E - content – Pro(Dr) L N RAI

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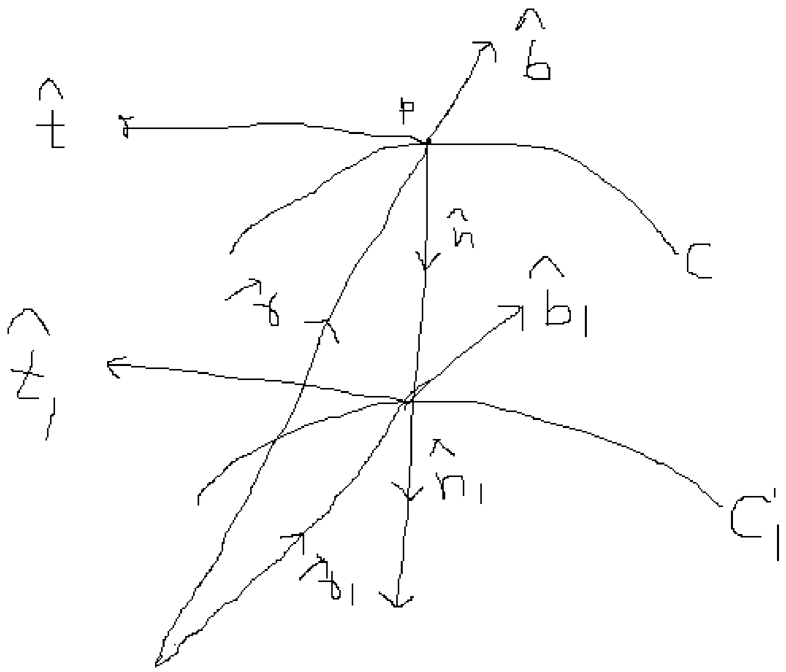
Topic- Differential Geometry

Define Betrand Curves and show that the distance between corresponding points of two curves is constant

Solution: Two curves C_0 and C_1 having their principal normals in common are called Betrand curves or conjugate curve



The distance between corespondingpoints of two curves is constant.

Proof

We take their principle normals in the same sense, so that $\hat{n}_1 = \hat{n}$ ....(1)

Let $\overrightarrow{r}$ be the position vector of a point on a curve C, the position vector $\vec{r}_1$ of a corresponding point P_1 on the associate Betrand curve C_1 and C is given by

$\vec{r}_1 = \vec{r} + \lambda\hat{n}$……………………(2)

where λ is a quantity which is function of 'S' and denotes the distance between two corresponding points of two curves

Differentiating (2) w.r.t 's' ,we get

$$\frac{d\vec{r}_1}{ds_1}.\frac{ds_1}{ds} = \hat{t} + \lambda'\hat{n} + \lambda(\boldsymbol{\zeta}\hat{b} - k\hat{t})$$

$$\hat{t}_1\frac{ds_1}{d} = (1 - \lambda k)\hat{t} + \lambda'\hat{n} + \lambda\boldsymbol{\zeta}\hat{b}\text{……………………}(3)$$

Taking dot product with (1) and (3) ,we get

λ'=0

⇒λ= constant