Exactly Soluble System

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LINEAR HARMONIC OSCILLATOR

The one dimensional motion of a point mass attracted to a fixed centre provides one of the fundamental problems of classical treatment (dynamics). The L.H.O in quantum mechanics is similarly of great importance. It is one of the simplest problems for which discrete energy levels are obtained; moreover, it finds practical application in many physical situations, for instance, the vibrations of individual atoms in molecules and in crystals.

When the particle oscillates about it means position under the action of a force which is directly proportional to the displacement at any instant from it means position, then the oscillating particle is called a harmonic oscillator. When the displacement is expressed in terms of single coordinates, the oscillator is called a linear harmonic oscillator and its motion is known as simple harmonic motion.

classical treatment :-

Let us consider a L.H.O, of mass m, oscillating about its mean position. If the mean position is the original of the coordinate axis, and the displacement from the original along x - axis is x, then restoring force (Fx), according to Hooke's law is

$$Fx = -K \cdot x \quad — (1)$$

where K is force constant. When x is positive, the force, Fx is in the $-x$ - direction and when x is negative, the force is in the $+x$ - direction.

Now according to Newton's second law

$$Fx = \frac{md^2x}{dt^2} \quad — (2)$$

then from equation (1) & (2)

$$m\frac{d^2x}{dt^2} = -kx \text{ or } \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0 \quad — (3)$$

the general Solution of this equation is

$$x = A \sin\left[\left(\frac{k}{m}\right)^{\frac{1}{2}} t + c\right] \quad — (4)$$

where A and c are constants.

Since the maximum and minimum value of the sine function are +1 and −1, the particle will have its maximum and minimum displacement +A and −A, Hence A represents the amplitude of motion. Further, since the motion is periodic in character, the conditions repeat themselves, exactly after each oscillation, i.e

$$(k/m)^{\frac{1}{2}} = 2\pi\nu_0 \text{ or } \nu_0 = \frac{1}{2\pi}(k/m)^{\frac{1}{2}} \quad — (5)$$

where ν_0 is the fundamental frequency of oscillation or vibration of the Particle. The motion of a particle under the influence of the restoring force F_x may be represented as the negative gradient of the Potential energy. In case of L.H.O the force is

$$F_x = -\left(\frac{dV}{dx}\right) = -kx \quad — (6)$$

Integrating, we get $\quad V = \frac{1}{2}kx^2 \quad — (7)$ (P.E)

The total energy E of the oscillator in one-dimensional oscillation then becomes

$$E = T + V = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 \quad — (7)$$

where V_x is velocity of the oscillator along x-direction. Using eq.(4) for x, we get

$$V_x = \frac{dx}{dt} = \left(\frac{k}{m}\right)^{\frac{1}{2}} A \cos\left[(k/m)^{\frac{1}{2}} t + c\right]$$

Equation (7) then becomes

$$E = \frac{1}{2}m(k/m)A^2\cos^2\left[\left(\frac{k}{m}\right)^{\frac{1}{2}} t + c\right] + \frac{1}{2}kA^2\sin^2\left[(k/m)^{\frac{1}{2}} t + c\right]$$

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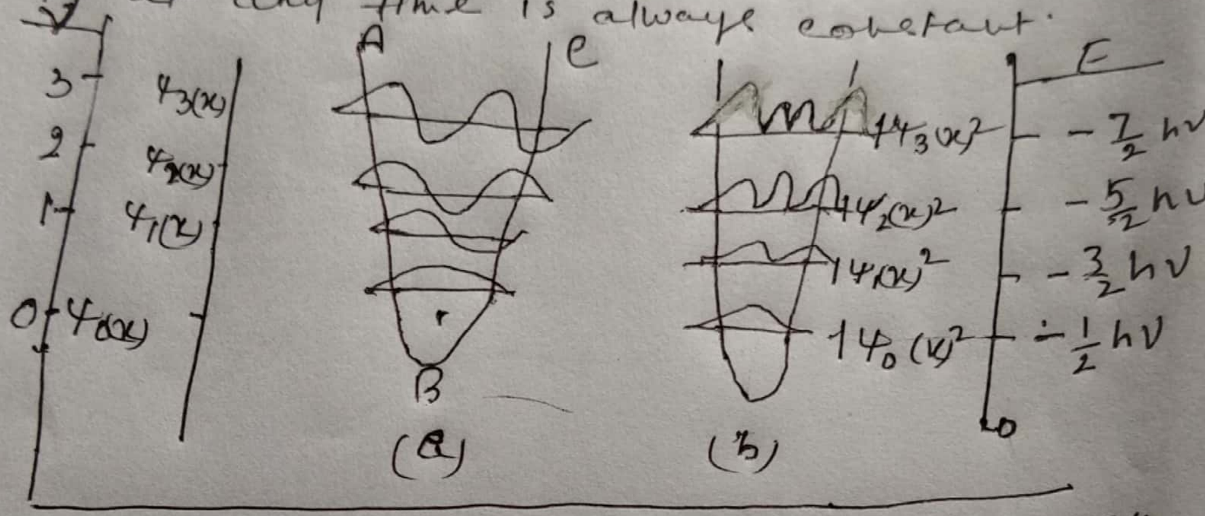
$$= \frac{1}{2} KA^2 \{\cos^2[(\frac{K}{m})^{\frac{1}{2}}t + C] + \sin^2[(\frac{K}{m})^{\frac{1}{2}}t + C]\} = \frac{1}{2}KA^2 \quad (8)$$

It is obvious that all positive values of E are allowed. The plot of potential energy V, versus displacement x, it will initially have only potential energy. As it moves, it gains kinetic energy, which becomes maximum at a position x = 0. The kinetic energy is then reconverted to potential energy, as it passes through to the other side. But the total energy at any time is always constant.



(a) Sharp curve ABC is the potential energy curve for classical harmonic oscillator. Thick curves are the allowed energy levels and wave functions for a quantum mechanical harmonic oscillator. (b) Probability density functions for a quantum mechanical harmonic oscillator.

Quantum Mechanical Treatment

Since it is one particle one-dimentional system its kinetic energy operator $\hat{T}$, as before, is written as:

$$\hat{T} = -\frac{h^2}{8\pi^2 m}\frac{d^2}{dx^2}$$

The classical potential energy contains x^2; it follows from Eq (7) that the operator

Corresponding to the the Potential energy V is the same as its classical value.

$$\hat{V} = \frac{1}{2} kx^2 \quad — (9)$$

The Hamiltonian operator for the system is

$$\hat{H} = \hat{T} + \hat{V} = -\frac{h^2}{8\pi^2 m} \cdot \frac{d^2}{dx^2} + \frac{1}{2} kx^2 \quad — (10)$$

Therefore, The Schrodinger equation becomes

$$-\frac{h^2}{8\pi^2 m} \cdot \frac{d^2\psi}{dx^2} + \frac{1}{2} kx^2 \psi = E\psi \quad (11)$$

This is Just the harmonic oscillator Potential. In general, any system instable equilibrium can be represented near the equilibrium position by means of a harmonic oscillator.
