**M.S c Mathematics –SEM 3 Differential Geometry**

**CC-13 Unit 1**

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**Define cylindrical helix and prove that necessary and sufficient condition for a curve to be helix is that its curvature and torsion are in a constant ratio.**

**Cylindrical Helix**

**A curve lies on a cylinder cuts the generators at a constant angle is called cylindrical helix.**

**Thus ,the tangent to a cylindrical helix makes a constant angle ($\alpha$ say with a fixed line called the axis of the helix.**

**Necessary and sufficient condition for a curve to be helix is that its curvature and torsion are in a constant ratio $\frac{K}{\zeta}$= constant .**

**Let $\hat{a}$ denotes the unit vector in the direction of the axis .By the definition of helix, we have**

$\hat{t}.\hat{a} = cos\ \alpha$= constant

Differentiating this w.r.t s we have

$\hat{t}'.\widehat{a} = 0 \Rightarrow K(\hat{n}.\hat{a}) = 0 \Rightarrow \hat{n}.\hat{a} = 0$

Here ,$k \neq 0$.

Hence $\hat{n}.\hat{a}$ =0 .This shows that $\hat{a}$ is orthogonal to $\hat{n}$ and hence $\hat{a}$ must lie in the rectifying plane (i.e the plane containing $\hat{t}$ and $\hat{b}$.

If $\overrightarrow{PB}$ represents the unit vector $\hat{a}$ then $|\overrightarrow{PB}|$=1

By Triangle's law of vectors ,we have

$$\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$$

$\hat{a} = cos\ \alpha\ \hat{t} + sin\ \alpha\ \hat{b}$

Differentiating w.r.t s , we get

$$0 = cos\ \alpha\ \hat{t}' + sin\ \alpha\ \hat{b}'$$

$0 = (cos\alpha\ K - sin\ \alpha\ \zeta)\hat{n}$

$\Rightarrow \boldsymbol{cos\ \alpha\ \ k}$ - $\boldsymbol{sin\ \alpha\ \zeta = 0}$

$$\boldsymbol{tan\alpha = \frac{K}{\zeta}}$$

**Sufficient condition**

**Let**

$\boldsymbol{\frac{K}{\zeta}}$**= constant = tan (let)**

**Then,**

$$\boldsymbol{Kcos\ \alpha - \zeta\ sin\ \alpha = 0}$$

$(\boldsymbol{Kcos\ \alpha - \zeta sin\ \alpha})\boldsymbol{\widehat{n}}$**= 0**

$\boldsymbol{K\widehat{n}\ cos\ \alpha\ +\ (\widehat{-\zeta n})}$**sin** $\boldsymbol{\alpha}$ **=0**

$\Rightarrow \boldsymbol{t'}$**cos** $\boldsymbol{\alpha}$**+** $\boldsymbol{\widehat{b'}}$ **+** $\boldsymbol{\widehat{b}'}$**sin**$\boldsymbol{\alpha}$**= 0**

$\boldsymbol{\frac{d}{ds}}(\boldsymbol{t'}$**cos** $\boldsymbol{\alpha}$**+** $\boldsymbol{\widehat{b'}}$ **+** $\boldsymbol{\widehat{b}'}$**sin**$\boldsymbol{\alpha}$**) = 0**

$\boldsymbol{t'}$**cos** $\boldsymbol{\alpha}$**+** $\boldsymbol{\widehat{b'}}$ **+** $\boldsymbol{\widehat{b}'}$**sin**$\boldsymbol{\alpha = \widehat{a}}$ **( say)**

$\boldsymbol{\hat{t}}.(\boldsymbol{t'}$**cos** $\boldsymbol{\alpha}$**+** $\boldsymbol{\widehat{b'}}$ **+** $\boldsymbol{\widehat{b}'}$**sin**$\boldsymbol{\alpha}) = \boldsymbol{\hat{t}\ \widehat{a}}$

**Cos** $\boldsymbol{\alpha = \hat{t}\ \widehat{a}}$

**Hence, the curve makes a constant angle with a fixed direction. Hence ,it is a helix.**