Simplex Method
(M.Sc. Sem-III)

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SIMPLEX METHOD FOR GOAL PROGRAMMING PROBLEM

The major steps of the simplex method for the linear goal programming problem are :

Step-1 : Identify the decision variables of the key decision and formulate the given problem is linear goal programming problem.

Step-2 : Determine the initial basic feasible solution and set up initial simplex table. Compute and $z_j - c_j$ values separately for each of the ranked goals : $P_1, P_2, ...$ and enter at the bottom of the simplex table. These are shown from bottom to top, i.e., first priority goal 'P_1' is shown at the bottom and least priority goal at the top.

Step-3 : Example $z_j - c_j$ values in the P_1-row first. If all $(z_j - c_j) \le 0$ at the highest priority levels, then the optimum solution has been obtained. If at least one $(z_j - c_j) > 0$ at a certain priority.

level and there is no negative entry at the higher unachieved priority levels, in the same column, then the current solution is not optimum.

Step-4 : If the target values of each goal in the solution column (x_B) is zero, the current solution & optimum.

Step-5 : Examine the positive values of $(z_j - c_j)$ of the highest priority (P_1) and choose the largest of these. The column corresponding to this value becomes the key column. Otherwise move to the next higher priority (P_2) and select the largest positive value of $(z_j - c_j)$ for determining the key column.

Step-6 : Determine the key row and key number (leading element) in the same way as in the Simplex Method.

Step-7 : Any positive value in the $(z_j - c_j)$ row which has negative $(z_j - c_j)$ under any lower priority rows are ignored. It is because deviations from the highest priority goal would be increased with the entry of this variable in the basis.

SAMPLE PROBLEM

The production manager of a company wants to schedule a week's production run for two products A and B each of which requires the labour and materials as shown below :

Product	Labour (in hours)	Material M_1 (in kgs.)	Material M_2 (in kgs.)
A	2	4	5
B	4	5	4
Available (per week)	600	1,000	1,200

The unit profit for A and B is Rs. 20 and Rs. 32 respectively.

The manager would like to maximise profit, but he is equally concerned with maintaining workforce of the division at nearly constant level in the interest of employee morale. The work, which consist of people engaged in production, sales, distribution and other general staff is consisted of

Since, we have to satisfy goal 1 and goal 2 simultaneously, the given problem may not have feasible solution. Further, in order to solve the given problem by simplex method, we introduce for deviational variables d_1^+ and d_1^- in goal 1 constraint and, d_2^+ and d_2^- in goal 2 constraints, where

$d_1^+ =$ number of rupees above the goal of Rs. 5,400.

$d_1^- =$ number of rupees below the goal of Rs. 5,400.

$d_2^+ =$ number of people aabove the work force goal of 108,

$d_2^- =$ number of people below the work force goal of 108.

Making use of slack variables $s_1 \geq 0,\ s_2 \geq 0,\ s_3 \geq 0$ in the first three constraints respectively, and the deviational variables in the fourth and fifth constraints; the goal linear programming problem is :

Minimize $z = d_1^- + d_2^- + 0.s_1 + 0.s_2 + 0.s_3 + 0.d_1^+ + 0.d_2^+$ subject to the constraints :

$$2x_1 + 4x_2 + x_1 = 600,\ 4x_1 + 5x_2 + s_2 = 1,000$$

$$5x_1 + 4x_2 + s_3 = 1,200,\ 20x_1 + 32x_2 + d_1^- - d_1^+ = 5,400$$

$$0.3x_1 + 0.75x_2 + d_2^- - d_2^+ = 108,$$

$$x_1,\ x_2,\ s_1,\ s_2, s_3 \geq 0 \text{ and } d_1^-,\ d_1^+,\ d_2^-,\ d_2^+ \geq 0$$

Solution by Simplex Method

Using simplex method an initial (starting) basic feasible solution is

$$s_1 = 600,\ s_2 = 1,000,\ s_3 = 1,200,\ d_1^- = 5,400, \text{ and } d_1^+ = 108$$

with I_5 as the initial basis matrix.

Initial Interation, Introduce y_2 and drop d_2^-.

			0	0	0	0	0	1	0	1	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	d^-_1	d^+_1	d^-_2	d^+_2
0	y_3	600	2	4	1	0	0	0	0	0	0
0	y_4	1,000	4	5	0	1	0	0	0	0	0
0	y_5	1,200	5	4	0	0	1	0	0	0	0
1	d^-_1	5,400	20	32	0	0	0	1	−1	0	0
1	d^-_2	108	0.3	0.75	0	0	0	0	0	1	−1
		z(=5508	20.3	32.75	0	0	0	0	−1	0	−1

Since, $z_1 - c_1 > 0$ and $z_2 - c_2 > 0,$ current solution is not optimum. As the largest of these two positive quantities is 32.75 corresponding to $z_2 - c_2 > 0,$ enters the basis. Further,

$$\text{Min.}\left\{\frac{x_{Bi}}{y_{i2}},\ y_{i2} > 0\right\} = \text{min.}\left\{\frac{600}{4}, \frac{1000}{5}, \frac{1200}{4}, \frac{5400}{32}, \frac{108}{0.75}\right\} = \frac{108}{0.75}.$$

This implies that d_2^- leaves the basis.

First lteration. Indroduce d_2^+ and drop y_3.

			0	0	0	0	0	1	0	1	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	d^-_1	d^+_1	d^-_2	d^+_2
0	y_3	24	2/5	0	1	0	0	0	0	−16/3	16/3
0	y_4	280	2	0	0	1	0	0	0	−20/3	20/3
0	y_5	624	17/5	0	0	0	1	0	0	−16/3	16/3
1	d^-_1	792	36/5	0	0	0	0	1	−1	−128/3	128/3
0	y_2	144	2/5	1	0	0	0	0	0	4/3	−4/3
		z(=792)	36/5	0	0	0	0	0	−1	−131/3	128/3

Since, $z_9 - c_9 = \frac{128}{3}$ is largest positive net evaluation, d_2^+ enters the basis. Further, min. $\left\{\frac{x_{Bi}}{y_a}, y_a > 0\right\}$ is $\frac{24}{16/3}$. This implies y_3 leaves the basis.

Second Iteration. Introduce y_1 and drop d_2^+.

c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	d^-_1	d^+_1	d^-_2	d^+_2
0	d^+_2	9/2	3/40	0	3/16	0	0	0	0	–1	1
0	y_4	250	3/2	0	–5/4	1	0	0	0	0	0
0	y_5	600	3	0	–1	0	1	0	0	0	0
1	d^-_1	600	4	0	–8	0	0	1	–1	0	0
0	y_2	150	1/2	1	1/4	0	0	0	0	0	0
		z(=600)	4	0	–8	0	0	0	–1	0	0

Clearly, y_1 enters the basis, since $z_1 - c_1 > 0$. Also, Min. $\left\{\frac{x_{Bi}}{y_a},\!. \, y_a > 0\right\} = \frac{9/2}{3/40}$ indicates d_2^+ leaves the basis.

Third Iteration. Introduce d_2^- and drop d_1^-.

c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	d^-_1	d^+_1	d^-_2	d^+_2
0	y_1	60	1	0	5/2	0	0	0	0	–40/3	40/3
0	y_4	160	0	0	–5	1	0	0	0	20	–20
0	y_5	420	0	0	–17/2	0	1	0	0	40	–40
1	d^-_1	360	0	0	–18	0	0	1	–1	160/3	–160/3
0	y_2	120	0	1	–1	0	0	0	0	20/3	–20/3
		z(=360)	0	0	–18	0	0	0	–1	157/3	–160/3

Clearly, d_2^- enters the basis because $z_8 - c_8 > 0$, and d_1^- leaves the basis.

Final Iteration. Optimum Solution.

c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	d^-_1	d^+_1	d^-_2	d^+_2
0	y_1	150	1	0	–2	0	0	1/4	–1/4	0	0
0	y_4	25	0	0	7/4	1	0	–3/8	3/8	0	0
0	y_5	150	0	0	5	0	1	–3/4	3/4	0	0
1	d^-_1	27/4	0	0	–27/80	0	0	3/160	–3/160	1	–1
0	y_2	75	0	1	5/4	0	0	–1/8	1/8	0	0
		z(=27/4)	0	0	–27/80	0	0	–157/160	–3/160	0	–1

Since, all $z_j - c_j \le 0$, an optimum solution is obtained. Hence, the optimum solution is :

$$x_1 = 150,\ x_2 = 75,\ d_2^- = \frac{27}{4} = 6.75,\ s_2 = 25 \text{ and } s_3 = 150 \text{ with the minimum of } z = 6.75.$$

This implies that the workforce shall be 108–6.75 (=101.25), with the employment goal being under-achieved to the extent of 6.75 people; while 25 kg. of material M_1 and 150 kg. of material M_2 would remain unutilised. The other variables are non-basic so that all the available labour hours shall be used and the profit goal be met exactly.

Note : The solution of the above problem can also be obtained graphically, where the goals are represented by washed lines.