E-content M.SC Semester-4 Elective Paper – Mathematical Method

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Defination

A Partial differential equation is a relation between the dependent variable Z (say0 and some of its partial derivative at every point (x,y, t,.....)

 $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$

Here Z is dependent variable & x ,y is independent variable

Sometimes this partial derivative also denoted as

$$z_x = \frac{\delta z}{\delta x}$$
, $z_y = \frac{\delta z}{\delta y}$, $z_{xx} = \frac{\delta^2 z}{\delta x^2}$, $z_{xy} = \frac{\delta^2 z}{\delta x \delta y}$, $z_{yy} = \frac{\delta^2 z}{\delta y^2}$

Example (i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ {Laplace Equation}

(ii)
$$\frac{\delta^2 z}{\delta x^2} = \frac{1}{C^2} \frac{\delta^2 z}{\delta t^2}$$
 [Wave equation]

In PDE Dependent Variable =1 , Independent variable ≥ 2

In ODE Dependent Variable ≥ 1 , Independent variable =1

Order of Partial Differential Equation

The order of a partial differential equation is to be the order of the derivative of highest order occurring in the partial Differential Equation.

Degree of Partial Differential Equation

The power of highest order derivative is known as degree of PDE

Example (i)
$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 1$$
 i.e $p^2 + q^2 = 1$

Order =1 Degree=2

Example (ii)) $\frac{\delta^4 z}{\delta x^4} + \frac{\delta^2 z}{\delta y^2} = 1$

Order =4 Degree=1

Classification of First Order Partial Differential Equation

Linear Equation : A first order partial differential equation is said to be linear if it is a linear in p, q, and z, i. e if it is of the form P(x, y) p + Q(x, y) q=R x, y) z + S(x, y)

Example y p - x q = xyz + x & p + q = z + x y

Semi- Linear Equation: A first order partial differential equation is said to be semi-linear equation if it is linear in p and q and the coefficient of P and Q is function of x & y only i.e if it is of the form

P(x, y)p + Q(x, y)q + R(x, y, z)

Example: $e^{x}p - xyq = xz^{2}$ and $yp + xq = \frac{x^{2}z^{2}}{y^{2}}$

Quasi- Linear Equation: A first order partial differential equation is said to be quasi-linear equation if it is linear in p and q and the coefficient of P and Q is function of x, y & z i. e if it is of the form

P(x, y, z)p + Q(x, y, z)q = R(x, y, z)

Example : $(x^{2}+y^{2})p-xyq = xz^{3} + y^{2}$

Non-Linear Equation: Partial differential equation of the form f(x, y, z, p, q) = 0 which do not come under the above three type are said to be non-linear equation.

Example: q = z, $p^2 + q^2 = 1$ are non-linear partial differential equation.

Note : Linear \rightarrow Semi - Linear \rightarrow Quasi - Linear