Kinetic theory for Plasma

Course: MPHYEC-01I Plasma Physics
(M.Sc. IV Sem)

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Lecture 3: Unit-IV

The Vlasov Equation

The Vlasov equation is the collisionless Boltzmann equation that directly incorporates the macroscopic smoothed internal electromagnetic fields as well as external fields.

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{1}{m_\alpha}[\mathbf{F}_{ext} + q_\alpha(\mathbf{E}_i + \mathbf{v} \times \mathbf{B}_i)] \cdot \nabla_v f_\alpha = 0$$

Where we have assumed that the plasma particle motions are governed by the applied external fields plus the macroscopic average internal fields, smoothed in space and time, due to the presence and motion of all plasma particles. Generally, the external forces are EM forces and; for magnetized plasma, the forces dominates over the internal EM forces.

The electromagnetic force follows the Maxwell's equation:

$$\nabla \cdot \mathbf{E}_i = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B}_i = 0$$

$$\nabla \times \mathbf{E}_i = -\frac{\partial \mathbf{B}_i}{\partial t}$$

$$\nabla \times \mathbf{B}_i = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}_i}{\partial t} \right)$$

Plasma charge density and the plasma current density are given by the expressions:

$$\rho(\mathbf{r},t) = \sum_\alpha q_\alpha \; n_\alpha(\mathbf{r},t) = \sum_\alpha q_\alpha \int_v f_\alpha(\mathbf{r},\mathbf{v},t) \; d^3v$$

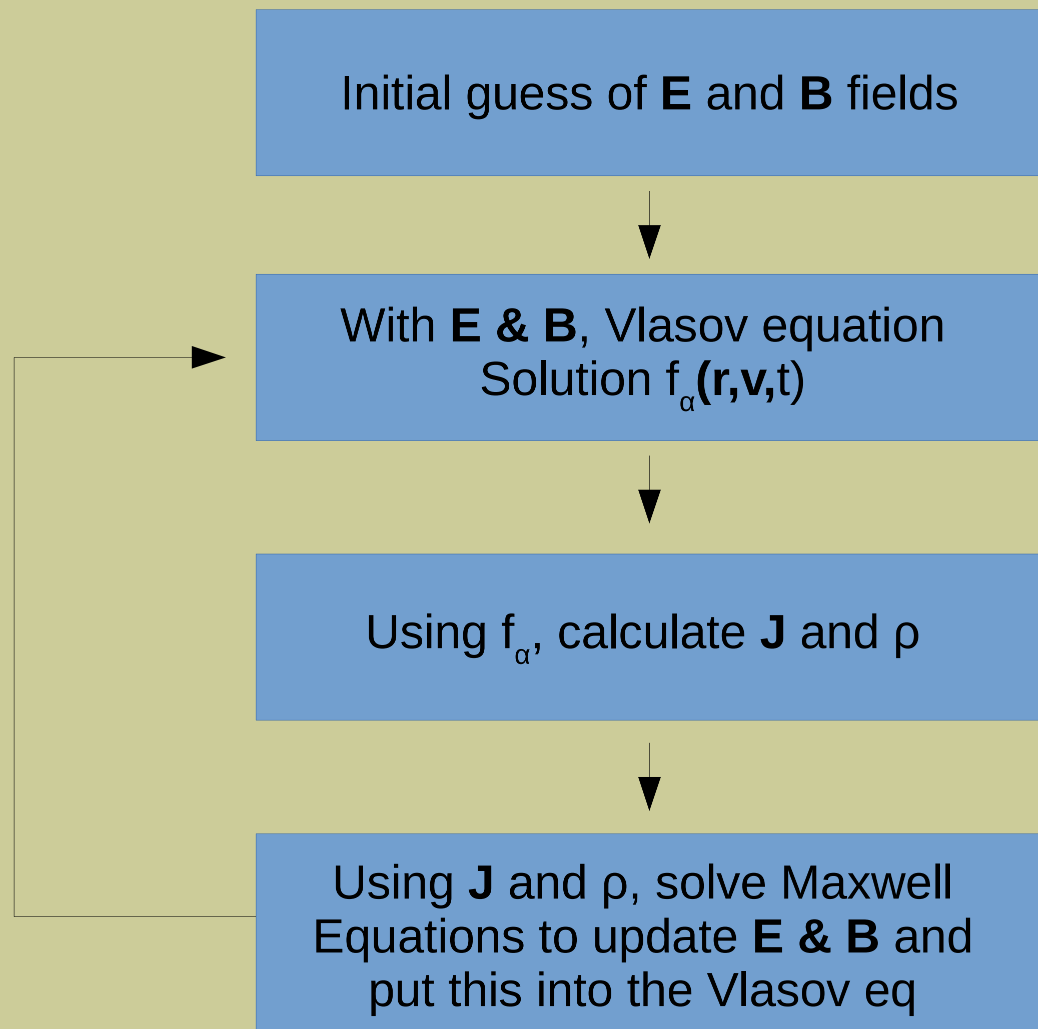
$$\mathbf{J}(\mathbf{r},t) = \sum_\alpha q_\alpha \; n_\alpha(\mathbf{r},t) \; \mathbf{u}_\alpha(\mathbf{r},t) = \sum_\alpha q_\alpha \int_v \mathbf{v} \; f_\alpha(\mathbf{r},\mathbf{v},t) \; d^3v$$

Here u_α(r, t) denotes the macroscopic average velocity for the particles of type α.

Vlasov equation complimented with Maxwell's eqautions form a complete set of equations and are the fundamental equations of kinetic theory.

Note: The Vlasov equation, due to absence of collision term, does, not take into account short-range collisions. However, it is not so restrictive becuase the effects of the particle interactions has already been included in the Lorentz force.

Self-consistent Approach to Solve the equations of Kinetic theory



Few Important Remarks

- The equilibrium distribution function is the time-independent solution of the Boltzmann equation in the absence of external forces.

- With the assumption of single species **uniformly distributated equilibrium** gas and **absence of external force**, the distributation function can be denoted by f(**v**) (i.e., only depends on **v**) and satifies the following condition:

$$\left(\frac{\delta f}{\delta t}\right)_{coll} = 0$$

- Hence, under equilibrium conditions, there are no changes in the distribution function as a result of collisions between the particles.

- Such a distributation function is known as "**the Maxwell-Boltzmann or Maxwellian velocity distribution function**" and expression of which is given as:

$$f(c) = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mc^2}{2kT}\right)$$

where n is number density, **u** is the average velocity (whole translation motion) **and v=c+u** with c representing the random motion of the particles.

Local Maxwell-Boltzmann Distribution Function

In many situations of interest we are dealing with a gas that, although not in equilibrium, is not very far from it. It is then a good approximation to consider that, in the neighborhood of any point in the gas, there is an equilibrium situation described by a local Maxwell-Boltzmann distribution function of the form:

$$f(\mathbf{r}, \mathbf{v}, t) = n(\mathbf{r}, t)\left[\frac{m}{2\pi k T(\mathbf{r}, t)}\right]^{3/2} \exp\left\{-\frac{m[\mathbf{v} - \mathbf{u}(\mathbf{r}, t)]^2}{2kT(\mathbf{r}, t)}\right\}$$

where the number density n, the temperature T, and the average velocity **u** are slowly varying functions of rand t.

Presence of more than one species

$$f_\alpha(v) = n_\alpha \left(\frac{m_\alpha}{2\pi k T_\alpha}\right)^{3/2} \exp\left[-\frac{m_\alpha(\mathbf{v}-\mathbf{u}_\alpha)^2}{2kT_\alpha}\right]$$

Each species has a Maxwellian distribution of velocities, but with its own density, average velocity, and temperature.

This doesn’t represent an equilibrium distribution for the system.

Only if the temperatures and average velocities of all species are equal will this be an equilibrium situation.

If two systems with different species and at different temperatures are brought together, then, as time passes, there will be a transfer of energy through collisions between the different species, until equilibrium is reached with the various species at the same temperature and mean velocity.

Equilibrium in Presence of an External Force

A gas under thermodynamic equilibrium and a conservative force is characterized by a distribution function that differs from the Maxwell-Boltzmann distribution by an exponential factor, known as the Boltzmann factor.

$$\mathbf{F}(\mathbf{r}) = -\nabla U(\mathbf{r})$$

$$f(\mathbf{r}, v) = f_0(v)\ \exp\left[-\frac{U(\mathbf{r})}{kT}\right]$$

$$= n_0 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{(\frac{1}{2}mv^2 + U)}{kT}\right]$$

$$n(\mathbf{r}) = n_0\ \exp\left[-\frac{U(\mathbf{r})}{kT}\right]$$

If we have external electricstatic field then:

$$\mathbf{E} = -\nabla\phi(\mathbf{r}) \qquad U(\mathbf{r}) = q\ \phi(\mathbf{r})$$

$$n(\mathbf{r}) = n_0\ \exp\left[-\frac{q\ \phi(\mathbf{r})}{kT}\right]$$

Thanks for the attention!