INVENTORY CONTROL-II
(M.Sc. Sem-III)

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PROBLEMS OF *EOQ* WITH PRICE BREAKS

In the real world, it is not always true that the unit cost of an item is independent of the quantity procured. Often, *discounts* are offered for the purchase of large quantities. These discounts take the form of *price breaks*.

Let us now consider a manufacturer who is encountered with a problem of determining in optimum production quantity for each production run and an optimal interval between successive rules. The following conditions are assumed to hold : (i) Demand is known and uniform, (ii) Shortages are not allowed, and (iii) Production for supply commodities is instantaneous.

Let Q be the lot size in each production run, D the total number of units produced or supplied over the time period, C_0 the cost per production run, K_1 the cost of manufacturing (or purchasing) per unit and I the monthly holding cost per unit.

This model when represented graphically for any one value of the unit purchase cost K_1, has the same shape as in figure. Evidently we have

$$1 = n \quad \text{and} \quad D = nQ$$

Now n is the total number runs during the entire time period.

The total amount cost for a discount level is given by

$$TC = [nQk_1 + nC_0] + \frac{1}{2} Q C_1$$

$$= \frac{D}{Q}(Qk_1 + C_0) + \frac{1}{2} Q (k_1 \times I)$$

$$= Dk_1 + \frac{D}{Q} C_0 + \frac{1}{2} Q k_1 I$$

For the optimum order quantity Q^0 that gives the minimum total annual

$$\frac{d}{dQ}\left[rC(Q)\right] = 0 \quad \text{and} \quad \frac{d^2}{dQ^2}\left[TC(Q)\right] > 0$$

$$Q^{\circ} = \sqrt{\frac{2\,D\,C_0}{k_1 1}}$$

Using this value of Q° in TC, we get

$$TC\,(Q^{\circ}) = D\,k_1 + \sqrt{2\,D\,C_0 k_1 I}$$

1. Problem of EOQ with One Price Break

There is only one price break (one quantity discount), the situation may be illustrated as shows :

Range of quantity	Purchase cost per unit
$0 \le Q_1 < b_1$	K_{11}
$b_1 \le Q_2 \le b_2$	K_{12}

Where b is that quantity at and beyond which the quantity discount applies and $K_{12} < K_{11}$.

The procedure for obtaining *EOQ* may be summarized in the following steps :

Step-1 : Compute Q_2°, i.e., optimum order quantity for the lowest price (highest discount), i.e., Q_2 and compare it with the quantity b_1.

Step-2 : If $Q_2^{\circ} > b_1$, then optimum order quantity will be Q_2°, i.e., $Q^{\circ} = Q_2^{\circ}$.

Step-3 : If $Q_2^{\circ} < b_1$, we cannot place order at the reduced price K_{12}. Therefore, in order to obtain the optimum order quantity, we need only to compare the total inventory cost for $Q = Q_1^{\circ}$ with $Q = b_1$.

The values of $TC\,(Q_1^{\circ})$ and $TC\,(b_1)$ may be determinede as follows :

$$TC\,(Q_1^{\circ}) = D\,K_{11} + \frac{D}{Q_1^{\circ}} C_0 + \frac{1}{2} Q_1^{\circ} \times K_{11} \times I$$

$$TC\,(b_1) = D\,K_{12} + \frac{D}{b} C_0 + \frac{1}{2} b_1 \times K_{12} \times I$$

If $TC\,(Q_1^{\circ}) > TC\,(b_1)$, then $Q^{\circ} = b_1$, otherwise $Q^{\circ} = Q_1^{\circ}$.

SAMPLE PROBLEMS

1. Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	*Unit cost (Rs.)*
$0 \le Q_1 < 500$	10.00
$500 \le Q_2$	9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and amount of ordering is Rs. 350.

Solution : We are given

$$C_0 = Rs.\ 350,\ D = 200 \text{ units per month, } I = \text{Re. } 0.2,\ K_{11} = Rs.\ 10 \text{ and } K_{12} = Rs.\ 9.25.$$

Step-1 : The highest discount available is Rs. 9.25. So, we compute Q_2^0 by taking consideration K_{12} as Rs. 9.25.

$$\therefore \qquad Q_2^0 = \sqrt{\frac{2 \times 350 \times 200}{9.25 \times 0.02}} \simeq 870 \text{ units.}$$

Now, $Q_2^0 = 870$ and b = 500 indicate that $Q_2^0 > b$, i.e., Q_2^0 is within the range of $Q_2^0 \ge 500$

Therefore, the optimum purchase quantity is given by

$$Q^0 \equiv Q_2^0 = 870 \text{ units.}$$

2. Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	*Unit cost (Rs.)*
$0 \le Q_1 < 800$	Re. 1.00
$800 \le Q_2$	Re. 0.98

The yearly demand for the product is 1,600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year.

Solution : We are given

D = 1,600 units per year, $C_0 = Rs.\ 5,\ I = \text{Re. } 0.10,\ K_{11} = \text{Re. } 1.00$ and $K_{12} = \text{Re. } 0.98$.

Step-1 : The highest discount avilable is $K_{12}\,(=\text{Re. } 0.98)$. So we compute Q_2° at K_{12}.

$$\therefore \qquad Q_2^\circ = \sqrt{\frac{2\times 5\times 1,600}{(0.10)\times(0.98)}} = 404 \text{ units.}$$

Now, $Q_2^\circ = 404$ and b = 800 indicate that $Q_2^\circ < b$.

Step-2 : Considering now $K_{11} = \text{Re. } 1.00;$ the optimum order quantity Q_1° is obtained as follows :

$$Q_1^\circ = \sqrt{\frac{2\times 5\times 1,600}{(0.10)\times(1.00)}} = 400 \text{ units.}$$

Since $Q_1^\circ = 400$ and b = 800 indicate that $Q_1^\circ < b,$ we compare the optimum cost of procuring the least quantity which will entitle us a price break (in this case, b = 800). Now,

$$TC\,(Q_1^\circ) = 1,600\times 1 + \frac{1,600}{400}\times 5 + \frac{1}{2}\times 400\times 1\times 0.10 = Rs.\ 1,640$$

$$TC\,(b) = 1,600\times 0.98 + \frac{1,600}{800}\times 5 + \frac{1}{2}\times 800\times 0.98\times 0.10 = Rs.\ 1,617.20$$

Since, $TC\,(Q_1^\circ) > TC\,(b);$ the optimum purchasde quantity is given by

$$Q^\circ = b = 800 \text{ units.}$$

3. A company uses 8,000 units of a product as raw material, costing Rs. 10 per unit. The administrative cost per purchase is Rs. 40. The holding costs are 28% of the average inventory. The company is following an optimal purchase policy and places orders according to *EOQ*. It has been offered a quantity discount of one per cent if it purchases its entire requirement only four times a year.

Should the company accept the offer of quantity discount of one per cent? If not, what minimum discount should the company demand?

Solution : We are given

D = 8,000 units, $C_0 =$ Rs. 40 per order, $C_1 =$ 28% of Rs. 10 = Rs. 2.80 per unit per year.

$$\therefore \qquad Q^\circ = \sqrt{\frac{2\times 40\times 8,000}{2.80}} = 478 \text{ units (approx.)}$$

Total cost associated with Q^o is

$$TC = D\times C + \frac{D\,C_0}{Q^\circ} + \frac{1}{2}Q^\circ\times C_1$$

$$= 8,000 \times 10 + \frac{8,000}{478} \times 40 + \frac{1}{2} \times 478 \times 0.28 \times 10$$

$$= 80,000 + 1,338.66 = Rs.\ 81,338.66.$$

When 1% discount is offered, the unit price will be 99% of Rs. 10, i.e., Rs. 9.90. In this case,

$$TC = 8,000 \times 9.90 + \frac{8,000}{2,000} \times 40 + \frac{2,000}{2} \times 0.28 \times 9.90$$

Since the order quantity = 2,000 units, and $C_1 = \text{Re}.\ 0.28 \times 9.90.$

Thus, $TC = 79,200 + 2,932 = Rs.82,132$

Since, the total cost comes out to be higher when one per cent discount is offered, the company should not accept the offer.

Now, let *x* be the percentage minimum discount acceptable to the company. Then it can be determined by setting the total cost equal to the total cost associatede with the policy of ordering *EOQ.*

Accordingly,

$$8,000 \times \left(\frac{100 - x}{100}\right) \times 10 + \frac{8,000}{2,000} \times 40 + \frac{1}{2} \times 2,000 \times 0.28 \times \left(\frac{100 - x}{100}\right) = \text{R}\ s.\ 81,338.66\ .$$

Or $$\left(\frac{100 - x}{100}\right) \times 82,800 = 81,338.66 - 160 = 81,178.66$$

Or $$100 - x = 100 \times \frac{81,178.66}{82,800} = 98.04$$

This gives *x* = 100 – 98.04 = 1.96 or 2 (approx.)

Hence, the minimum discount acceptable to the company is 2%.

PROBLEMS

4. The annual demand for an item of inventory whose price is Rs. 10 per unit is 2,400 units, ordering cost per order is Rs. 350 and inventory holding costs are 2% per month. The supplier offers a quantity discount of 75%, if the quantity ordered is 400 units or more. The rate of discount will increase to 12.5%, if the order is for 3,000 units or more. Find out the economic order quantity.

5. A factory needs 36,000 units (annually) of a component that cost Rs. 2 per unit. Cost of each order placing is Rs. 25 and inventory carrying cost is Rs. 10 per year.

(i) Find economic lot size and the total inventory cost.

(ii) What is the time between placing of orders?

(iii) The supplier offers 2% discount if a single order is placed. Should the company accept it?

6. For one of the boughtout items, the following are the relevant data :

Ordering cost = Rs. 500, Holding cost = 40%, Cost per item = Rs. 100, Annual demand = 1,000. The purchase manager placed five orders of equal quantity in one year, in order to avail the discount of 5% on the cost of items. Work out the gain or loss to the organization due to his ordering policy for this item.

7. A Purchase Manger has decided to place order for a minimum quantity of 500 numvbers of a particular item in order to get a discount of 10%. From the past records, it was found out that in the last year, 8 orders each of size 200 units were placed. Given the ordering cost = Rs. 500 per order, inventory carrying cost of 40% of the memory value and the price of the item of Rs. 400 per unit. Is the Purchase Manager justified in his decision? What is the effect of his decision to the company?