M.S c Mathematics –SEM 3 Rigid Dynamics

CC-13 Unit 1

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Generalised Co-ordinates :

The minimum number of co-ordinates required to describe the configuration of the dynamical system at any given time is called the generalised co-ordinates of the system.

Following are the examples

(i) A dynamical system be a simple pendulum of length 1 ; the corresponding generalised co-ordinate is θ ,the angular displacement from the vertical.

(ii) A particle on the surface of a sphere; generalised co-ordinates are $\theta, \emptyset$, where $\theta, \emptyset$ are the polar-co-ordinates on the surface.

Degree of freedom

The number of generalised co-ordinates required to describe the configuration of a system is called the degree of freedom.

Holonomic and Non-Holonomic dynamical system

A dynamical system is called holonomic if it is possible to give arbitrary and independent variations to the generalised co-ordinates of the system without violating constraints , otherwise it is non- holonomic .

Example :

Let $q_1{'}q_2{'}$ ....q_n be n- generalised co-ordinates of a dynamical system . Then , for a holonomic system ,we can change q_r to $q_r + \partial q_r$ without making any change in the remaining $(n - 1)$ co-ordinates.

For a dynamical system, prove that $T + V = Constant$

Lagrange's Equation of motion for holonomic conservative dynamical system is

$$\frac{d}{dt}\left(\frac{\partial T}{\dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = -\frac{\partial V}{\partial q_r}; r = 1, 2, \dots \dots n$$

$$\textit{Multiplying both side by } \dot{q}_r, \textit{we get}$$

$$\sum_{r=1}^{n} \dot{q}_r \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \sum_{r=1}^{n} \dot{q}_r \frac{\partial T}{\partial q_r}$$

$$= -\sum_{r=1}^{n} \dot{q}_r \frac{\partial V}{\partial q_r}$$

$$or \ \sum_{r=1}^{n} \frac{d}{dt}\left(\dot{q}_r \frac{\partial T}{\partial q_r}\right)$$

$$-\sum_{r=1}^{n}\left(\ddot{q}_r \frac{\partial T}{\partial \dot{q}_r} + \dot{q}_r \frac{\partial T}{\partial q_r}\right)$$

$$= -\sum_{r=1}^{n} \dot{q}_r \frac{\partial V}{\partial q_r} \ldots \ldots \ldots (1)$$

$$\textit{But Tis a homogeneous quadratic function in } \dot{q}_1, \dot{q}_2, \ldots \ldots \ldots \ldots \dot{q}_n \textit{ then}$$

$$\sum_{r=1}^{n} \dot{q}_r \frac{\partial V}{\partial q_r}$$

$$= 2T \ldots \ldots \ldots \ldots (2)[\textit{using Euler's theorem}]$$

$$\textit{Also}$$

$$T = T(q_1, q_2 \ldots \ldots q_n, \dot{q}_1, \dot{q}_1, \dot{q}_1 \ldots \ldots \dot{q}_1$$

$$\frac{dT}{dt} = \sum_{r=1}^{n} \frac{\partial r}{\partial q_r}\dot{q}_r + \sum_{r=1}^{n} \frac{\partial T}{\partial \dot{q}_r}\ddot{q}_r \ldots\ldots(3)$$

Also we have

$$V = V(q_{1,}q_2 \ldots \ldots . q_n) \; \textit{then}$$

$$\frac{dV}{dt} = \sum_{r=1}^{n} \frac{\partial V}{\partial q_r}\dot{q}_r \ldots \ldots \ldots (4)$$

$\textit{using } (2), (3), (4), \textit{in equation } (1), \textit{we get}$

$$\frac{d}{dT}(2T) - \frac{dT}{dt} = -\frac{dV}{dt}$$

$$2\frac{dT}{dt} - \frac{dT}{dt} = -\frac{dV}{dt}$$

$$\frac{dT}{dt} = -\frac{dV}{dt}$$

$$\frac{d}{dt}(T + V) = 0$$

$$\Rightarrow T + V = \textit{Constant}$$

Proved