

E-content M.SC Semester-2 CC-10

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Theorem

Let a be the sum of two relatively prime squares .Then there exists an integer b such that $a \mid b^2 + 1$.

Proof

Let $a = u^2 + v^2$, $u > v$, $(u, v) = 1$ and $\frac{u}{v} = [a_1, \dots, a_n]$. Then form the continued fraction as under

And consider its convergent $\frac{P_n}{q_n} = < a_n, a_{n-1}, \dots, \dots, \dots, \dots, a_1 >$

Therefore $\frac{P_n}{P_{n-1}} = < a_1, \dots, \dots, \dots, \dots, a_n > = \frac{a}{b}$

This implies $P_n = u$ and $P_{n-1} = v$

We know $P_{2n} = P_{n-1}^2 + P_n^2$

Also $q_{2n} = P_{2n-1}$(3)

Now the last two convergent of (1) are connected by the relation

$$P_{2n}q_{2n-1} - P_{2n-1}q_{2n} = (-1)^{2n} = 1$$

It follows from (2) and (3) that

$$a_{q_{2n-1}} - P_{2n-1}^2 = 1$$

Putting $b = P_{2n-1}$ we have

$$a_{q_{2n-1}} = b^2 + 1$$

Thus a divides $b^2 + 1$

