Exactly Soluble System (II)
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LANGUERRE POLYNOMIAL

Leguerre Polynomials (1834-1886) which is Second-order linear differential equation. This equation has nonsingular solutions only if n is a non-negative integer. The Laguerre Polynomials arise in quantum mechanics, in the radial part of the solution of the Schrodinger equation for a one-electron atom. They also describe the static wigner functions of oscillator system in quantum mechanics in phase space, In the quantum mechanics the schrodinger equation for the hydrogen-like atom is exactly solvable by separation of variables in spherical Co-ordinate. The radial part of the wave fuction is a Laguerre Polynomial.

A useful set of Polynomials, the Lagurre functions, is given by the solution to the Laguerre equation

$$x\frac{d^2L_\alpha}{dx^2} + (1-x)\frac{dL_\alpha}{dx} + \alpha L_\alpha = 0$$

and for $\alpha = n$, the associated Lagurre Polynomials,

$$L_n^K(x) = (-1)^K \frac{d^K}{dx^K} L_{n+K}(x)$$

$$x\frac{d^2L_n^K}{dx^2} + (K+1-x)\frac{dL_n^K}{dx} + \alpha L_n^K = 0$$

Associated Laguerre equation by differentiating the Laguerre equation K times. For the Laguerre equation, we assume a solution of the form —

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$$L_\alpha = \sum_{s=0}^{\infty} a_s x^s$$

then

$$\frac{dL_\alpha}{dx} = \sum_{s=1}^{\infty} s a_s x^{s-1}$$

$$\frac{d^2L_\alpha}{dx^2} = \sum_{s=2}^{\infty} s(s-1) a_s x^{s-2}$$

So that $\sum_{s=2}^{\infty} s(s-1) a_s x^{s-2} + (1-x) \sum_{s=1}^{\infty} s a_s x^{s-1} + \alpha \sum_{s=0}^{\infty} a_s x^s = 0$

$$\sum_{s=2}^{\infty} s(s-1) a_s x^{s-1} + \sum_{s=1}^{\infty} s a_s x^{s-1} - \sum_{s=1}^{\infty} s a_s x^s + \alpha \sum_{s=0} a_s x^s = 0$$

$$\sum_{m=1}^{\infty} m(m+1) a_{m+1} x^m + \sum_{m=0}^{\infty} m+1\, a_{m+1} x^m - \sum_{m=1}^{\infty} m a_m x^m + \alpha \sum_{m=0}^{\infty} a_m x^m = 0$$

The $m=0$ term is

$$a_1 + \alpha a_0 = 0$$

for all $m > 0$

$$(m(m+1) a_{m+1} + (m+1) a_{m+1} - m a_m + \alpha a_m) x^m = 0$$

$$(m+1)^2 a_{m+1} + (\alpha - m) a_m = 0$$

and therefore $a_{m+1} = -\frac{\alpha - m}{(m+1)^2} a_m$

for $m = 0$ this formula also gives $a_1 = -\alpha a_0$ so we may extend this formula to all m. Iterate this ~~f~~ series.

$$a_m = -\frac{\alpha - m + 1}{m^2} a_{m-1}$$

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$$= (-1)^2 \frac{(\alpha - m+1)(\alpha - m+2)}{m^2 (m-1)^2} a_{m-2}$$

$$= (-1)^K \frac{(\alpha - m+1)(\alpha - m+2) \cdots (\alpha - m+K)}{} \alpha_{m-k}$$

So that for $k = m$

$$a_m = (-1)^m \frac{(\alpha - m+1)(\alpha - m+2) \cdots \alpha}{m^2 \cdots 1^2} a_0$$

$$= (-1)^m \frac{\Gamma(\alpha+1)}{\lfloor m \lfloor m \, \Gamma(\alpha - m+1)} a_0$$

where the Γ function satisfies

[Γ Tau]

$$\Gamma(m) = \lfloor(m-1)$$

$$\Gamma(\alpha+1) = \alpha \Gamma(\alpha)$$

The Solution is

$$L_\alpha(x) = \sum_{m=0}^{\alpha} \frac{\Gamma(\alpha+1)}{\lfloor m \lfloor m \, \Gamma(\alpha - m+1)} (-1)^m x^m$$