Dr Abhik Singh

Guest Assistant Professor

PG Dept of Mathematics

e- Content –M. Sc –Semester –I

Modern Algebra–CC-1

Binary Composition or Binary Relation

Let S be a non-empty set . A binary relation * on S is a mapping $*: S \times S \to S$ defined as $*(a, b) = a * b$

Whether a composition is a binary composition on a set or not ,it depends on the set.

For example : usual addition '+' is binary composition for the set of rationals but not a binary composition for the set of irrational numbers as the sum of two irrational numbers may not be irrational.

Algebraic structure

A set G equipped with one or more binary operation is called algebraic structure .Let G be a non-empty set and $'*', '0'$ etc are binary operation on G then $(G, *, 0)$ etc is called algebraic structure.

Groupoid or Quasi Group

Let G be any non-empty set and * is a binary operation than the structure (G, *) is called a groupoid or Quasi –Group.

If the binary operation * in the set G satisfies the commutative property.

$$a * b = b * a \quad \forall\ a, b \in G$$

Then, the structure (G,*) is said to be a commutative grouped.

Remark

A groupoid with identity element is called loop

Example

(i) The structure (N , +) , (N,.), (Z, +) ,(Z , .) ,(Q , +), (Q , .) , (R , +) , (R , .) , (C , +) ,(C, .) are all commutative group.

(ii) The set N is not a groupoid with respect to the operation '-'.

Since if $a, b \in N$ then a-b is not always a natural number

Semi-Group

A set equipped with associative binary operation is said to be semi-group or If G be set and * be an operation on G s.t $\forall$ a, b $\in$ G , $a * b \in G$ -closure.

Or

Let G be a non-empty set and * be a binary operation defined on it, then the structure (G, *) is said to be a semi group if $a * (b * c) = (a * b) * c \quad \forall\, a, b, c \in G$ (Associative)

Remarks

A semi –group is an associative groupoid .

If the operation * also satisfies the commutative property , the structure (G,*) is called a commutative semi-group.

(i) The structure (N , +) , (N,.), (Z, +) ,(Z , .) ,(Q , +), (Q , .) , (R , +) , (R , .) , (C , +) ,(C, .) are commutative semi-group.

(ii) The structure (Z,-) ,(Q,-),(R,-), (C, -) are not semi-groups.Since associative law is not satisfied.

Monoid

Let G be a non-empty and * be a binary operation defined on it. Then the structure (G ,$*$) is said to be a monoid if the following axioms are satisfied.

(i) Associative law

$$a * (b * c) = (a * b) * c \quad \forall\, a, b, c \in G$$

(ii) Existence of Identity : There exist an element, denoted by e and called the identity, in G such that $a * e = a = e * a \quad \forall\ a \in G.$

Remarks

(i) A semi-group having the identity element is called a monoid.

(ii) If the operation * is also commutative ,then the structure (G,*) is called a commutative monoid.

(iii) Every monoid has a unique identity element.

Example (a)

The structure (Z, +) ,(Z , .) ,(Q , +), (Q , .) , (R , +) , (R , .) , (C , +) ,(C, .) are monoids.

(b) The structure (N,+) is a semi-group ,but not a monoid, since the additive identity (i.e 0) does not exists in N.

Groups

Let G be a non-empty set and $' * '$ be a binary operation defined on it ,then the structure (G,$*$) is said to be a group if the following axioms are satisfied

(i) Closure property

$\boldsymbol{a * b \in G \ \forall a, b \in G}$

(ii) Associativity

The operation $*$ is associative on G i.e

$$\boldsymbol{a * (b * c) = (a * b) * c \quad \forall\, a, b, c \in G}$$

(iii) Existence of Identity

There exists an element $\boldsymbol{e \in G}$ such that $\boldsymbol{a * e = a = e * a \ \forall \ a \in G}$, e is called identity of $*$ in G.

(iv) Existence of Inverse

For each element $\boldsymbol{a \in G}$ there exist an element $\boldsymbol{b \in G}$ such that the element b is called the inverse of alement a with respect to $*$, and we write $\boldsymbol{b = a^{-1}}$