M.S c Mathematics –SEM 3 Functional Analysis- CC-11 Unit 3

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Theorem

Let B be a Banach space and suppose that M and N are two closed linear subspaces of B such that B= M$\oplus$N. If z=x+y is the unique representations of a vector in B as a sum of vectors in M and N, then the mappings P defined by P(z)= x i a projection on B whose range and null spaces are M and N.

Proof

Let us consider P is continuous .

If B' denotes the linear space B equipped with the new norm $||\ \ ||'$.

Defined by $||\, Z\, ||' = ||x|| + ||y||$

Then B' is a Banach space

$||P(z)|| = ||x||$(by definition of P)

$$\leq ||x|| + ||y|| = ||z||$$

Hence P is bounded and therefore continuous as a mapping of B' into B.

It suffices to prove that B′ and B have the same topology .

If T denotes the identity mapping of B′ and B have the same topology.

If T denotes the identity mapping of B′ onto B, then

$$||T(z)|| = ||z|| = ||x + y||$$
$$\leq ||x|| + ||y|| = ||z||$$

This shows that T is continuous and one-one liner transformation of B′ onto B.

It follows that T is a homeomorphism and so B and B′ have the same topology.