}{\sqrt{n}} \quad ...(3.18)$$

(vi) Standard Deviation of Standard Deviation. For a multiple sample data, the standard deviation of the standard deviation is given as

$$\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}} \quad ...(3.19)$$

3.8 CHARACTERISTICS OF EXPERIMENTAL DATA

During measurement of any quantity, scattered data is obtained and this variation can be controlled by taking all care in all manipulations and by holding conditions as steady as possible during the period of measurement. But even with maximum care an unavoidable uncertainty remains. So no measurement can be carried out with absolute definiteness and as the measurements are made closer to limits, presence of smaller disturbances becomes more evident.

All known errors from data such as known systematic effects, calibration etc., should be removed first, before applying statistical methods as they are based on laws of chance, and not on consistent factors. Statistical analysis allows us to determine the best value possible from the given data and set the limits of uncertainty inherent in the scatter of the data.

The distribution of data in a set of readings may be presented in several ways, one of which is a block diagram or *histogram*. Table 3.2 shows a set of 60 current readings, that were taken at small intervals and recorded to the nearest of hundredth of an ampere. The nominal value of measured current is 10.00 A.

TABLE 3.2 *Tabulation of Current Readings*

Current Reading in Amperes	*Number of Readings*
9.97	1
9.98	3
9.99	13
10.00	23
10.01	15
10.02	4
10.03	1
Total	60

In Fig. 3.2, a histogram for Table 3.2 is shown, in which the number of readings are plotted against each observed current reading.

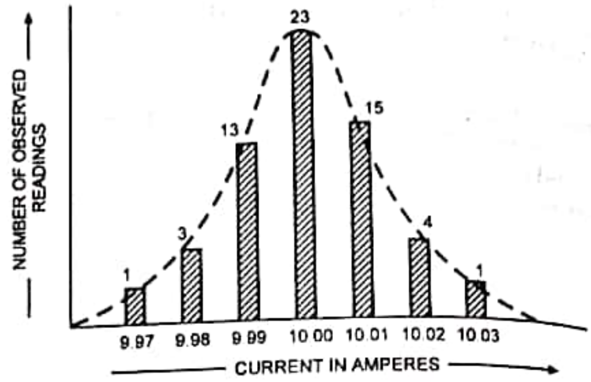


Fig. 3.2

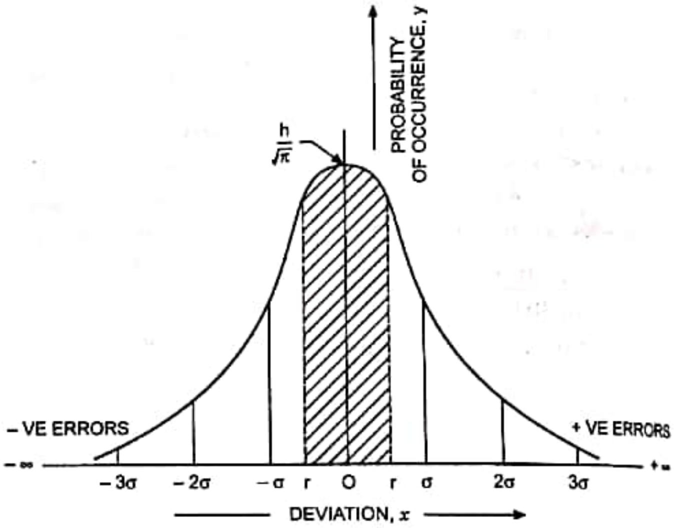


Fig. 3.3 *Curve For Normal Law of Error*

From the figure it is obvious that the largest number of readings (23) occurs at the central value of 10.00 A while the other readings are placed more or less symmetrically on either side of central value. If more readings are taken at smaller increments of 0.005 A intervals (200 readings), then the distribution of observations will remain approximately symmetrical about the central value and the shape of the histogram will be approximately the same as before. With more and more data taken at smaller and smaller increments, the contour of the histogram will finally become a smooth curve, as shown in Fig. 3.2 by the dashed line. The bell shaped curve is known as a *Gaussian curve*. The sharper and narrower the curve, the more definitely an observer may state that the most probable value of the true reading is the central value or mean value.

The normal, or Gaussian, law of errors is the basis for the major part of study of random effects.

3.9 NORMAL LAW OF ERROR

When measurement of a quantity is carried out, the determinations are always finite and limited in number. Random effects cancel each other completely in an infinite set of measurements but it is not true for a small set of measurements. Hence in a limited set of measurements.

mean of sample is not necessarily the mean of the larger set, and the standard deviation as obtained from the limited set of measurements may not be the standard deviation of a larger sample or of the universe. Generally precision increases with the size of the sample but the means and the standard deviation of various samples have the property of scatter around the universe values.

The distribution between limited set and universe is a basic concept in statistical study. It was considered formerly that reliable results could be obtained only from a large sample but advances have been made in the field of small sample theory, so that more precise conclusions can be drawn from a small sample.

A probability distribution, expresses the likelihood of a particular event or of a deviation of a particular amount. A type of variation may be characterised by a curve or equation which gives the relative probability of a particular result, or of a deviation of a given amount. There are several types of variations that occur in statistical work for different purposes and 'normal law of error' is one of them and is the basis for the major part of the study of random effects.

The assumptions made for the normal law of error are as follows:

(i) All observations include a large number of small random disturbing effects or random errors.
(ii) These random errors can be either positive or negative.
(iii) There is an equal probability of positive and negative random errors.

So it can be expected that measurement observations include + ve and – ve errors in more or less equal amounts so that the total error will be small and the mean value will be the true value of the measured variable.

The possibilities as to the form of the error distribution curve can be stated as follows:

(i) Small errors are more probable than large errors.
(ii) Large errors are very improbable.
(iii) There is an equal probability of positive and negative errors so that the probability of a given error will be symmetrical about the zero value.

A normal curve for error distribution is shown in Fig. 3.3 in which it can be seen that errors are distributed symmetrically. The normal curve may be regarded as the limiting form of a histogram shown in Fig. 3.2, when the data becomes increasingly large, gathered in smaller and smaller bands.

One form of the equation for the normal law of error may be written in terms of a simple equation. This equation is,

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2} \quad ...(3.20)$$

where x is the magnitude of deviation from mean, y is the probability of occurrence of deviation x (number of readings at any deviation x) and h is a constant often referred as *precision index*.

The above Eq. (3.20) leads to a curve of the form shown in Fig. 3.3, which may be seen in a general way to match the requirements of the probability behaviour as outlined in the assumptions given above. The abscissa represents the magnitude of deviation from the central value, and the ordinate is the probability of the occurrence in an infinite set of an observation with a deviation of this magnitude. The presence of the square, x^2 in the exponent provides symmetry for plus and minus deviations, and provides flatness of slope at the origin.

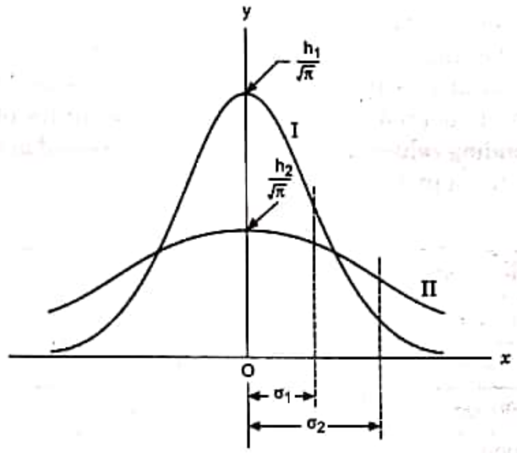


Fig. 3.4 *Comparison of Two Frequency Distribution Curves With Different Degrees of Dispersion*

The parameter h reveals the spread or scatter of the measured values about O. The larger the value of h, the greater peak the distribution curve has. A larger h implies higher precision and consistency in measurement and hence it is treated as *precision index*. For $x = 0$, the ordinate $y = h/\sqrt{\pi}$. If the two curves shown in Fig. 3.4 are compared, it is noticed that the curve with the larger value of h has a greater central probability, and drops off more rapidly with increase of x. Curve II has a lower h, a lower central value, and a slower drop-off at the sides. (Both curves I and II have the same unit area under them). In summary, a larger value of h means a more closely grouped set of observations, with smaller dispersion, and should, therefore, be considered as a better set, with respect to the control of random effects.

The value of h is given as

$$h = \frac{1}{\sigma\sqrt{2}} \quad ...(3.21)$$

If we substitute $h = \frac{1}{\sigma\sqrt{2}}$ in Eq. (3.20), we have

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} \quad ...(3.22)$$

This form of equation is particularly useful, as σ is the quantity we ordinarily know and are interested in. σ is a quantity of the same units as the observed quantity and its mean. This makes it easy to visualize the dispersion in comparison with the mean.

3.10 PROBABLE ERROR

The area under the Gaussian probability curve shown in Fig. 3.3 within certain limits represents the number of cases among the observations within those deviation limits, expressed as a fraction of the total cases. The area between the limits $-\infty$ to $+\infty$ represents the entire number of observations and taken as unity.

The area under the curve between $-\sigma$ and $+\sigma$ limits represents the number of observations that differ from the mean by no more than the standard deviation. This area can be determined by integration in series, and for normally dispersed data, following the Gaussian distribution, the value is found to be 0.68, so 68 per cent of the cases for the normally dispersed data lie between the limits of $\pm\sigma$. Corresponding values of other deviations, expressed in terms of σ, are given in Table 3.3.

TABLE 3.3

Deviation (±)	*Fraction of Total Area Included*
0.6745σ	0.5000
1.0000σ	0.6828
2.0000σ	0.9546
3.000σ	0.9972
1.96σ	0.9500

If, for example, a large number of capacitors having nominal value of 10 μF are measured and the mean value is found to be 10.00 μF with a standard deviation of 0.02 μF, then we know that on an average 68% of all the capacitors have values lying between limits of ± 0.02 μF of the mean. There is then approximately a two to one chance that any capacitor, selected from the lot at random, will lie within these limits. If larger odds are required, then deviation may be extended to a limit of $\pm 2\sigma = \pm 0.04$ F. According to Table 3.3, this now includes 95% of all the cases, giving 21 to 1 odds and any capacitor selected at random lies within ± 0.04 μF of the mean value of 10.00 μF.

If ordinates are erected at deviations of $\pm r = 0.6745\,\sigma$, as shown in Fig. 3.3, half the area under the curve is enclosed between these limits. The quantity r is called the *probable error (PE)*. This value is probable, as shown, in the sense that there is an even chance that any one observation will have a random error no greater than $\pm r$.

The *probability of occurrence* can be stated in terms of *odds* which is the number of chances that a particular reading will occur when the error limit is specified. The odds can be determined as follows:

$$\text{Probability of occurrence} = \frac{\text{Odds}}{\text{Odds} + 1} \quad \text{...(3.23)}$$

The odds that the observation lies between $\pm\sigma$ limits are

$$\frac{\text{Odds}}{\text{Odds} + 1} = 0.6828$$

$$\text{or Odds} = 2.15 : 1$$

Probable error has been used in experimental work to some extent but standard deviation is more convenient in statistical work and is preferred.

3.11 MEASUREMENT DATA SPECIFICATION

After making statistical analysis of multi-sample data, the results of the measurements are to be specified. The results are expressed as deviations about a mean value. The deviations may be expressed as follows:

1. ***Standard Deviation.*** The result is expressed as $\bar{x} \pm \sigma$. The error limit in this case is the standard deviation which means that 68.28% (or about two-thirds) of all the readings have values which lie between limits of $\pm\sigma$ and the odds are 2.15 to 1. Thus there is approximately a two to one chance (or possibility) that a new observation will be within these limits.

2. ***Probable Error.*** The result is expressed as $\bar{x} \pm r$ or $\bar{x} \pm 0.6745\sigma$. It means that 50% or half of all the readings lie within these limits and odds are 1 to 1. There is an even chance that any one observation will be within these limits.

3. ***± 2σ Limits.*** The result is expressed as $\bar{x} \pm 2\sigma$. In this case, probability range is increased, approximately 95 per cent of all the readings fall within these limits and odds are 21 to 1.

4. ***± 3σ Limits.*** The result is expressed as $\bar{x} \pm 3\sigma$. The probability in this case is 0.9972 which means that 99.72% of all the readings fall within these limits i.e. practically all the readings are included in these limits. The odds of any observation falling within these limits are 356 to 1.

Example 3.27. The following set of 10 measurements was recorded during an experiment. Calculate the precision of fourth measurement.

Measurement No.	1	2	3	4	5	6	7	8	9	10
Quantity	98	102	101	97	100	103	98	106	107	99

[*U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2006-07*]

Solution: Arithmetic mean of the set of 10 measurements

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}{10}$$

$$= \frac{98 + 102 + 101 + 97 + 100 + 103 + 98 + 106 + 107 + 99}{10}$$

$$= \frac{1011}{10} = 101.1$$

Precision for fourth measurement

$$= 1 - \left|\frac{x_4 - \bar{x}}{\bar{x}}\right| = 1 - \left|\frac{97 - 101.1}{101.1}\right|$$

$$= 1 - 0.04055 = 0.95945 \quad \textbf{Ans.}$$

Example 3.28. A circuit was tuned for resonance by eight different students, and the values of resonant frequency in kHz were recorded as 532, 548, 543, 535, 546, 531, 543

and 536. Calculate (i) the arithmetic mean (ii) deviations from mean (iii) the average deviation (iv) the standard deviation and (v) variance.

[*Rajasthan Technical Univ. Electronic Measurements and Instrumentation, 2006-207; U.P.S.C. I.E.S. Electrical Engineering-I, 2013*]

Solution: (*i*) Arithmetic mean,

$$\bar{x} = \frac{532 + 548 + 543 + 535 + 546 + 531 + 543 + 536}{8}$$

$$= \frac{4{,}314}{8} = 539.25 \text{ kHz} \quad \textbf{Ans.}$$

(*ii*) Deviations from mean,

$$\left.\begin{aligned} d_1 &= 532 - 539.25 = -7.25 \text{ kHz}\\ d_2 &= 548 - 539.25 = +8.75 \text{ kHz}\\ d_3 &= 543 - 539.25 = +3.75 \text{ kHz}\\ d_4 &= 535 - 539.25 = -4.25 \text{ kHz}\\ d_5 &= 546 - 539.25 = +6.75 \text{ kHz}\\ d_6 &= 531 - 539.25 = -8.25 \text{ kHz}\\ d_7 &= 543 - 539.25 = +3.75 \text{ kHz}\\ d_8 &= 536 - 539.25 = -3.25 \text{ kHz}\end{aligned}\right\} \textbf{Ans.}$$

(*iii*) Average deviation,

$$D = \frac{\Sigma |d|}{n}$$

$$= \frac{7.25 + 8.75 + 3.75 + 4.25 + 6.75 + 8.25 + 3.75 + 3.25}{8}$$

$$= \frac{46.00}{8} = 5.75 \text{ kHz} \quad \textbf{Ans.}$$

(*iv*) Standard variation,

$$s = \sqrt{\frac{\Sigma d^2}{n-1}}$$ because number of readings is 8, which is less than 20

$$= \sqrt{\frac{(-7.25)^2 + (+8.75)^2 + (3.75)^2 + (-4.25)^2 + (+6.75)^2 + (-8.25)^2 + (+3.75)^2 + (-3.25)^2}{8-1}}$$

$= 6.54$ kHz **Ans.**

(*v*) Variance, $V = s^2 = (6.54)^2 = 42.772 \text{ (kHz)}^2$ **Ans.**

Example 3.29. The following 10 observations were recorded when measuring a voltage:

1	2	3	4	5	6	7	8	9	10
41.7	42	41.8	42	42.1	41.9	42.5	42	41.9	41.8

Find (*i*) mean (*ii*) standard deviation (*iii*) probable error of one reading. [*UP.S.C. I.E.S. Elec. Engineering-I, 2003*]

Solution: (*i*) Arithmetic mean,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}{10}$$

$$= \frac{41.7 + 42 + 41.8 + 42 + 42.1 + 41.9 + 42.5 + 42 + 41.9 + 41.8}{10}$$

$$= \frac{419.7}{10} = 41.97 \quad \textbf{Ans.}$$

Deviations from the mean

$$\begin{aligned} d_1 &= 41.7 - 41.97 = -0.27\\ d_2 &= 42 \;\; - 41.97 = +0.03\\ d_3 &= 41.8 - 41.97 = -0.17\\ d_4 &= 42 \;\; - 41.97 = +0.03\\ d_5 &= 42.1 - 41.97 = +0.13\\ d_6 &= 41.9 - 41.97 = -0.07\\ d_7 &= 42.5 - 41.97 = +0.53\\ d_8 &= 42 \;\; - 41.97 = +0.03\\ d_9 &= 41.9 - 41.97 = -0.07\\ d_{10} &= 41.8 - 41.97 = -0.17\end{aligned}$$

(*ii*) Since the number of reading is 10, which is less than 20, the standard deviation is calculated from the equation

$$s = \sqrt{\frac{\Sigma d^2}{n-1}}$$

Standard deviation,

$$s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2 + d_8^2 + d_9^2 + d_{10}^2}{10-1}}$$

$$= \sqrt{\frac{(-0.27)^2 + (+0.03)^2 + (-0.17)^2 + (+0.03)^2 + (+0.13)^2 + (-0.07)^2 + (+0.53)^2 + (+0.03)^2 + (-0.07)^2 + (-0.17)^2}{9}}$$

$$= \sqrt{\frac{0.4410}{9}} = 0.221 \quad \textbf{Ans.}$$

(*iii*) Probable error of one reading,

$r = 0.6745 \times s = 0.6745 \times 0.221 = 0.149$ **Ans.**

Example 3.30. The following 10 observations were recorded when measuring a voltage:

41.7, 42.0, 41.8, 42.0, 42.1, 41.9, 42.0, 41.9, 42.5, 41.8.

Find: (*i*) the mean, (*ii*) the standard deviation, (*iii*) the probable error of one reading, (*iv*) the probable error of mean and (*v*) range.

[*Pb. Univ. Elec. Measurements-I, January 1991*]

Solution:

(*i*) Arithmetic mean, $\bar{x}$ = 41.97 V **Ans.**

(*ii*) Standard deviation, s = 0.221 V **Ans.** } As worked out in Example 3.29

(*iii*) Probable error of one reading,

$r = 0.6745\, s = 0.6745 \times 0.221 = 0.149$ V **Ans.**

(*iv*) Probable error of mean,

$$r_m = \frac{r}{\sqrt{n-1}} = \frac{0.149}{\sqrt{10-1}} = 0.0497 \text{ V} \quad \textbf{Ans.}$$

(*v*) Range = 42.5 − 41.7 = 0.8 V **Ans.**

Example 3.31. A set of independent 10 measurements were made to determine the weight of a lead shot. The weights in gramme were:

1.570, 1.597, 1.591, 1.562, 1.577, 1.580, 1.564, 1.586, 1.550, 1.575.

Determine the (*i*) arithmetic mean, (*ii*) average deviation, (*iii*) standard deviation, (*iv*) variance, (*v*) probable error of one reading, (*vi*) probable error of the mean.

[*Pb. Univ. Elec. Measurements-I, July 1993*]

Solution: (*i*) Arithmetic mean,

$$\bar{x} = \frac{1.570 + 1.597 + 1.591 + 1.562 + 1.577 + 1.580 + 1.564 + 1.586 + 1.550 + 1.575}{10}$$

≈ 1.5752 gramme **Ans.**

Deviations from the mean

$d_1 = 1.570 - 1.5752 = -0.0052$

$d_2 = 1.597 - 1.5752 = +0.0218$

$d_3 = 1.591 - 1.5752 = +0.0158$

$d_4 = 1.562 - 1.5752 = -0.0132$

$d_5 = 1.577 - 1.5752 = +0.0018$

$d_6 = 1.580 - 1.5752 = +0.0048$

$d_7 = 1.564 - 1.5752 = -0.0112$

$d_8 = 1.586 - 1.5752 = +0.0108$

$d_9 = 1.550 - 1.5752 = -0.0252$

$d_{10} = 1.575 - 1.5752 = -0.0002$

(*ii*) Average deviation,

$$D = \frac{0.0052 + 0.0218 + 0.0158 + 0.0132 + 0.0018 + 0.0048 + 0.0112 + 0.0108 + 0.0252 + 0.0002}{10}$$

$= 0.011$ gramme **Ans.**

(*iii*) Since the number of readings is 10 which is less than 20, standard deviation is calculated by the following equation

$$s = \sqrt{\frac{\Sigma d^2}{n-1}}$$

Standard deviation,

$$s = \sqrt{\frac{(-0.0052)^2 + (+0.0218)^2 + (+0.0158)^2 + (-0.0132)^2 + (+0.0018)^2 + (+0.0048)^2 + (-0.0112)^2 + (+0.0108)^2 + (-0.0252)^2 + (-0.0002)^2}{10-1}}$$

$= 0.01426$ gramme **Ans.**

(*iv*) Variance,

$V = (s)^2 = (0.01426)^2 = 2.033 \times 10^{-4}$ gramme^2 **Ans.**

(*v*) Probable error,

$r = 0.6745s = 0.6745 \times 0.01426 = 0.0096$ gramme **Ans.**

(*vi*) Probable error of mean,

$$r_m = \frac{r}{\sqrt{n-1}} = \frac{0.0096}{\sqrt{10-1}} = 0.0032 \text{ gramme} \quad \textbf{Ans.}$$

3.12 CONFIDENCE LEVEL

As mentioned in Art. 3.10, the area under the normal distribution curve between the limits $-\infty$ to $+\infty$ represents the entire number of observations and taken as unity. In actual practice, we usually specify a certain range of acceptable values of scatter or dispersion from the mean and find the probability that the measured values lie in that range. This probability can be evaluated by determination of area under the normal distribution curve in the specified range. This probability expressed in a percentage is termed as *confidence level*. A range of deviation, from the mean value within which a certain fraction of all values is expected to lie, is called the *confidence interval*.

For instance, the probability of occurrence of the measured value x in the range $(\bar{X} + \sigma)$ to $(\bar{X} - \sigma)$ obviously, is the area under the normal distribution curve in the specified range of $(\bar{X} + \sigma)$ to $(\bar{X} - \sigma)$. Using Table 3.3 we find that the integral Gaussian probability in the range of $(\bar{X} \pm \sigma)$ is 0.6828. Thus, it can be said that chances are better than 2 : 1 (68.28 : 31.72 or 2.15 : 1) that the measured value lie in the range of $(\bar{X} \pm \sigma)$. Alternatively, it can be said that the confidence level in the confidence interval of $(\bar{X} \pm \sigma)$ is 68.28%.

The percentage probability of error is defined as 100 minus the confidence level. In general, we accept errors up to 5% i.e. 95% confidence level for which confidence interval is $\bar{X} \pm 1.96\sigma$. However, where human life is involved, we insists on low probability of error, of the order of 1 per cent. This gives a confidence level of 99%, for which confidence interval is $(\bar{X} + 2.576\sigma)$.

If the number of observations is large and the errors involved are random and follow the normal Gaussian distribution, the various confidence intervals about the mean value $\bar{X}$ are given below in tabular form (Table 3.4).

TABLE 3.4

Confidence Level	*Confidence Interval*	*Values Lying Outside Confidence Interval*
0.500	$\bar{X} \pm 0.674\sigma$	1 in 2
0.800	$\bar{X} \pm 1.282\sigma$	1 in 5
0.900	$\bar{X} \pm 1.645\sigma$	1 in 10
0.950	$\bar{X} \pm 1.960\sigma$	1 in 20
0.990	$\bar{X} \pm 2.576\sigma$	1 in 100
0.999	$\bar{X} \pm 3.291\sigma$	1 in 1000

In case the number of observations is small (say less than 20) and the standard deviation is not accurately known, the confidence intervals are to be broadened.

Here the standard deviation s, determined from Eq. (3.16) is multiplied by a suitable factor for establishment of confidence interval.

In order to have confidence intervals for mean of a group of observations from the corresponding intervals for an individual observation, the later is divided by $\sqrt{n}$ i.e.

Confidence interval of mean

$$= \frac{\text{Confidence interval of individual observation}}{\sqrt{n}} \quad ...(3.24)$$

Example 3.32. A 100.00 V reference dc source is used for calibration of a digital multimeter in the 200 V range. The uncertainty quoted by a lab of higher echelon in national calibration network in the calibration certificate of reference standard is ± 0.01 V. The observations made during calibrations are 100.2, 100.3, 100.2 and 100.1 V respectively. Find the assigned value and uncertainty associated with measurements (Assume a confidence level 95% and corresponding student factor, *t* = 2.78).

[U.P.S.C. I.E.S. Electrical Engineering-I, 2004]

Solution: The four observations including uncertainty of ±0.01 V are 100.2 ±0.01 V, 100.3 ±0.01 V, 100.2 ±0.01 V and 100.1 ±0.01 V.

i.e.	100.21 V	100.19 V
	100.31 V	100.29 V
	100.21 V	100.19 V
	100.11 V	100.09 V

$$\text{Arithmetic mean, } \bar{X} = \frac{100.21 + 100.31 + 100.21 + 100.11}{4} = 100.21 \text{ V}$$

$$\text{and } \bar{X} = \frac{100.19 + 100.29 + 100.19 + 100.09}{4} = 100.19 \text{ V}$$

Assuming arithmetic mean,

$$\bar{X} = 100.21$$

Deviations from the mean,

$$d_1 = 100.21 - 100.21 = 0$$
$$d_2 = 100.31 - 100.21 = +0.10$$
$$d_3 = 100.21 - 100.21 = 0$$
$$d_4 = 100.11 - 100.21 = -0.10$$

$$\text{Standard deviation, } s = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{0^2 + (+0.10)^2 + 0^2 + (-0.10)^2}{4-1}} = \sqrt{\frac{0.02}{3}} = 0.08165 \text{ V}$$

Standard deviation of mean

$$= \frac{s}{\sqrt{n}} = \frac{0.08165}{\sqrt{4}} = 0.040825 \text{ V}$$

For a confidence level of 95%, student factor $t = 2.78$

True mean of voltage under measurement lies between the limits of

$\bar{X} \pm 2.78 \times$ standard deviation of mean

$$= 100.21 \pm 2.78 \times 0.040825$$
$$= 100.21 \pm 0.1135$$

The assigned value is 100.21 and the uncertainty limits lie between 100.0965 V and 1̸0.3235 V. **Ans.**

3.13 DATA REJECTION

In most of the practicals, it is found that some of the data points are significantly different from the majority of the data. In case such data points were obtained under abnormal conditions involving gross blunders and the person performing experiment is sure about their dubious nature, they can be discarded straight away. However, a data cannot be rejected simply on the ground that it is different from the others. We must follow certain standard mathematical method for rejecting/retaining any experimental data. The commonly used methods are:

1. 3σ limits
2. Use of confidence intervals
3. Chauvenet's criterion

1. Data Rejection Based Upon ±3σ Limits. As already mentioned in Art. 3.11, the probability that the reading will be within ±3σ limits of central value is 0.9972 which is very high. So any reading not lying within ±3σ limits can be discarded straight away.

2. Data Rejection Based Upon Confidence Intervals. In this method, a data point having deviation from the mean exceeding four times the probable error of a single reading is discarded. It means discarding of a data outside a confidence interval for a single reading at a confidence level of 0.993.

A better criterion is of discarding a data that lies outside the interval corresponding to confidence level of 0.99 for a single observation. This criterion does not involve the evaluation of probable error when the set of data points are small and standard deviation is not accurately known. In this method, not more than 1 reading in 100 would lie outside this range.

A still better criterion is to use the confidence interval corresponding to a confidence level of 0.95 so as to scrutinize the measurement procedure used.

3. Chauvenet's Criterion. In this method, it is assumed that the number of observations made is large enough that the results follow a normal Gaussian distribution. We may make use of this distribution for computation of probability that a given reading will deviate by a certain amount from the mean. This criterion specifies that a reading may be rejected if the probability of obtaining the particular deviation from the mean is less than $1/2n$. The values of the ratio of deviation to standard deviation for different values of n, according to this criterion, are given below in tabular form (Table 3.5). The reading that has the ratio of its deviation to the standard deviation exceeding the limits given in Table 3.5 is discarded (Refer to Example 3.33).

TABLE 3.5 *Chauvenets Criterion For Data Rejection*

Number of Readings	*Ratio of Maximum Acceptable Deviation To Standard Deviation, d_{max}/σ*
2	1.15
3	1.38
4	1.54
5	1.65
6	1.73
7	1.80
10	1.96
15	2.13
25	2.33
50	2.57
100	2.81
300	3.14
500	3.29
1000	3.48

Example 3.33. The following 10 observations were recorded when measuring a voltage.

1	2	3	4	5	6	7	8	9	10
41.7	42	41.8	42	42.1	41.9	42.5	42	41.9	41.8

Point out any reading that can be rejected by applying Chauvenet's criterion.

Solution: Arithmetic mean $\bar{x}$, deviations from the mean d_1, d_2, ..., d_{10}, standard deviation, as worked out in Example 3.29 are as follows:

$$\bar{x} = 41.97;$$

Deviations from the means are:

$d_1 = -0.27$, $d_2 = +0.03$, $d_3 = -0.17$, $d_4 = +0.03$, $d_5 = +0.13$, $d_6 = -0.07$, $d_7 = +0.53$, $d_8 = +0.03$, $d_9 = -0.07$ and $d_{10} = -0.17$

Standard deviation, $s = 0.221$

The ratios of deviation to standard deviation for different observations are computed as below:

$$\frac{|d_1|}{s} = \frac{0.27}{0.221} = 1.222$$

$$\frac{|d_2|}{s} = \frac{|d_4|}{s} = \frac{|d_8|}{s} = \frac{0.03}{0.221} = 0.136$$

$$\frac{|d_3|}{s} = \frac{|d_{10}|}{s} = \frac{0.17}{0.221} = 0.769$$

$$\frac{|d_5|}{s} = \frac{0.13}{0.221} = 0.588$$

$$\frac{|d_6|}{s} = \frac{|d_9|}{s} = \frac{0.07}{0.221} = 0.317$$

$$\frac{|d_7|}{s} = \frac{0.53}{0.221} = 2.398$$

Assuming that the ratio of maximum deviation to standard deviation should not exceed 1.96 (Refer to Table 3.5, according to which for 10 readings the ratio of deviation to standard deviation is not to exceed 1.96), the reading no. 7 i.e. 42.5 having ratio of deviation to standard deviation as 2.398 is to be discarded.

3.14 COMBINATIONS OF VARIANCES, STANDARD DEVIATIONS AND PROBABLE ERRORS OF COMPONENTS

3.14.1. Combination of Variances. If X is a function of several component variables, each of which is subject to random effects, then we have

$$X = f(x_1, x_2, \ldots x_n)$$

and if $x_1, x_2, \ldots x_n$ are independent variates, then for small variations in $x_1, x_2, \ldots$ from their mean value, denoted by $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$, the resulting variation of X from its mean value for any one determination is given as

$$\Delta X = \frac{\partial X}{\partial x_1}\Delta x_1 + \frac{\partial X}{\partial x_2}\Delta x_2 + \ldots \quad \ldots(3.25)$$

ignoring differentials of higher order, where Δx_1 ... are the variations occurring in that particular determination.

By squaring this equation, we have

$$(\Delta X)^2 = \left(\frac{\partial X}{\partial x_1}\right)^2 (\Delta x_1)^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 (\Delta x_2)^2 + \ldots + 2\left(\frac{\partial X}{\partial x_1}\right)\left(\frac{\partial X}{\partial x_2}\right)(\Delta x_1 + \Delta x_2) + \ldots \quad \ldots(3.26)$$

Now, if the variations of x_1, x_2 etc., are independent, as assumed, positive values of one increment are equally likely to be associated with positive or negative values of other increments, so the sum of the cross product terms tends to be zero in repeated observations. By definition of variance V as mean-square error, the mean of $(\Delta X)^2$ for repeated observations becomes the variance of X, denoted by V_x, and, therefore, we may write

$$V_x = \left(\frac{\partial X}{\partial x_1}\right)^2 V_{x_1} + \left(\frac{\partial X}{\partial x_2}\right)^2 V_{x_2} + \ldots \left(\frac{\partial X}{\partial x_n}\right)^2 V_{x_n} \quad \ldots(3.27)$$

since in repeated measurements $(\Delta x_1)^2$ tends to the mean value V_{x1} etc.

Thus the above equation may be written as

$$V_x = V_{x_1} + V_{x_2} + \ldots + V_{x_n} \quad \ldots(3.28)$$

This shows that the component variances are additive, with the weighting factors $\left(\frac{\partial X}{\partial x_1}\right)^2$, etc., which express the relative influence of the various components on the combined function. The weighted variance can be written as

$$V_{x_1} = \left(\frac{\partial X}{\partial x_1}\right)^2 V_{x_1} \quad \ldots(3.29)$$

3.14.2. Combinations of Standard Deviations. The standard deviation of X may be determined from Eq. (3.27) and may be expressed either in variances, or in the standard deviations, of the components. Thus the standard deviation of X is σ_x and may be expressed as

$$\sigma_x = \sqrt{V_x}$$

$$= \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 V_{x_1} + \left(\frac{\partial X}{\partial x_2}\right)^2 V_{x_2} + \ldots + \left(\frac{\partial X}{\partial x_n}\right)^2 V_{x_n}}$$

$$= \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \ldots \left(\frac{\partial X}{\partial x_n}\right)^2 \sigma_{x_n}^2} \quad \ldots(3.30)$$

From above Eq. (3.30), it is obvious that both component standard deviations are additive with weighing factors $\left(\frac{\partial X}{\partial x_1}\right)^2$ etc. which express the relative influence of the various components on the combined function.

Thus we can write

$$\sigma_x = \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 + \ldots \sigma_{x_n}^2} \quad \ldots(3.31)$$

where σ_{x_1} is the weighted standard deviation of x_1,

$$\sigma_{x_1} = \left(\frac{\partial X}{\partial x_1}\right)^2 \sigma_{x_1} \quad \ldots(3.32)$$

The above expressions are valid only if component quantities $x_1, x_2, ..., x_n$ are independent of each other and that the increments are small, so that terms of order higher than the first may be neglected. The latter condition is usually not a serious limitation in engineering applications, in which it is generally possible to keep random effects under fairly close control.

3.14.3. Combinations of Probable Errors. As we have seen that probable error (PE) is proportional to standard deviation and is given as $r = 0.6745\,\sigma$, therefore, probable error in X from Eq. (3.30) is given as

$$r_x^2 = \left(\frac{\partial X}{\partial x_1}\right)^2 r_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 r_{x_2}^2 + ... + \left(\frac{\partial X}{\partial x_n}\right)^2 r_{x_n}^2 \quad ...(3.33)$$

where r_{x_1}, r_{x_2} etc. are the probable errors of x_1, x_2 etc.

The contribution of probable error of x_1 to the total error in X is $\left(\frac{\partial X}{\partial x_1}\right)^2 r_{x_1}^2$ and this contribution may be written in another form as $r_{x_1}^2$. Thus the Eq. (3.33) may be written as

$$r_x = \sqrt{r_{x_1}^2 + r_{x_2}^2 + ... + r_{x_n}^2} \quad ...(3.34)$$

where the weighted probable error of x_n becomes as

$$r_{x_n} = \left(\frac{\partial X}{\partial x_n}\right)^2 r_{x_n} \quad ...(3.35)$$

Example 3.34. A resistance is determined by voltmeter-ammeter method. The voltmeter reads 100 V with a probable error of ± 12 V and the ammeter reads 10 A with a probable error of ± 2 A. Determine the value of probable error in the calculated value of resistance.

Solution: By Ohm's law, resistance $R = \frac{\text{Voltage V}}{\text{Current I}}$

So weighted probable error in the resistance due to voltage V is,

$$r_{RV} = \frac{\partial R}{\partial V} r_V = I^{-1} r_V = \frac{r_V}{I} = \pm\frac{12}{10} = \pm 1.2\ \Omega$$

Similarly weighted probable error in the resistance due to current I is,

$$r_{RI} = \frac{\partial R}{\partial I} r_I = \frac{V}{I^2} r_I = \frac{100}{(10)^2} \times (\pm 2) = \pm 2\ \Omega$$

From Eq. (3.34), probable error in the calculated resistance is,

$$r_R = \sqrt{(r_{RV})^2 + (r_{RI})^2} = \sqrt{(1.2)^2 + (2)^2}$$

$$= \pm 2.332\ \Omega \quad \textbf{Ans.}$$

3.15 PROPAGATION OF UNCERTAINTIES

The analysis for uncertainties in measurements, when many variables are involved is carried out on the same basis as is carried out for error analysis when the results are expressed as standard deviations or probable errors.

Let X be a function of several variables i.e.

$$X = f(x_1, x_2, ... x_n)$$

where $x_1, x_2, ... x_n$ are independent variables with the same degree of odds.

Let u_x be the effective uncertainty and $u_{x_1}, u_{x_2}, ... u_{x_n}$ be the uncertainties in the independent variables $x_1, x_2 ... x_n$ respectively. The uncertainty in the result will be as given by equation

$$u_x = \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 (u_{x_1})^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 (u_{x_2})^2 + ... \left(\frac{\partial X}{\partial x_n}\right)^2 (u_{x_n})^2} \quad ...(3.36)$$

The above expression is known as *quadratic error* or *propagation law* for incidental errors. In contrast to the linear error propagation law which holds good for controllable errors the above law applies to all errors with $r \pm$ sign and which are equally likely to be positive or negative.

Example 3.35. A certain resistor has a voltage drop of 110.2 V and a current of 5.3 A. The uncertainties in the measurements are ± 0.2 V and ± 0.06 A respectively. Calculate the power dissipated in the resistor and the uncertainty in power.

[U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2006-07]

Solution: Power dissipated, $P = V \times I$ ...(i)

$= 110.2 \times 5.3 \cong 584.06$ W **Ans.**

Differentiating partially Eq. (i) w.r.t V and I we have

$$\frac{\partial P}{\partial V} = I$$

$$\frac{\partial P}{\partial I} = V$$

Resultant uncertainty,

$$u_x = \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 u_V^2 + \left(\frac{\partial P}{\partial I}\right)^2 u_I^2}$$

$$= \sqrt{I^2 u_V^2 + V^2 u_I^2}$$

Substituting VI = P in above equation we have

$$u_x = \sqrt{P^2\left(\frac{u_V}{V}\right)^2 + P^2\left(\frac{u_I}{I}\right)^2}$$

or $$\frac{u_x}{P} \times 100 = \sqrt{\left(\frac{u_V}{V}\right)^2 + P^2\left(\frac{u_I}{I}\right)^2} \times 100$$

$$= \sqrt{\left(\frac{0.2}{110.2}\right)^2 + \left(\frac{0.06}{5.3}\right)^2} \times 100$$

$$= \pm 1.1465\% \quad \textbf{Ans.}$$

Example 3.36. A power factor of a circuit is determined by $\cos\phi = \frac{P}{VI}$ where P is the power in watts, V is the voltage in volts and I is the current in amperes. The relative error in power, current and voltage are respectively

± 0.5%, ± 1% and ±1%. Calculate the relative error in power factor. Also calculate the uncertainty in power factor if the errors were specified as uncertainties.

[U.P.S.C. I.E.S. Electrical Engineering-I, 2011]

Solution: Power factor,

$$\cos\phi = \frac{P}{V \times I} \quad ...(i)$$

Relative limiting error of power factor

$$= \pm\left[\frac{\delta P}{P} + \frac{\delta V}{V} + \frac{\delta I}{I}\right] \times 100$$

$$= \pm [0.5 + 1 + 1] = \pm 2.5\% \quad \text{Ans.}$$

Differentiating partially expression (i) w.r.t. P, V and I we get

$$\frac{\partial \cos\phi}{\partial P} = \frac{1}{VI} \quad ...(ii)$$

$$\frac{\partial \cos\phi}{\partial V} = \frac{-P}{V^2 I} \quad ...(iii)$$

and $$\frac{\partial \cos\phi}{\partial I} = \frac{-P}{VI^2} \quad ...(iv)$$

Resultant uncertainty,

$$u_x = \sqrt{\left(\frac{\partial \cos\phi}{\partial P}\right)^2 u_P^2 + \left(\frac{\partial \cos\phi}{\partial V}\right)^2 u_V^2 + \left(\frac{\partial \cos\phi}{\partial I}\right)^2 u_I^2}$$

$$= \sqrt{\left(\frac{1}{VI}\right)^2 u_P^2 + \left(\frac{-P}{V^2 I}\right)^2 u_V^2 + \left(\frac{-P}{VI^2}\right)^2 u_I^2}$$

Substituting $\frac{P}{VI} = \cos\phi$ in above equation, we have

$$u_x = \sqrt{(\cos\phi)^2\left(\frac{u_P}{P}\right)^2 + (\cos\phi)^2\left(\frac{u_V}{V}\right)^2 + (\cos\phi)^2\left(\frac{u_I}{I}\right)^2}$$

or $$\frac{u_x}{\cos\phi} \times 100 = \sqrt{\left(\frac{u_P}{P}\right)^2 + \left(\frac{u_V}{V}\right)^2 + \left(\frac{u_I}{I}\right)^2}$$

$$= \sqrt{(0.5)^2 + 1^2 + 1^2} = \pm 1.5\% \quad \text{Ans.}$$

Example 3.37. Resistors R_1 and R_2 have the nominal values 3 Ω and 6 Ω and tolerance of ± 10% and ± 5% respectively. What would be the tolerance in the equivalent resistance value when R_1 and R_2 are connected in parallel.

[U.P.S.C. I.E.S. Electrical Engineering-I, 2002]

Solution: Resistance $R_1 = (3\ \Omega \pm 10\%) = (3 + 0.3)\ \Omega$

Resistance $R_2 = (6\ \Omega \pm 5\%) = (6 + 0.3)\ \Omega$

Resultant resistance, $R = \frac{R_1 R_2}{R_1 + R_2}$

Differentiating partially w.r.t R_1 and R_2 respectively, we have

$$\frac{\partial R_1}{\partial R} = \frac{R_2}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2} = \frac{6}{3+6} - \frac{6 \times 3}{(6+3)^2} = 0.4444$$

$$\frac{\partial R_2}{\partial R} = \frac{R_1}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2} = \frac{3}{3+6} - \frac{6 \times 3}{(6+3)^2} = 0.1111$$

Hence uncertainty in equivalent resistance,

$$u_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 u_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 u_{R_2}^2}$$

$$= \pm\sqrt{(0.4444)^2 \times 0.3^2 + (0.1111)^2 \times 0.3^2}$$

$$= \pm 0.1374\ \Omega$$

Effective resistance in parallel

$$= \frac{R_1 R_2}{R_1 + R_2} \pm u_R = \frac{3 \times 6}{3 + 6} \pm 0.1374 = (2 \pm 0.1374)\ \Omega$$

Example 3.38. Two sets of large number of 20 kΩ and 30 kΩ resistors are used to make a large number of 12 kΩ and 50 kΩ resistors choosing one from each group. If the standard deviations of the two sets of resistors of 20 kΩ and 30 kΩ are respectively 5% and 10%. Find the standard deviations of the combined resistor sets of 12 kΩ and 50 kΩ. [U.P.S.C. I.E.S. Electrical Engineering-I, 2010]

Solution: Let R_1 = 20 kΩ and R_2 = 30 kΩ

Standard deviation of 20 kΩ resistor,

$$\sigma_{R_1} = \frac{5}{100} \times 20 \cong 1\ k\Omega$$

Standard deviation of 30 kΩ resistor,

$$\sigma_{R_2} = \frac{10}{100} \times 30 = 3\ k\Omega$$

Resistance of 50 kΩ is made by series combination of resistances R_1 and R_2

So $R_{se} = R_1 + R_2 = 20 + 30 = 50\ k\Omega$

Differentiating partially w.r.t. R_1 and R_2 respectively we have

$$\frac{\partial R_{se}}{\partial R_1} = 1$$

and $$\frac{\partial R_{se}}{\partial R_2} = 1$$

Standard deviation of 50 kΩ resistance,

$$\sigma_{R_{se}} = \sqrt{\left(\frac{\partial R_{se}}{\partial R_1}\right)^2 \sigma_{R_1}^2 + \left(\frac{\partial R_{se}}{\partial R_2}\right)^2 \sigma_{R_2}^2}$$

$$= \sqrt{1^2 \times 1^2 + 1^2 \times 3^2} = \pm 3.16\ k\Omega$$

or $$\%\sigma_{R_{se}} = \pm\frac{3.16}{50} \times 100 = \pm 6.32\% \quad \text{Ans.}$$

Resistance of 12 kΩ is made by parallel combination of resistances R_1 and R_2

So $$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12\ k\Omega$$

Differentiating partially w.r.t. R_1 and R_2 respectively we have

$$\frac{\partial R_P}{\partial R_1} = \frac{R_2}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2}$$

$$= \frac{30}{20 + 30} - \frac{20 \times 30}{(20 + 30)^2} = 0.36$$

$$\frac{\partial R_P}{\partial R_2} = \frac{R_1}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2}$$

$$= \frac{20}{20 + 30} - \frac{20 \times 30}{(20 + 30)^2} = 0.16$$

Standard deviation of 12 kΩ resistance,

$$\sigma_{R_P} = \sqrt{\left(\frac{\partial R_P}{\partial R_1}\right)^2 \sigma_{R_1}^2 + \left(\frac{\partial R_P}{\partial R_2}\right)^2 \sigma_{R_2}^2}$$

$$= \sqrt{0.36^2 \times 1^2 + 0.16^2 \times 3^2}$$

$$= \sqrt{0.36} = \pm 0.6 \text{ k}\Omega$$

Percentage standard deviation of 12 kΩ resistance

$$= \pm \frac{0.6}{12} \times 100 = \pm 5\% \quad \textbf{Ans.}$$

3.16 SPECIFICATIONS OF INSTRUMENTS AND THEIR SIGNIFICANCE

In the purchase of instruments, the user frequently has some choice between competing makes, and may like to go either for the best possible performance within a certain price range or for an instrument which satisfies certain minimum requirements at the lowest possible cost. In either case it is desirable to use a performance specification to ensure that the instrument purchased adequately meets his needs. While the needs of the purchasers vary widely, depending upon the use to which the instrument is to be put, consideration should be given to a number of factors which describe the instrument's appearance, its performance under operating conditions, and its ability to withstand the various hazards to which one may reasonably expect it to be subjected. These factors include such items as appearance and workmanship of the case, mounting dimensions and interchangeability (in case of a panel or switch board instrument), length of scale, rated accuracy of indications, response time and damping of the moving system, effects of various influences (such as temperature, external field and position), power consumption, ability to withstand shock or vibration, and effect of humidity on operation.

The writing of such specifications demands skill and judgement, and a considerable amount of technical knowledge about the instruments as well as about the requirements of the possible applications involved.

EXERCISES

1. Explain the terms "accuracy", "sensitivity" and "resolution" as used for indicating instruments.
[J.N. Technological Univ. Hyderabad Electronic Measurements & Instrumentation, February/March-2012]
2. Define accuracy, precision, absolute error and relative accuracy of a measurement. [U.P.S.C. I.E.S. Electronics and Telecommunication Engineering-I, 2009]
3. Define and explain: (*i*) Accuracy (*ii*) Precision (*iii*) Resolution (*iv*) Linearity. [R.G. Technical Univ. Electronic Instrumentation, December-2010]
4. What do you understand by the terms "Accuracy" and "Precision"? How the two differ from each other?
[G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2011-12]
5. What is error of an instrument? Discuss about various types of "errors" in instrument.
[U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2013-14]
6. Explain the different types of errors that may occur in measurements. [U.P.S.C. I.E.S. Elec. Engineering-I, 1993]
7. What are the different types of errors? Describe their sources briefly. [U.P.S.C. I.E.S. Elec. Engineering-I, 1997]
8. What are the various types of errors occurring in electrical measurements? Explain them.
[G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2012-13]
9. Explain about different types of errors that occur in instruments. How they can be minimized?
[J.N. Technological Univ. Hyderabad Electronic Measurements and Instrumentation, May-2011]
10. What are different types of systematic errors? Discuss.
[U.P. Technical Univ. Electrical and Electronics Measurements and Instruments, 2013-14]
11. Define random errors and systematic errors.
[M.D. Univ. Electrical Measurements and Measuring Instruments, May-2007]
12. Define random errors and explain how are they analysed statistically. [Pb. Univ. Elec. Measurement-I, December 1991; June-1993]
13. How the limiting error is calculated when
(*i*) Two or more variables are added.
(*ii*) Product of two or more variable.
(*iii*) Composite factor.
[G.B. Technical Univ. Electronic Instrumentation and Measurements, 2012-13]
14. What is meant by arithmetic mean, average deviation and standard deviation?
[G.B. Technical Univ. Electronic Instrumentation and Measurements, 2010-11]
15. Define: (*i*) mean value (*ii*) deviation (*iii*) variance.
[U.P. Technical Univ. Electronic Measurements and Instrumentation, 2009-10]
16. Explain with a histogram, the normal distribution of errors. What is probable error and what is its significance?
[U.P.S.C. I.E.S. Electrical Engineering-I, 2012]
17. Define the following for Gaussian distribution of data: (*i*) Precision index (*ii*) Probable error (*iii*) Standard deviation of mean (*iv*) Standard deviation of standard deviation.
[Rajasthan Technical Univ. Electronic Measurements and Instrumentation, Februry-2011]
18. Differentiate giving suitable examples:
(*i*) Probable errors (*ii*) random errors, (*iii*) systematic errors.
[Pb. Univ. Elec. Measurements-I, December 1992]
19. Define probable errors and explain how they are statistically dealt with. [Pb. Univ. Elec. Measurements-I, January 1991]
20. Explain in detail the limiting and probable errors in measurement giving suitable examples for both.
[Rajasthan Technical Univ. Electronic Measurements and Instrumentation, 2007]
21. Differentiate between the probable errors and random errors. [M.D. Univ. Electrical Measurements and Measuring Instruments, December-2009]
22. Give the meanings of the following terms:
(*i*) Precision (*ii*) Accuracy (*iii*) Standard deviation, and (*iv*) Probable error. [U.P.S.C. I.E.S. Elec. Engineering-I, 1994]

SHORT ANSWER TYPE QUESTIONS WITH ANSWERS

Q. 1. What is meant by an absolute error of measurement?

Ans. The difference between the measured value A_m and the true value A of the unknown quantity or measurand is known as the absolute error of measurement δA i.e.

$$\delta A = A_m - A$$

Q. 2. What is meant by reading correction and how is it related to absolute error?

Ans. The difference of true value A and measured value A_m of the measurand is known as the reading correction. Absolute error and reading correction are of the same magnitude but are of opposite sign.

Q. 3. What is difference between absolute error and relative error? **[U.P. Technical Univ., 2013-14]**

Ans. Absolute error δA is equal to the difference of the measured value A_m and the true value A of the measurand.

i.e. $\delta A = A_m - A$.

But relative error is equal to the ratio of absolute error to the true value of the quantity under measurement.

i.e. Relative error, $\varepsilon_r = \dfrac{\delta A}{A} = \dfrac{\varepsilon_0}{A} = \dfrac{\text{Absolute error}}{\text{True value}}$.

Q. 4. How is relative error expressed?

Ans. Relative error is defined as the ratio of absolute error to the true value of the measurand and is expressed in fraction or percentage.

Q. 5. What is meant by limiting error?

Ans. The limits of the deviations from the specified value, as mentioned by the manufacturer of the equipment/apparatus is known as limiting error.

Q. 6. Define the term resolution. **[Pb. Technical Univ., December-2004]**

Ans. The resolution of any instrument is the smallest change in the input signal (quantity under measurement) which can be detected by the instrument. It may be expressed as an actual value or as a fraction or percentage of the full-scale value.

Q. 7. Explain the difference between Accuracy and Precision. **[M.D. Univ., December-2010]**

OR

Differentiate between Accuracy and Precision. **[Pb. Technical Univ., May-2009]**

OR

What is the difference between Accuracy and Precision of a measuring instrument? **[U.P.S.C. I.E.S. ETE-I, 2007]**

Ans. Accuracy is defined as the degree of exactness (closeness) of a measurement compared to the expected (desired) value, whereas precision is a measure of the consistency or repeatability of measurements, i.e. successive readings do not differ. (Precision is the consistency of the instrument output for a given value of input).

In brief, accuracy can be defined as *conforming to truth* and precision can be defined as *sharply* or *closely defined.*

Q. 8. Define relative accuracy of measurement.

Ans. Accuracy measured as percentage of true value is known as relative accuracy.

Q. 9. Why do errors occur during measurements?

Ans. Measurement is a process of comparing the unknown quantity with an accepted standard quantity. It involves insertion of a measuring instrument into the system under consideration and observing the resulting response. The measurement thus obtained is a quantitative measure of the so called 'true value' (since it is very difficult to define the true value, the term expected value is used). Any measurement is affected by many variables, therefore, the results rarely reflect the expected value. For instance, insertion of a measuring device into the circuit under consideration always disturb the circuit, causing the measurement to differ from the expected value.

Q. 10. Name the categories of static errors.

Ans. Static errors can be grouped in three categories viz. gross errors, systematic errors and random or accidental errors. Systematic errors may further be categorised as instrumental errors, environmental errors and observational errors.

Q. 11. What is meant by systematic errors? **[Pb. Technical Univ., December-2009]**

Ans. Systematic errors remain constant or change according to a definite law on repeated measurement of the given quantity. These errors can be evaluated and their influence on the results of measurement can be eliminated by the introduction of proper corrections.

Q. 12. Differentiate between static and dynamic errors in instruments.

Ans. Static error is defined as the difference between the measured value and true value of a quantity i.e.

$$\delta A = A_m - A$$

where δA is error, A_m is measured value of quantity and A is true value of quantity.

Dynamic errors are caused by the instruments not responding fast enough to follow the variations in a measured variable.

Q. 13. How do random errors differ from systematic errors?

Ans. Systematic errors remain constant or change according to a definite law on repeated measurement of the given quantity whereas random errors, also called the accidental errors, are of variable magnitude and sign and do not obey any known law.

Q. 14. What is meant by "arithmetic mean", "average deviation" and "standard deviation"?

Ans. **Arithmetic mean** is the sum of the number of observations divided by the number of observations i.e.

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x}{n}$$

where $\bar{x}$ is the arithmetic mean and $x_1, x_2, \dots, x_n$ are the readings taken and n is the number of readings taken.

Average deviation is the sum of the scaler values (without sign) of the deviations divided by the number of readings i.e.

$$\text{Average deviation, } D = \frac{|d_1| + |d_2| + \dots + |d_n|}{n} = \frac{\sum d}{n}$$

where $d_1, d_2 \dots d_n$ are the deviations of the readings from the mean.

Standard deviation, also known as *root mean square deviation*, of an infinite number of data is the square root of the sum of all the individual deviations squared, divided by the number of readings

i.e. Standard deviation, $\sigma = \sqrt{\dfrac{d_1^2 + d_2^2 + d_3^2 + ... + d_n^2}{n}}$.

Q. 15. What is difference between "standard deviation" and "variance"?

Ans. Standard deviation is the root mean square deviation whereas variance is the mean square deviation.

So variance, V = Mean square deviation = σ^2.

Q. 16. What is probable error?

Ans. The probable error is given as $r = 0.6745\ \sigma$, where σ is the standard deviation. The quantity r is called the probable error in the sense that there is an even chance that any one observation will have a random error not greater than $\pm r$.

Q. 17. What is the expression for probable error if the readings of measurement follow Gaussian distribution?

Ans. The probable error is given as $r = 0.6745\ \sigma$, where σ is the standard deviation.

PROBLEMS

1. The measured value of a resistance is 111 Ω whereas its true value is 110 Ω. Determine absolute error of measurement and relative error. [Ans. 1 Ω; 1%]
2. The capacitance of a capacitor is specified as 200 μF ± 5 per cent by manufacturer. Determine the limits of capacitance between which it is guaranteed. [Ans. 200 ± 10 μF]
3. A moving coil voltmeter has a uniform scale with 100 divisions and gives full-scale reading of 200 V. The instrument can read upto 1/5th of a scale division with fair degree of certainty. Determine the resolution of the instrument in volt. [Ans. 0.4 V]
4. A 600 V voltmeter is specified to be accurate within ± 2.5% of full-scale deflection. Calculate the limiting error when the instrument is used to measure a voltage of 400 V.
[U.P. Technical Univ. Electronics Measurements and Instrumentation, 2006-07]
[Ans. ± 3.75%]
5. A wattmeter having a range of 500 W has an error of ± 1.5 per cent of full-scale deflection. If the true power is 50 W, what would be the range of the reading?
[U.P. Technical Univ. Electronics Measurements and Instrumentation, 2007-08]
[Ans. (50 ± 7.5) watts or 42.5 W to 57.5 W]
6. A 20 V dc voltage is measured by analog multimeter. Multimeter is on the 25 V range and its specified accuracy is ± 2%. Determine the measurement accuracy.
[B.P. Univ. of Technology Orrisa Electronics Instrumentation and Measurement, 2008]
[Ans. 2.5%]
7. The value of resistance r was determined by measuring current I flowing through the resistance with an error $\varepsilon_1 = \pm 1.5$ per cent and power loss P in it with an error $\varepsilon_2 = \pm 1$ per cent.

   Determine the maximum possible relative error to be expected on measuring resistance r calculated from formula $r = P/I^2$ [Ans. ± 4%]
8. Three resistors have the following ratings

   $R_1 = 47\ \Omega \pm 4\%$, $R_2 = 65\ \Omega \pm 4\%$; $R_3 = 55\ \Omega \pm 4\%$

   Determine the magnitude and limiting errors in ohms and in percentage of the resistance of these resistors connected in series. [Ans. 167 Ω, ± 6.68 Ω; ± 4%]
9. A voltmeter reading of 70 V on its 100 V range and an ammeter reading of 80 mA on its 150 mA range are used to determine the power dissipation in a resistor. Both these instruments are guaranteed to be accurate within ± 1.5% at full-scale deflection. Determine power and limiting error of the power.
[U.P. Technical Univ. Measurements and Instrumentation, 2002-03]
[Ans. 5.6 W; 4.956%]
10. The wattmeter is used to measure power in the circuit with the help of the following equation $P^2 = E^2/R$, where limiting values of voltage and resistance are, E = 200 V + 1% and R = 1,000 Ω ± 5%.

    Calculate (*i*) the nominal power consumed (*ii*) the limiting error of power in watts and per cent.
[U.P. Technical Univ. Elec. Measurements and Measuring Instruments, 2004-05]
[Ans. (*i*) 40 W (*ii*) 2.8 W; ± 7%]
11. Explain the limiting error. A 4 dial decade resistance box has its accuracy specified as follows:

    (*i*) $X_1 \times 1{,}000\ \Omega \pm 0.1\%$ (*ii*) $X_2 \times 100\ \Omega \pm 0.1\%$
    (*iii*) $X_3 \times 10\ \Omega \pm 0.5\%$ (*iv*) $X_4 \times 1\ \Omega \pm 1\%$

    If R across the terminal of decade resistance box is 4,739 Ω, then determine the relative limiting error involvement in this measurement?
[G.B. Technical Univ. Electronics Instrumentation and Measurements, 2012-13]
[Ans. ± 0.1024%]
12. Two resistances R_1 and R_2 are connected in parallel with $R_1 = 10\ k\Omega \pm 5\%$ and $R_2 = 5\ k\Omega \pm 10\%$. Calculate the percentage error and range of combined resistance.
[U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2009-10]
[Ans. 21.667%; 2.611 to 4.055 kΩ]
13. Two resistances $R_1 = 1\ k\Omega \pm 1\%$ and $R_2 = 500\ \Omega \pm 1\%$ are connected in parallel. Find the limiting error in ohms and in percentage for the total resistance.
[G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2011-12]
[Ans. ± 3%; ± 10 Ω]
14. Two resistors have the following ratings: $R_1 = 100\ \Omega \pm 5\%$ and $R_2 = 200\ \Omega \pm 5\%$. Calculate
    (*i*) the magnitude of error in each resistor.
    (*ii*) the limiting error in ohms when the resistors are connected in series.
    (*iii*) the limiting error in ohms when the resistors are connected in parallel.
[J.N. Technological Univ. Hyderabad Electronic Measurements and Instrumentation, February/March-2012]
[Ans. (*i*) ± 5 Ω; ± 10 Ω; (*ii*) ± 15 Ω; (*iii*) ± 10 Ω]
15. The resistance of an unknown resistor is determined by a Wheatstone bridge. The solution for the unknown resistance is stated as $R_4 = \dfrac{R_1 R_2}{R_3}$. The limiting values of various resistances are:

    $R_1 = 500 \pm 1\%$, $R_2 = 615 \pm 1\%$, $R_3 = 100 \pm 0.5\%$

Calculate (i) the nominal value of unknown resistance, (ii) its limiting error in per cent, and (iii) its limiting error in ohm. [U.P.S.C. I.E.S. Elec. Engineering-I, 1993]

[Ans. (i) 3,075 Ω (ii) ± 2.5% (iii) ± 76.875 Ω]

16. The unknown resistance is determined by Wheatstone bridge and is given by

$$R_X = \frac{R_2 R_3}{R_1}$$

where $R_1 = 100 \pm 0.4\%$, $R_2 = 800 \pm 0.7\%$; $R_3 = 825 \pm 0.5\%$ Ω. Determine R_X and its limiting error.

[M.D. Univ. Electrical Measurements and Measuring Instruments, May-2007]

[Ans. 6,600 Ω; ± 1.6%; 105.6 Ω]

17. During the measurement of a capacitor, following ten readings were obtained:

1.002, 0.998, 1.005, 1.009, 0.995, 0.997, 1.004, 1.008, 1.003, 0.994 μF

Calculate: (a) the arithmetic mean (b) deviations from the means (c) average deviation and (d) standard deviation.

[U.P.S.C. I.E.S. Electrical Engineering-I, 2005]

[Ans. (a) 1.0015 μF

(b) +0.0005 μF, −0.0035 μF, +0.0035 μF, +0.0075 μF, −0.0065 μF, −0.0045 μF, +0.0025 μF, +0.0065 μF, +0.0015 μF, −0.0075 μF,

(c) 0.0044 μF (d) 0.00527573 μF]

18. Ten samples of a steel wire were tested on a universal test machine. The breaking strength in tonnes was 4.3, 4.5, 4.7, 4.2, 4.5, 4.6, 4.4, 4.6, 4.9, 4.5. Determine (i) mean value and (ii) standard deviation.

[M.D. Univ. Measurements and Instrumentation 6th Semester, 2011]

[Ans. 4.52 tonnes; 0.199 tonne]

19. A circuit was tuned for resonance by eight different students and the values of resonant frequency in kHz was recorded as 432, 447, 444, 435, 446, 444, 436 and 441. Calculate (i) standard deviation and (ii) variance.

[Rajasthan Technical Univ. Electronic Measurements and Instrumentation, February-2011]

[Ans. (i) 5.605 kHz; (ii) 31.41 $(kHz)^2$]

20. The following 10 observations were recorded when measuring a voltage: 31.6, 31.0, 31.7, 31.0, 32.1, 31.9, 31.0, 31.8, 32.5 and 31.8 volts. Find (i) the probable error of one reading (ii) the probable error of mean.

[Rajasthan Technical Univ. Electronic Measurements and Instrumentation, February-2011]

[Ans. (i) 0.3443 V; (ii) 0.1148 V]

21. The following ten readings are taken of a certain physical length: 5.30 m, 5.73 m, 6.77 m, 5.26 m, 4.53 m, 5.45 m, 6.09 m, 5.64 m, 5.81 m, 5.75 m. Calculate:

(i) Mean and standard deviation.

(ii) Using the Chauvenet's criterion, test the data points for possible consistency.

(iii) Eliminate the questional points and calculate area standard deviation for the adjusted data.

[Anna Univ. Chennai (TN) Measurements and Instrumentation, April/May-2011]

[Ans. (i) 5.633 m; 0.5827 m (ii) 0.5827 m]

22. A random sample was taken of the internal mean diameters of nuts produced by a manufacturer. The mean was 7.5 mm with a standard deviation of 0.1 m. If the nuts having a diameter of 7.735 and 7.583 are acceptable what percentage are rejected?

[M.D. Univ. Measurements and Instrumentation 6th Semester, 2011]

[Ans. 54%]

23. Define and distinguish between precision and accuracy. The power factor of a circuit is defined by:

$$\cos\phi = \frac{P}{VA}$$

where P is the power in watts, V voltage in volts and A is the current in amperes. The relative error in power, current and voltage are ± 5%, ± 1% and ± 1% respectively. Calculate the relative error in power factor. Also calculate the uncertainty in the power factor if the errors were specified as uncertainties.

[M.D. Univ. Electronics Measurements and Instrumentation, 2004-05]

[Ans. 7%; 5.196%]

24. Two resistors R_1 and R_2 are connected in parallel. The values of resistances are:

$R_1 = 1$ kΩ ± 1% $R_2 = 500$ Ω ± 1%

Assuming given errors as uncertainty, calculate the uncertainty in the combined resistance.

[U.P. Technical Univ. Elec. Measurements and Measuring Instruments, 2005-06]

[Ans. (333.33 ± 2.48) Ω]

25. Find the value of $(R_1 + R_2)$, considering the errors in their values as (i) limiting error (ii) standard deviation when

$R_1 = (100 \pm 2\%)$ ohms, $R_2 = (200 \pm 2.5\%)$ ohms

[U.P.S.C. I.E.S. Electrical Engineering-I, 1997]

[Ans. Limiting error = ± 7 Ω; ± 2.333%; 9.8995 Ω]

26. The measurement of resistance of a resistor gives the following results:

101.2, 101.7, 101.3, 101.0, 101.5, 101.3, 101.2, 101.4, 101.3, 101.1 ohms.

Assuming that the random errors are present calculate (i) arithmetic mean (ii) the standard deviation of the readings, (iii) the probable error of average of 10 readings.

[Pb. Univ. Elec. Measurements-I, December 1999]

[Ans. 101.3, 0.2 Ω, 0.04497 Ω]

27. Following readings were obtained in respect of measurement of a resistor:

0.903, 0.895, 0.892, 0.899, 0.901, 0.890, 0.906, 0.897, 0.912, 0.892, 0.898 and 0.902 Ω.

Determine (a) the arithmetic mean (b) the standard deviation of the readings (c) the probable error of mean value.

[Ans. (a) 0.8989 Ω (b) 0.00637 (c) 0.138%]

28. Given the following set of voltage measurements taken from a voltmeter, find their (i) average value (ii) average deviation (iii) standard deviation and (iv) probable error.

Data: 153 V, 162 V, 157 V, 161 V, 155 V.

[Ans. (i) 157.6 V (ii) 3.12 V (iii) 3.847 Ω (iv) 2.595 Ω]

29. Two resistors R_1 and R_2 are connected in series and then in parallel. The value of resistances are:

$R_1 = 100.0 \pm 0.1$ Ω; $R_2 = 50 \pm 0.05$ Ω

Calculate the uncertainty in the combined resistance for both series and parallel arrangements.

[U.P.S.C. I.E.S. Elec. Engineering-I, 1994]

[Ans. (i) 0.1118 Ω; (ii) 0.0248 Ω]

30. Two sets of large number of 10 kΩ and 15 kΩ resistors are used to make a large number of 6 kΩ and 25 kΩ resistors choosing one from each group. If the uncertainties of the two sets of resistors (10 kΩ and 15 kΩ) are 5% and 10% respectively, find the uncertainties of the combined resistor-sets (6 kΩ and 25 kΩ).

[U.P.S.C. I.E.S. Elec. Engineering-I, 2000]

[Ans. (25 ± 1.58) kΩ in series combination; (6 ± 0.3) kΩ in parallel combination]

□□□