M.S c Mathematics –SEM 3 Rigid Dynamics

CC-13 Unit 1

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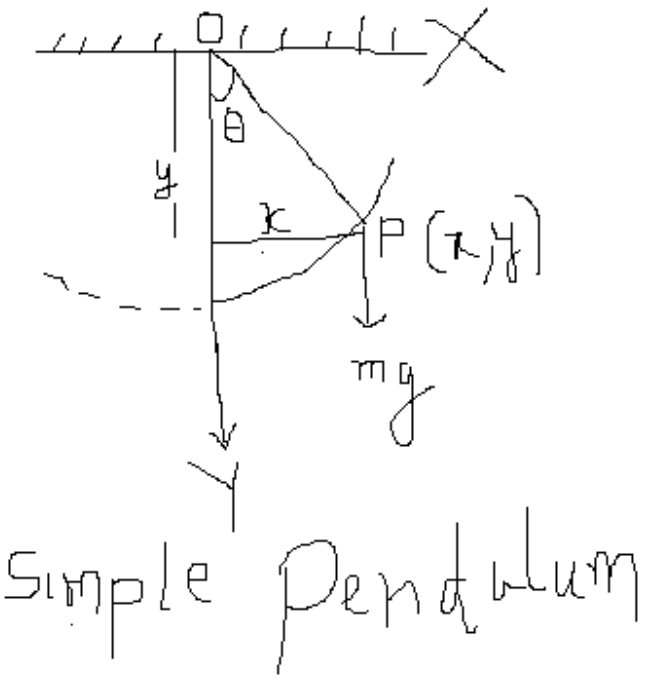
Topic- Equation of motion for a simple pendulum

Equation of motion for a simple pendulum of length l and mass of the bob 'm' by using Lagrange's equation

Solution:

Let θ be the angular displacement from the vertical OY at any time t. This is shown below



The dynamical system consists of a single particle of mass m and generalised co-ordinate

We have Lagrange's equation of motion

$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta}\right) - \frac{\partial L}{\partial \theta} = 0$.....(1)

Here

$T = \frac{1}{2} ml^2 \dot{\theta}^2$ and $V = -mgLcos\theta$

$L = T - V$

$= \frac{1}{2} ml^2 \dot{\theta}^2 + mgLcos\theta$

Substituting the value of L in equation (1) ,we obtain

$\frac{d}{dt}\left(ml^2 \dot{\theta}^2\ \right) + mgLcos\theta\ = 0$

$ml^2 \dot{\theta}^2 + mgLcos\theta\ = 0$

$\ddot{\theta} + \frac{g}{l} sin\theta = 0$.....(ii)

This is the required equation of motion.