Lagrangian and Eulerian flow (1)

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Fluid comprises of individual molecules and its physical properties like density and viscosity are extremely non-uniform from a microscopic point of view. However in a microscopic scale, the fluid is considered as a continuum and molecule details are not taken into account. The fluid flow is describing in two ways on the basis of considered fluid as point form or rigid system; Lagrangian and Eulerian.

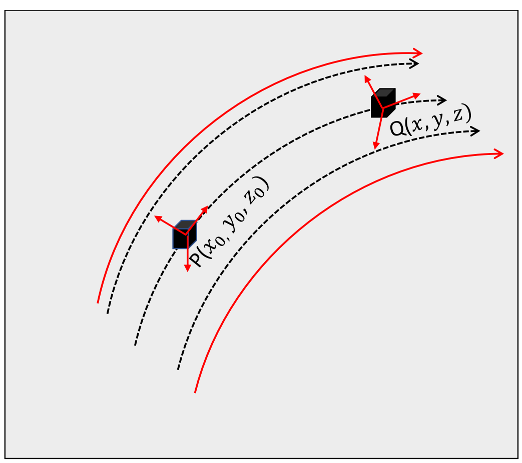

Figure 1: Schematic of the fluid flow particle.

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0.1 Lagrangian and Eulerian approach of flow field

- (i) Lagrangian: Identify any individual fluid particles and study of effect of change of velocity, viscosity, density, pressure, temperature, concentration etc at each point and each instant during the motion of the fluids, Plotting the position of an individual, thorough times gives the path line of the parcel. Let (x_0, y_0, z_0) be the position of chosen individual parcel at a given time $(t = t_0)$at point P. After time $(t = t_1)$, the position of same parcel be $r = (x, y, z)$ at point Q. as shown (Fig.1).

$$x = f_1(x_0, y_0, z_0, t), \quad y = f_2(x_0, y_0, z_0, t), \quad z = f_3(x_0, y_0, z_0, t) \tag{1}$$

Let $\mathbf{q} = (u, v, w)$ and $\mathbf{a} = (a_1, a_2, a_3)$ be the velocity and acceleration of parcel of fluid respectively, Then

$$u = \frac{\partial x}{\partial t} \quad v = \frac{\partial y}{\partial t}, \quad u = \frac{\partial z}{\partial t}$$

and

$$a_1 = \frac{\partial u}{\partial t} \quad a_2 = \frac{\partial v}{\partial t}, \quad a_3 = \frac{\partial w}{\partial t}$$

- (ii) Eulerian method:- In this method select any point fixed in space occupied by the fluid and study the changes which take place in physical properties as the fluid passes through this point. Here point represent fixed volume(continuum) in space. This type phenomena visualized by sitting on the bank of the river watching the water passes the fixed location. By (Fig.1) at selected contnuum at $Q(x, y, z)$ fluid passes as time passes. $\mathbf{q} = (u, v, w)$ be velocity at $Q(x, y, z)$ at time t. then,we

$$u = F_1(x, y, z, t), \quad v = F_2(x, y, z, t), \quad w = F_2(x, y, z, t)$$

where

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}$$

Initial Condition

$$x = x_0, \quad y = y_0, \quad z = z_0 \quad \text{at} \quad t = t_0 = 0$$

$$\int dx = \int F_1 dt \quad \implies x = f_1(x_0, y_0, z_0, t_0)$$

$$\int dy = \int F_2 dt \quad \implies y = f_2(x_0, y_0, z_0, t_0)$$

$$\int dz = \int F_3 dt \quad \implies z = f_3(x_0, y_0, z_0, t_0)$$

If $\psi = \psi(x, y, z, t)$ be any physical quantity in Eulerian fluid motion, then

$$\psi = \psi\left(f_1(x_0, y_0, z_0, t_0, f_2(x_0, y_0, z_0, t_0, f_3(x_0, y_0, z_0, t_0, t)\right)$$

which represent Lagrangian fluid flow.

- **Lagrangian to Eluerian**

Let $h(x_0, y_0, z_0, t)$ be some physical quantity involving Lagrangian description.

$$h = h(x_0, y_0, z_0, t)$$

There exist a function by implicit function theorem

$$x_0 = g_1(x,y,z,t), \quad y_0 = g_2(x,y,z,t), \quad z_0 = g_2(x,y,z,t)$$

then

$$h(x,y,z) = h(g_1(x,y,z,t), g_2(x,y,z,t), g_2(x,y,z,t))$$

Example: The velocity components for fluid flow can be given by $u = x + y + 2t, \quad v = 2y + t$. Find the displacement of fluid particle in the Lagrangian system.

Solution:- Given $u = \frac{dx}{dt} = x + y + 2t$, and $v = \frac{dy}{dt} = 2y + t$

$$y = e^{2t}\left(\int te^{-2t}dt + K_1\right) = e^{2t}k_1 - \frac{1}{4}(2t+1)$$

and

$$\frac{dx}{dt} - x = e^{2t}k_1 + \frac{1}{4}(6t-1) \quad \Longrightarrow \quad x = k_2e^t + e^{2t}k_1 - (6t+5)/4$$

if $x = x_0, \quad y = y_0, \quad z = z_0$, at $t = t_0 = 0$ then

$$\begin{cases} x = (x_0 - y_0 + 1)e^t + (y_0 + 1/4)e^{2t} - (6t+5)/4 \\ y = (y_0 + 1/4)e^{2t} - (2t+1)/4 \end{cases}$$

Assignment

1. The velocities at a point in a fluid in the Eluerian system are given by

$$u = x + y + z + t, \quad v = 2(x+y+z) + t, \quad w = 3(x+y+z) + t$$

Find the displacement of fluid in Lagrangian system?

2. The velocities at a point in a fluid in the Eluerian system are given by

$$u = 2x + 2y + 3t, \quad v = x + y + /2t$$

Find the displacement of fluid in Lagrangian system?

All the best...

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