Hydrogeology

Principles of Groundwater Flow

Lecture 3

1

Hydrostatic pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad [ML^{-1}T^{-2}]$$

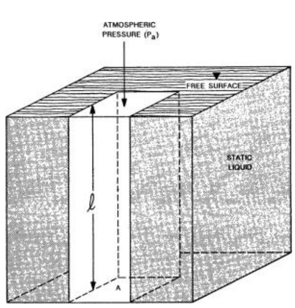
$$\text{Hydrostatic pressure} = \frac{\text{Weight of fluid column}}{\text{Area}}$$

$$\text{By definition } \gamma = \rho g \quad \frac{\text{mass}}{\text{volume}} \cdot g = \frac{\text{weight}}{\text{volume}}$$

where γ is weight density or specific weight $\left(\frac{\text{weight}}{\text{volume}}\right)$

ρ is mass density $\left(\frac{\text{mass}}{\text{volume}}\right)$, and

g is acceleration due to gravity.



The total force acting at the bottom of the prism with area A is

$$P_t A = P_a A + \gamma_f \ell A$$

Dividing both sides by the area, A ,of the prism

$$P_t = P_a + \gamma_f \ell .$$

By convention, in hydraulics and fluid mechanics we generally do not work with total pressure, but with "gage" pressure--that is, the pressure exerted by the static liquid alone. Atmospheric pressure is regarded as an environmental constant that need not be taken into account explicitly. From (2) the pressure exerted by the static column of liquid P_f is

$$P_f = \gamma_f \ell .$$

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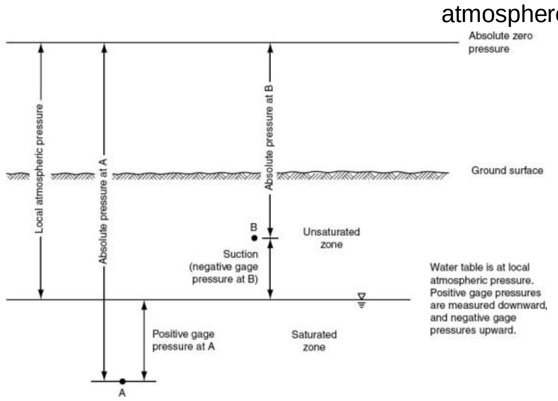
Hydrostatic Pressure

Absolute pressure = Local atmospheric pressure + Gage pressure

Top of the atmosphere



Thus a positive suction corresponds to a negative gage pressure. The dimensions of pressure are F/L^2, that is Newton per square meter or pascal (Pa), kiloNewton per square meter or kilopascal (kPa) in SI units

Point A is in the saturated zone and the gage pressure is positive. Point B is in the unsaturated zone and the gage pressure is negative. This negative pressure is referred to as a *suction or tension.*

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Hydraulic Head

$$P_f = \gamma_f l.$$

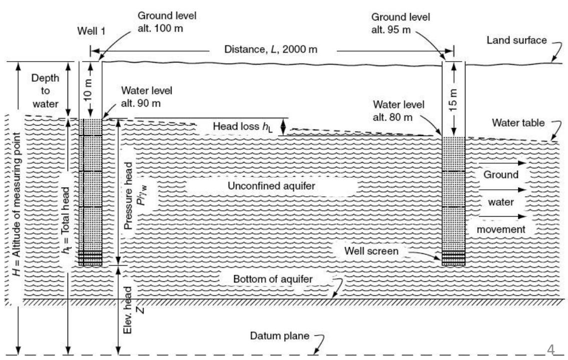
the length of the fluid column l may be expressed as

$$l = \frac{P_f}{\gamma_f}.$$

The law of hydrostatics states that pressure p can be expressed in terms of height of liquid h measured from the water table (assuming that groundwater is at rest or moving horizontally). This height is called the pressure head: For point A the quantity h is positive whereas it is negative for point B.

$$h = P_f/Y_f = P_f/\rho g$$

If the medium is saturated, pore pressure, p, can be measured by the pressure head, $h = p_f/\gamma_f$ in a piezometer, a nonflowing well. The difference between the altitude of the well, H, and the depth to the water inside the well is the total head, h_i, at the well.



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Energy in Groundwater

- Groundwater possess mechanical energy in the form of kinetic energy, gravitational potential energy and energy of fluid pressure.
- Because the amount of energy vary from place-to-place, groundwater is forced to move from one region to another in order to neutralize the energy differences.
- Kinetic energy: energy due to its motion

$$E_k = \frac{mv^2}{2}$$

- v is velocity in m/sec; m is mass in kg
- E_k is energy in kg.m^2/sec^2 which is N.m and this is 1 Joule (also the unit of work W = F.d)

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- Potential Energy: Energy acquired by the work is done when a mass of water is elevated to a height (z) above a datum. This is energy is due to the position of the fluid mass with respect to the datum.

$$E_g = mgz$$

- Fluid mass has another source of potential energy due to the pressure of the surrounding fluid acting upon it.

$$P = \frac{F}{A}$$

- Unit of pressure is N/m^2 which can be rewritten as N.m/m^3 or joule/m^3
- Hence pressure can be considered as potential energy per unit volume of fluid.
- For a unit volume of fluid mass m is numerically equal to its density ρ.

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- The total energy of a unit volume of fluid is the sum of the three energy components

$$E_{tv} = \frac{\rho v^2}{2} + \rho g z + P$$

- E_{tv} is the total energy per unit volume
- The total energy can be calculated for unit mass by dividing by ρ.

$$E_{tm} = \frac{v^2}{2} + gz + \frac{P}{\rho}$$

- This is known as Bernoulli Equation
- For a steady flow of a frictionless, incompressible fluid along a smooth line of flow, the total energy per unit mass is constant.

$$E_{tm} = \frac{v^2}{2} + gz + \frac{P}{\rho} = constant$$

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- The energy per unit weight for steady flow is

$$\frac{v^2}{2g} + z + \frac{P}{\rho g} = constant$$

- This equation expresses all the energy terms in joules/newton or m thus a unit a length.
- This total energy is the **total mechanical energy per unit weight** of groundwater and is also known as the **hydraulic head (h)** and the advantage is that it is measured in the field or in a lab in units of length.
- Piezometer is used to measure the total energy of a fluid flowing a through point which is located at an elevation z, with a velocity v, and is acted upon by a fluid pressure, P.

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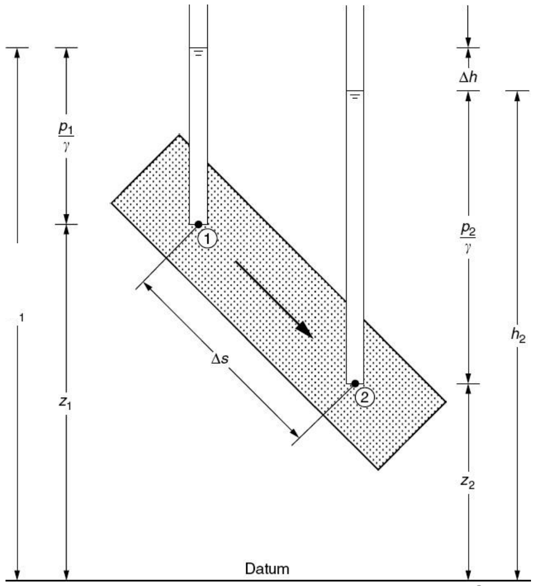
Bernoulli Equation States:

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} = h$$

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + \Delta h$$

$$\Delta h = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_1 - h_2$$

$$h = \frac{P}{\gamma} + z$$



- The amount of energy developed as kinetic energy by flowing groundwater is very small compared to the other two.
- Hence by dropping the velocity term the hydraulic head(h) becomes $z + \frac{P}{\rho g} = h$
- For a fluid at rest, the pressure at a point is equal to the weight of the overlying water per unit cross-sectional area hence

$$P = \rho g h_p$$

$$h = z + h_p$$

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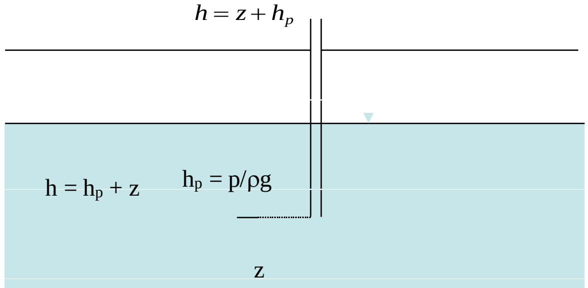

Fluid Potential

- Hence the total head is equal to the sum of the elevation head and the pressure head
- **Potential ---** A physical quantity capable of measurement at every point in a flow system, whose properties are such that flow always occurs from regions in which the quantity has less higher values to those in which it has lower values regardless of the direction in space.
- E.g: Heat conducts from high temperature to low temperature : Temperature is a potential
- Electricity flows from high voltage to low voltage : Voltage is a potential
- **Fluid potential and hydraulic head:** Fluids flow from high to low fluid potential
- Flow direction is away from location where mechanical energy per unit mass of fluid is high to where it is low.
- How does this relate to measurable quantity?

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- Force potential(Φ): total mechanical energy per unit mass.
- Hence h is also a potential
- Hydraulic head is total mechanical energy per unit weight.

$$\Phi = gz + \frac{P}{\rho}$$

$$\Phi = gz + \frac{\rho g h_p}{\rho}$$

$$\Phi = g(z + h_p)$$

$$\Phi = gh$$

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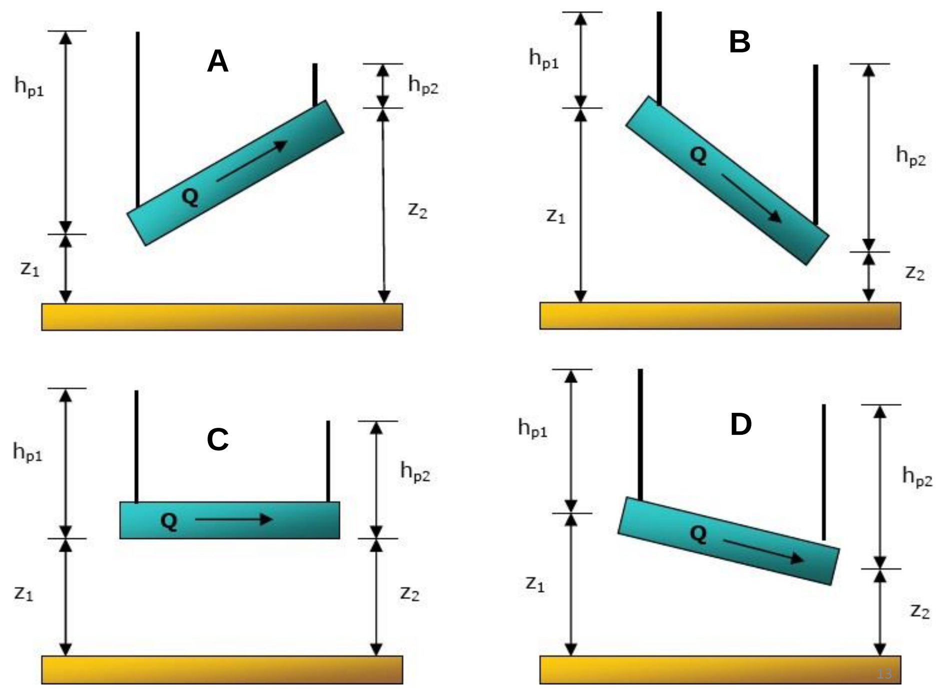
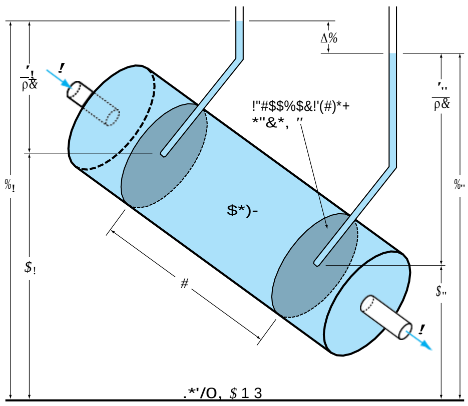


Darcy's Law

- In 1856 in Dijon, France, Henry Darcy conducted his now famous experiment of pouring water through sediment---packed pipes to see how much would flow through them in a given amount of time [volume of flow per unit time]: Flow through porous media.

$$Q = KA\frac{h_1 - h_2}{L}$$



- Flow through column is Q in L^3/T – the most important quantity

$$\frac{Q}{A} = -K\frac{h_2 - h_1}{L} = -K\frac{h_2 - h_1}{l_2 - l_1} \Rightarrow q = -K\frac{dh}{dl}$$

Q/A (q) is specific discharge or Darcian Velocity (unit of L/T)

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Darcy's Law

$$Q = qA = KA\frac{\partial h}{\partial x} = KAi = \frac{k\rho g}{\mu}Ai$$

- q is a conceptual velocity called the specific discharge or flow rate per unit area [LT^{-1}] also known as the Darcy velocity, μ is the dynamic viscosity, and k is the intrinsic permeability. The hydraulic head, h, is the sum of the elevation head z and the pressure head p/γ_w
- The one-dimensional form of Darcy's law is

$$q = -\frac{K\left[\left(\frac{p_1}{\gamma_w} + z_1\right) - \left(\frac{p_2}{\gamma_w} + z_2\right)\right]}{L}$$

- where subscripts 1 and 2 refer to the points at which the pressure heads and the elevation heads are considered, respectively, and L is the distance between these points.

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- When water flows through an open channel the discharge Q is equal to the product of velocity and the cross-sectional area.
- One can use the same reasoning in Darcy's Law for flow through porous medium.

$$Q = qA$$

$$q = -K\frac{dh}{dl}$$

- However, in porous medium the cross-sectional area of flow is much smaller than the aquifer dimension.
- It is equal to the product of area and porosity.

$$V_s = -\frac{K}{n}\frac{dh}{dl}$$

- Where V_s is the average liner velocity of water flowing through the pores.

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Specific Discharge

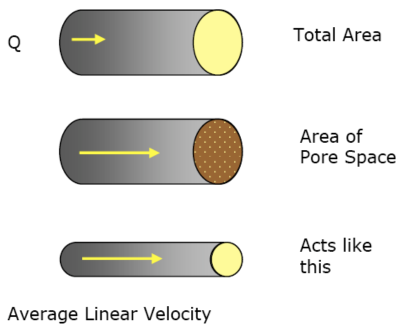
- Remember Darcian Velocity is not an actual velocity; it is discharge per unit area (area is TOTAL cross section)

$$\bar{v} = \frac{q}{\phi}$$

$\bar{v}$ is the average linear pore water velocity. ϕ Is the effective or average porosity

$\bar{v}$ is larger than the Darcian Velocity.

$$q = \phi\bar{v}$$



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- The coefficient K (hydraulic conductivity) has units of L/T or velocity, also termed as coefficient of permeability.

$$\frac{Q}{A\frac{\partial h}{\partial l}} = K$$

- Specific weight (γ) is the force exerted by gravity on a unit volume of the fluid.

$$K = K_i\left(\frac{\gamma}{\mu}\right) = K_i\left(\frac{\rho g}{\mu}\right)$$

- K_i is the intrinsic permeability with a dimensions of (L^2)area
- In some industries darcy is used as a unit of intrinsic permeability.
- 1 darcy = 9.87×10^{-9} cm^2
- While K_i is representative of the properties of porus media alone, K is a function of properties of both the porous medium and the fluid passing through it.

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Hydraulic Conductivity (K)

- K describe the relative ease with which a particular liquid will flow through the medium.
- combines both medium and fluid properties,

$$K = k\rho g/\mu.$$

- **k is the intrinsic permeability (L^2), a property of media only**
- ρ is the mass density (M/L^3)
- µ is the dynamic viscosity (M/LT) and measures the resistance of fluid to shearing that is necessary for flow
- This parameter has the dimension of velocity, generally cm/sec or feet per day
- Will revisit this aver introducing Darcy’s Law.

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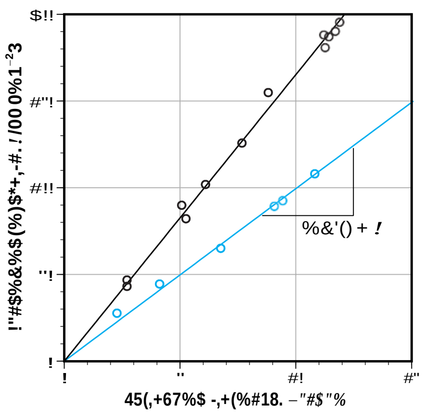
Hydraulic Conductivity

- Hydraulic conductivity (K) is the slope of the line relating specific discharge (q) with hydraulic gradient (dh/dl)
- For a given set of medium and fluid, q can be measured by varying dh/dl and plotted to obtain the slope.
- For a fluid with constant density and viscosity, K will only increase with the coarseness of the material.



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Transmissivity (T)

- The term, *transmissivity* is used to describe the ease with which water moves through a large porous medium body such as a horizontal or layered aquifer .
- Transmissivity, *T (sometimes called transmissibility) is simply the product* of hydraulic conductivity and saturated thickness of the aquifer, *T = Kb, b is the thickness*
- *T* has the dimensions of L^2/T.
- This dimensional characteristic derives from the definition of transmissivity is "the volume of water per unit time passing through a unit width area of aquifer perpendicular to flow integrated over the thickness of the aquifer," or $[L^3/(TL^2)]L$.
- Transmissivity is usually reported in units of square feet per day or square meters per day.

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Anisotropy and Heterogeneity

- If the parameters differ in value directionally at a point, the medium is then said to be *anisotropic or if they* are equal in all directions at any point in the porous medium then it is known to be isotropic.
- These directional properties are described three-dimensionally in cartesian tensor notation. Vectors require 3 components and tensors require 9 components.
- **Isotropy – Having the same value in all directions. K is a scalar.**

⟶ **K** ⟶

$$q_x = K\frac{\partial h}{\partial x} \qquad q_y = K\frac{\partial h}{\partial y} \qquad q_z = K\frac{\partial h}{\partial z}$$

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- The value of hydraulic conductivity is measured in the lab using a permeameter
- There are two leading varieties, constant--head and falling-head.
- Constant---head is generally used for noncohesive materials like sand and rocks.
- Falling-head is used for cohesive sediments with low conductivities and hence much smaller volume of water moves through the materials.
- In a constant---head, a chamber with an overflow provides a supply of water at a constant head. So water moves through the sample at a steady rate.
- And the head should never be more than 0.5 times the length of the sample.
- In using any of these permeameters it is critical that the samples are completely saturated as presence of air bubbles in the sample with reduce the area of flow.
- Also the sample must be tightly packed against the sidewalls of the container to avoid water flowing through the sides.

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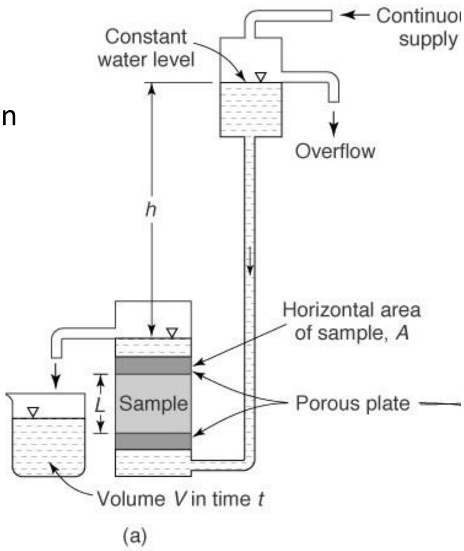
Constant---head Permeameter

- In this case, K is given by
  - V = vol of water discharging in time
  - L = length of the sample
  - A = cross-sectional area
  - h = hydraulic head
  - K = hydraulic conductivity

$$K = \frac{V}{A}\frac{L}{th}$$



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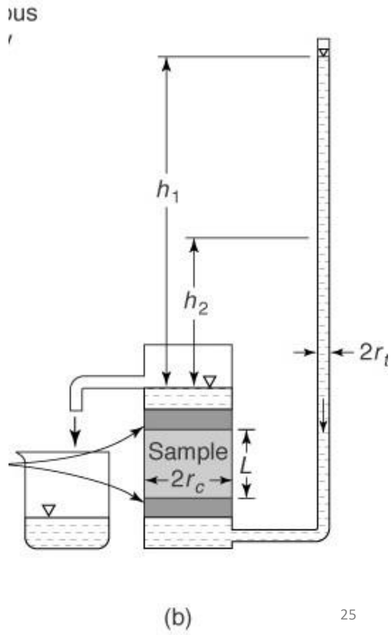
Falling-head Permeameter

- Here a falling head tube is attached to the permeameter
  - h_0 = initial water level above the outlet in the falling head tube
  - h = water level measure aver some time t.
  - d_t= inside diameter of the falling head tube
  - d_c = diameter of the sample
  - L = length of the sample

$$K = \frac{d_t^2 L}{d_c^2\, t} \ln\left(\frac{h_0}{h}\right)$$



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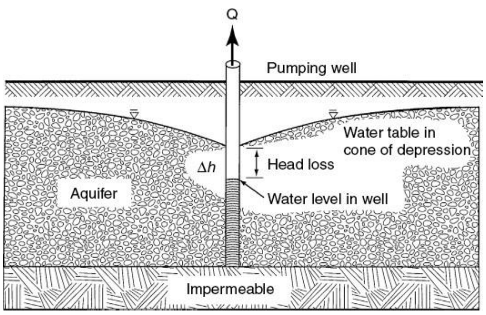
Head loss

- Groundwater flow is not laminar throughout.
- Flow is turbulent near the discharge point (well or spring)- for maintaining a constant discharge rate.
- The higher energy required for turbulent flow is obtained by a greater head gradient---**head loss or well loss**. To accomplish this, the water level in the well will have to drop below the level required for laminar flow
- Well loss means extra energy is needed which reduces the efficiency of the pumping system and drives up the cost of pumping water



Groundwater may flow for some distance over a wide area in the laminar state, but as it approaches a discharge point (e.g., a spring or well) much narrower than its upgradient flow field, the flow velocity will increase to maintain the same volumetric discharge rate

In addition, because a greater pressure drop occurs with head loss, gasses will be more likely to come out of solution and cause precipitation of lime (CaCO3) and oxides (Fe_2O_3 and MnO_2) onto the well screen which causes clogging of the screen and further reduces the efficiency of the well

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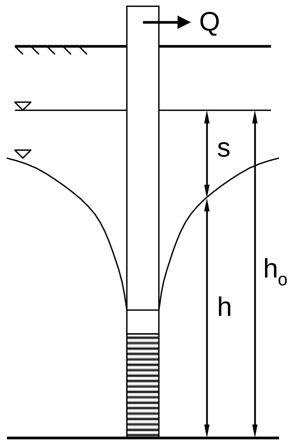
Pumping Well Terminology



- **Static Water Level** [SWL] (h_o) is the equilibrium water level before pumping commences
- **Pumping Water Level** [PWL] (h) is the water level during pumping
- **Drawdown** ($s = h_o$ --- h) is the difference between SWL and PWL
- **Well Yield** (Q) is the volume of water pumped per unit time
- **Specific Capacity** (Q/s) is the yield per unit drawdown

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Pumping Well Terminology

- The drawdown observed in a pumping well has two component parts:
  - **aquifer loss**
    - drawdown due to laminar flow in the aquifer
  - **well loss**
    - drawdown due to turbulent flow in the immediate vicinity of the well through the screen and/or gravel pack

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Darcy's Law shows that

- Q is in direction of decreasing head
- The proportionality constant is K (hydraulic conductivity),
- flow is from higher to lower hydraulic head (negative hydraulic gradient)
- Q is a flow per unit cross section and is **not** the actual velocity of groundwater flow.
- Darcy's law is a macroscopic law. It doesn't tell you about the flow through individual pores.

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Limitations of Darcy's Law

- Darcy's Law is assumed to be effective in porous media flow and the flow is assumed to be laminar, that is, the Reynold's number (R_e) for underground water movement ranges between 1 to 10.
- $R_e = (\rho v d)/\mu$
- where ρ is water density, v is average pore velocity, d is average pore diameter, and μ is dynamic viscosity of water at a given temperature.
- the pore size in a medium can range over even a few orders of magnitude in very heterogeneous media---it is unlikely that laminar flow occurs in all pores at the same time

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