Equation of motion under impulsive forces (9)

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1 Impulsive action

Let sudden velocity changes be produces at the boundaries of an in-compressible fluid or that impulsive forces be made to act to its interior. Then it is known that the impulsive pressure at any point is the same in every direction. Moreover the disturbances produced in both cases are propagated instantaneously throughout the fluid.

2 Equation of motion under impulsive forces

Let S be an arbitrary small closed surface drawn in the in-compressible fluid enclosing a volume V. Let I be the impulsive body forces per unit mass. Let this impulse change the velocity at $p(r,t)$ of V instantaneously from q_1 to q_2 and let it produce impulsive pressure on the boundary S. Let $\widetilde{\omega}$ denote the inpulsive pressure on the element δS. Let n be the unit outward drawn normal at δS.
Let ρ be the density of the fluid.

We now apply Newton's second law for inpulsive motion to the fluid enclosed by S, namely,

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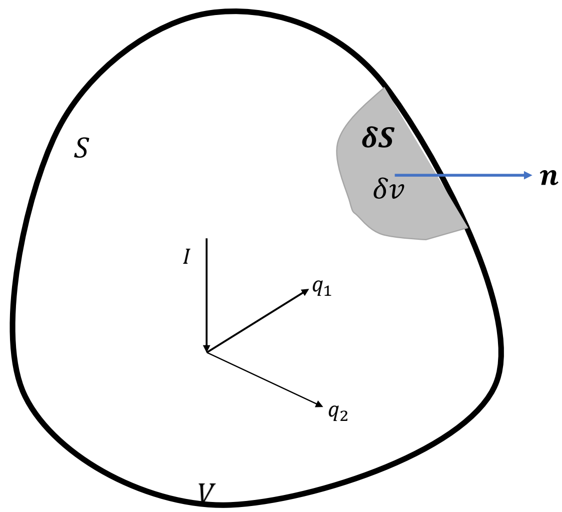


Total impulse applied = chance of momentum

$$\therefore \qquad \int_V I\rho dV - \int_S n\widetilde{\omega} dS = \int_V \rho(q_2 - q_1) dV \tag{1}$$

$$\int_S n\widetilde{\omega} dS = \int_V \nabla\widetilde{\omega} dV \quad \text{by the guess divergence theorem}$$

$$\therefore \text{From(1)} \qquad \int_V \left[I\rho - \nabla\widetilde{w} - \rho(q_2 - q_1)\right] dV = 0 \tag{2}$$

since V is an arbitrary small volume, (2) gives

$$I_\rho - \nabla\widetilde{\omega} - \rho(q_2 - q1) = 0 \qquad \text{or} \qquad q_2 - q_1 = I - (1/\rho)\nabla\widetilde{\omega} \tag{3}$$

corollary 1. *Let $I = 0$ (i.e. external impulsive body forces are absent) whereas impulsive pressures be absent. Then (3)reduces to*

$$q_2 - q_1 = (1/\rho)\nabla\widetilde{\omega} \tag{4}$$

$$\textit{from (4)} \qquad \nabla.(q_2 - q_1) = \nabla.\left[-(1/\rho)\nabla\widetilde{\omega}\right]$$

$$\textit{or} \qquad \nabla.q_2 - \nabla.q_1 = -\left(1/\rho\right)\nabla^2\widetilde{\omega} \tag{5}$$

for the in-compressible fluid, the equation of continuity gives

$$\nabla.q_2 = \nabla.q_1 = 0 \tag{6}$$

Making use of (5) and (6)

$$\nabla^2\widetilde{\omega} = 0 \qquad \textit{(Laplace equation)} \tag{7}$$

corollary 2. *Let $q_1 = 0$ and $I = 0$ so that the motion is started from rest by the application of impulsive pressure at the boundaries but without use of external impulsive body forces.*

Then, writing $q_2 = q$ (3) reduces to

$$q = -\nabla\left(\widetilde{\omega}/\rho\right) \tag{8}$$

showing that there exists a velocity potential $\phi = \widetilde{\omega}/\rho$ and the motion is irrotational.

corollary 3. *Let $I = 0$ i.e. let there be no extraneous impulses. Further, let ϕ_1 and ϕ_2 denote the velocity potential just before and just after the impulsive action. Then*

$$q_1 = -\nabla\phi \qquad \text{and} \qquad q_2 = -\nabla\phi_2 \tag{9}$$

Then (3) reduces to

$$-\nabla\phi_2 + \nabla\phi_1 = -\left(1/\rho\right)\nabla\widetilde{\omega} \qquad \text{or} \qquad \nabla\widetilde{\omega} = \rho\left(\phi_2 - \phi_1\right) \text{ integrating, when } \rho \text{ is constant}$$

$$\widetilde{\omega} = \rho\left(\phi_2 - \phi_1\right) + C. \tag{10}$$

The constant C may be omitted by regrading as an extra pressure and constant throughout the fluid.

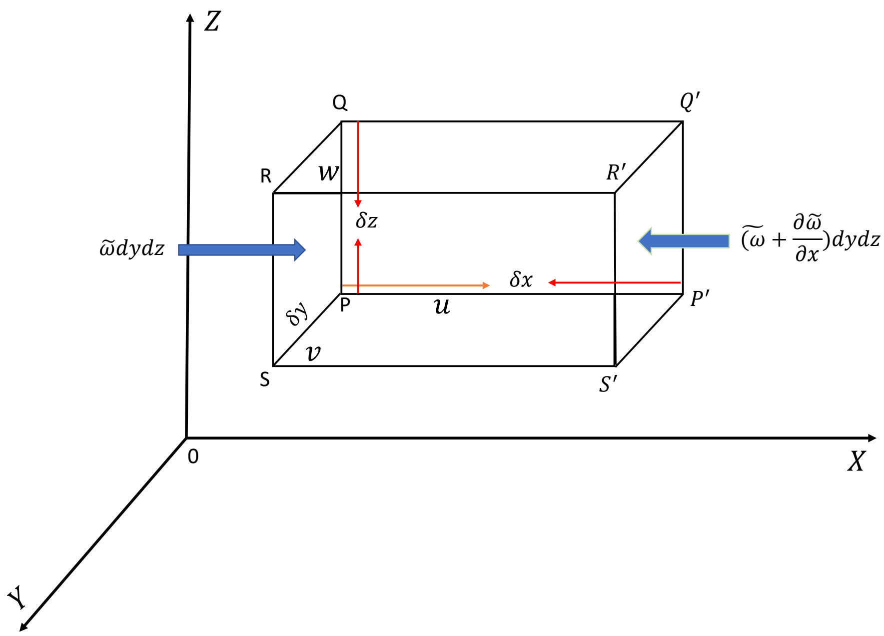
$$\therefore \qquad \widetilde{\omega} = \rho\phi_2 - \rho\phi_1 \tag{11}$$

corollary 4. *Take $\phi_1 = 0$ and $\rho = 1$ in cor.(3). Then we find that any actual motion, for which a single valued velocity potential exists, could be produced instantaneously from test by applying appropriate impulses. We then also note that the velocity potential is the impulsive pressure at any point.*

It is also easily seen that when a state of rotational motion exists in a fluid, the motion could neither be created nor destroyed by impulsive pressures.

3 Equation of motion under impulsive forces (Cartesian form)

Let there by a fluid particle at $P(x, y, z)$ and let ρ be the density of the incompressible fluid.
Let u_1, v_1, w_1 and u_2, v_2, w_2 be the velocity components at the point P just before and just after the impulsive action. Let I_x, I_y, I_z be the components of the external impulsive forces per unit mass of the fluid. construct a small parallelepiped with edges of lengths $\delta x, \delta y, \delta z$ parallel to their respective coordinate axes, having P at one of the angular points as shown in figure. Let $\widetilde{\omega}$ denote the impulsive pressure at P. Then, we have



$$\text{Force on the face} PQRS = \widetilde{\omega}\delta y\delta z = f(x,y,z) \quad \text{say} \tag{12}$$

$$\therefore \qquad \text{Force on the face} P'Q'R'S' = f(x+\delta x,y,z) = f(x,y,z) + \delta x \frac{\partial}{\partial x} f(x,y,z) + \dots \tag{13}$$

$$\therefore \text{The net force on the opposite faces } PQRS \text{ and } P'Q'R'S'$$

$$= f(x,y,z) - \left[\delta x \frac{\partial}{\partial x} f(x,y,z) + \dots\right]$$

$$= -\delta x \frac{\partial}{\partial x} f(x,y,z), \text{to the first order of approximation}$$

$$= -\delta x \frac{\partial}{\partial x}(\widetilde{\omega}\delta y\delta z), \text{using}(12)$$

$$= -\delta x\delta y\delta z \frac{\partial \widetilde{\omega}}{\partial x}, \text{which will act along the x-axis.} \tag{14}$$

Again, the impulse on the elementary parallelepiped along the x-axis due to external impulsive body forces I_x

$$= \rho\delta x\delta y\delta z I_x. \tag{15}$$

$$\text{Finally, the change in momentum along x-axis } = \rho\delta x\delta y\delta z(u_2 - u_1) \tag{16}$$

We, now apply Newton's second law for impulsive motion to the fluid enclosed by the parallelopiped, namely, Total impulse applied along x-axis = change of momentum along x-axis.

$$\therefore -\delta x\delta y\delta z\frac{\partial\widetilde{\omega}}{\partial x}+\rho\delta x\delta y\delta z I_x=\rho\delta x\delta y\delta z(u_2-u_1)$$

$$\rho(u_2-u_1)=\rho I_x-(\partial\widetilde{\omega}/\partial x) \tag{17}$$

$$\rho(v_2-v_1)=\rho I_y-(\partial\widetilde{\omega}/\partial y) \tag{18}$$

$$\rho(w_2-w_1)=\rho I_z-(\partial\widetilde{\omega}/\partial z) \tag{19}$$

Equation (17), (18) and (19) are the required equations of motion of an incompressible fluid under impulsive forces.

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