Charged Particle in Uniform Electromagnetic Field

Course: MPHYEC-01I Plasma Physics (M.Sc. IV Sem)

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Lecture 4: Unit-I

Charged Particle in Uniform Static Electrocmagnetic Field

- Equation of motion of a charged particle q in an static **E** and **B:**

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- Decomposing **E** and **v** into components which are parallel and perpendicular to **B**

$$\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$$

$$\mathbf{E} = \mathbf{E}_{\parallel} + \mathbf{E}_{\perp}$$

- The equations of motion for the parallel and normal components are

$$m\frac{d\mathbf{v}_{\parallel}}{dt} = q\mathbf{E}_{\parallel}$$

$$m\frac{d\mathbf{v}_{\perp}}{dt} = q(\mathbf{E}_{\perp} + \mathbf{v}_{\perp} \times \mathbf{B})$$

- Particle has a constant acceleration along B.

- Solution of the parallel component equation is straightforward:

$$\mathbf{v}_{\parallel}(t) = \left(\frac{q\mathbf{E}_{\parallel}}{m}\right)t + \mathbf{v}_{\parallel}(0)$$

$$\mathbf{r}_{\parallel}(t) = \frac{1}{2}\left(\frac{q\mathbf{E}_{\parallel}}{m}\right)t^2 + \mathbf{v}_{\parallel}(0)t + \mathbf{r}_{\parallel}(0)$$

- For solving the normal component equation, we define normal component as:

$$\mathbf{v}_{\perp}(t) = \mathbf{v}'_{\perp}(t) + \mathbf{v}_E$$

- $\mathbf{v}_E$ is a constant velocity in plane normal to **B.** New normal component represents the normal velocity of the particle in a frame which is moving with $\mathbf{v}_E$.

- Now we put the above expression of normal component of velocity in the equation of motion in the normal plane to **B**.

$$\mathbf{E}_\perp = -\left(\frac{\mathbf{E}_\perp \times \mathbf{B}}{B^2}\right) \times \mathbf{B}$$

$$m\frac{d\mathbf{v}'_\perp}{dt} = q\left(\mathbf{v}'_\perp + \mathbf{v}_E - \frac{\mathbf{E}_\perp \times \mathbf{B}}{B^2}\right) \times \mathbf{B}$$

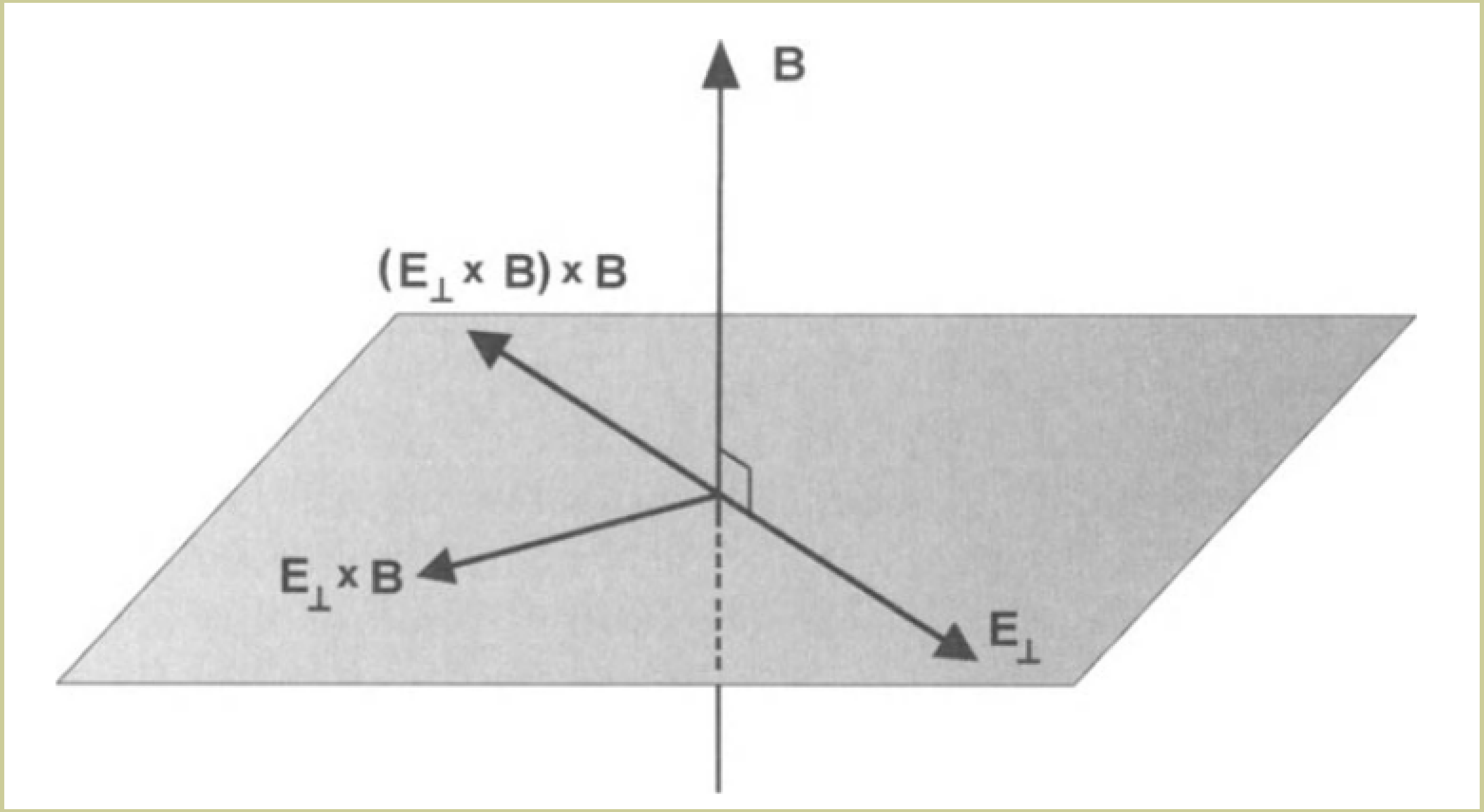


Figure is from “Fundaments of Plasma Physics” by Bittencourt.

Define $\mathbf{v}_E$ as:

$$\mathbf{v}_E = \frac{\mathbf{E}_\perp \times \mathbf{B}}{B^2}$$

Then from the equation motion written in the previous slide, we get

$$\mathbf{v}'_\perp = \boldsymbol{\Omega}_c \times \mathbf{r}_c$$

Total velocity is then given as:

$$\mathbf{v}(t) = \boldsymbol{\Omega}_c \times \mathbf{r}_c + \frac{\mathbf{E}_\perp \times \mathbf{B}}{B^2} + \frac{q\mathbf{E}_\parallel}{m} t + \mathbf{v}_\parallel(0)$$

- Cyclotron cicular Motion in plane normal to **B**
- Constant drift velocity of the guiding center
- Constant acceleration of the guiding center along **B**
- Initial value of the parallel component of velocity

Solution in Cartesian Geometry

We choose a cartesian system such that the z-axis along the direction of **B.**

$$\mathbf{B} = B\widehat{\mathbf{z}}$$

$$\mathbf{E} = E_x\widehat{\mathbf{x}} + E_y\widehat{\mathbf{y}} + E_z\widehat{\mathbf{z}}$$

Then equation of motion is

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}[(E_x + v_y B)\widehat{\mathbf{x}} + (E_y - v_x B)\widehat{\mathbf{y}} + E_z\widehat{\mathbf{z}}]$$

Manipulations lead to:

$$\frac{d^2 v_x}{dt^2} + \Omega_c^2 v_x = \Omega_c^2 \frac{E_y}{B}$$

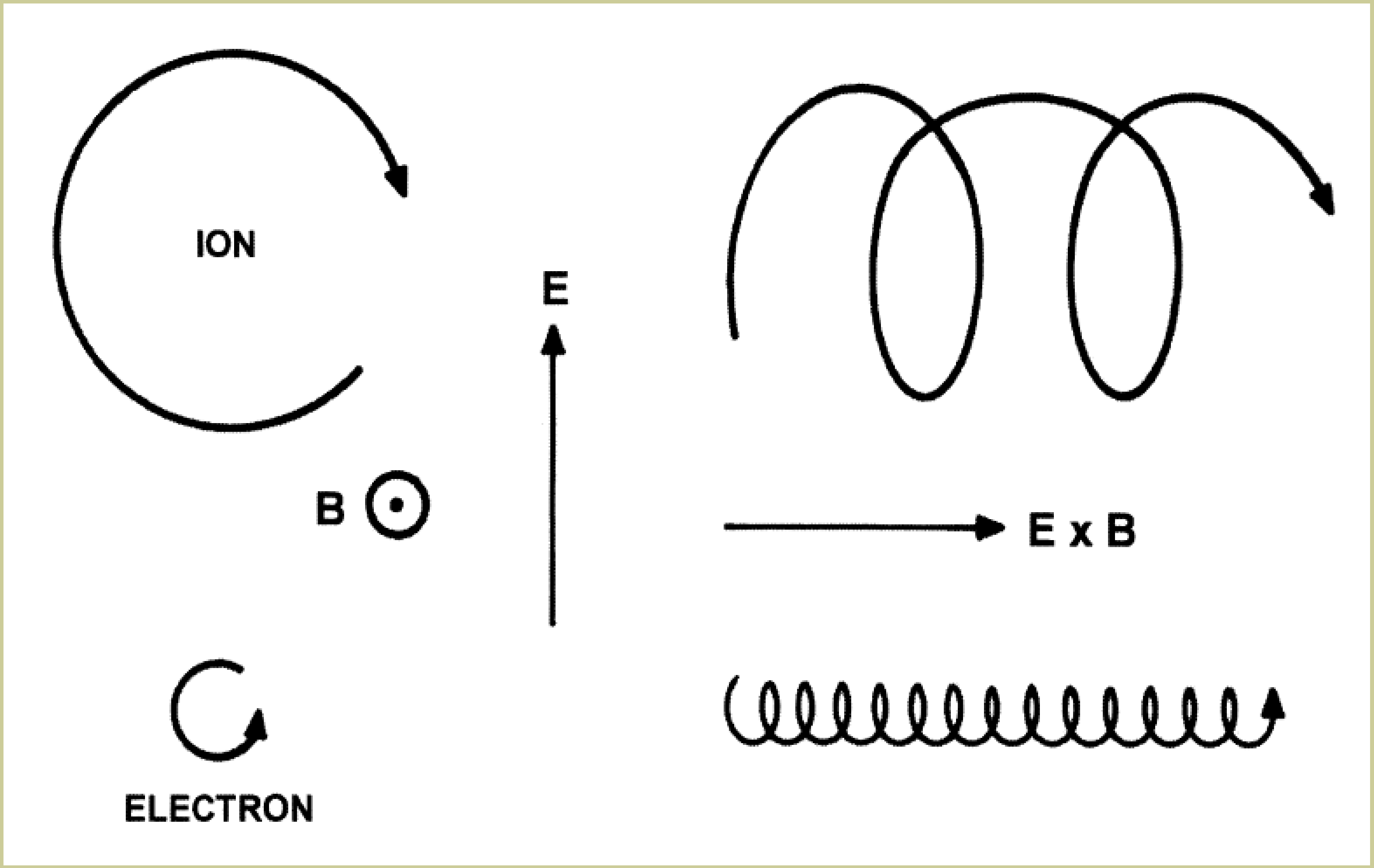
$$v_x(t) = v'_\perp \sin(\Omega_c t + \theta_o) + \frac{E_y}{B}$$

$$v_y(t) = \frac{1}{\Omega_c}\frac{dv_x}{dt} - \frac{E_x}{B} = v'_\perp \cos(\Omega_c t + \theta_o) - \frac{E_x}{B}$$

$$\mathbf{v}_E = \frac{E_y}{B}\hat{\mathbf{x}} - \frac{E_x}{B}\hat{\mathbf{y}}$$

$$x(t) = -\frac{v'_\perp}{\Omega_c} \cos(\Omega_c t + \theta_o) + \frac{E_y}{B}t + X_o$$

$$y(t) = \frac{v'_\perp}{\Omega_c} \sin(\Omega_c t + \theta_o) - \frac{E_x}{B}t + Y_o$$



Cycloidal trajectories of ions and electrons in uniform electric and magnetic fields. The electric field **E** along with magnetic field **B** gives rise to a drift velocity in the direction given by **E x B**. The figure is from "Fundamentals of Plasma Physics" by Bittencourt.

Thanks!