M.S c Mathematics –SEM 3 Differential Geometry

CC-13 Unit 1

E-content – Pro(Dr)L N RAI

HOD, PG Department of Mathematics, Patna University, Patna.

Topic- Serret Frenet formula and find its expression for $\frac{d\hat{t}}{ds}$, $\frac{d\hat{b}}{ds}$, $\frac{d\hat{n}}{ds}$

Solution

The following set of three relations involving space derivatives of the fundamental unit vectors $\hat{t}, \hat{n}, \hat{b}$ are known as serret Frenet formula

(i) $\frac{d\hat{t}}{ds} = k\hat{\eta}$

(ii) $\frac{d\hat{b}}{ds} = -\zeta\hat{\eta}$

(iii) $\frac{dn}{ds} = \zeta\hat{b} - K\hat{t}$

Proof of (i)

We take $\hat{t}=\overrightarrow{r'}2$ is the unit tangent vector to curve at the point P. Since , $\hat{t}$ is of

constant magnitude and it is perpendicular to its derivative $\hat{t}'$.

$|\hat{t}| = \overrightarrow{r'^2}$=1$\Rightarrow \hat{r'}\hat{r'} = 1$

$Differentiating$ we get

$$\hat{r'}.\hat{r''} = 0$$

$\hat{t}.\hat{t}'$=0

$Also$, the vector $\hat{t} = \hat{r}''$ lies in the oscillating plane perpendicular to $\hat{t}$ implies that $\hat{r}''$ is collinear with $\hat{n}$.

$Also$

$|\vec{r}''| = k$ so that we have $\vec{r}''$=$\mp k\,\hat{n}$.

We choose the direction of $\hat{n}$ such that curvature k is always positive i.e. we take $\vec{r}''$=$k\,\hat{n}$ or

$$\frac{d\hat{t}}{ds} = k\hat{\eta}$$

Proof of (ii)

We have $\hat{t}.\hat{b} = 0$

Differentiating w.r t ‘s’ ,we get $\hat{t}.\hat{b} = 0$

Further,

$\hat{b}.\hat{b} = 1$

$2\hat{b}.\hat{b} = 0$

Hence $\hat{b}'$ **is perpendicular to** $\hat{b}$**.**

Thus $\hat{b}'$ **is collinear with** $\hat{n}$

Thus ,

$$\frac{d\hat{b}}{ds} = \mp\zeta\hat{\eta}$$

$Since$ **,** $\hat{b}$ **has the opposite direction to** $\hat{n}$**.**

So **, negative sign is taken i.e** $\hat{b}'$**=-** $\zeta\hat{\eta}$

$$\frac{d\hat{b}}{ds} = \mp\zeta\hat{\eta}$$

Proof of (iii)

We know that $\hat{b}$ **x** $\hat{t}$**= n**

Differentiating w. r. t 's ', we get

$$\frac{d\hat{n}}{ds} = \frac{d\hat{b}}{ds}\text{x}\hat{t} + \hat{b}\text{ x }\frac{d\hat{t}}{ds}$$

$$=\frac{d\hat{b}}{ds}\text{ x }\hat{t} + \hat{b}\text{ x (k}\hat{n}\text{)=(}\zeta\hat{\eta}\text{)x }\hat{t} + \hat{b}\text{ x (k}\hat{n}\text{)}$$

$= -\zeta(\hat{n} \times \hat{t}) + k(\hat{b} \times \hat{n})$

$=- \zeta(-\hat{b}) + k(-\hat{t})$

$= \zeta\hat{b} - k\hat{t}$

Type equation here.